An Empirical Study in The Relationship between
Crude Oil and Gold Futures

by

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Date: September 3, 2013
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Abstract

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This paper analyzes the relationship between crude oil and gold futures. The data used in this paper are from January 2000 to December 2012. The methodology used in this study includes several statistical tests including GARCH and TGARCH models. The results imply that the prices of crude oil and gold are highly correlated. However, the returns of the two commodities are not obviously correlated. On the other hand, the volatility of crude oil price return has an effect on the volatility of gold price returns.
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Chapter 1: Introduction

1.1 Purpose of Study

Crude oil and gold are hotly debated topics in recent decades. Crude oil is used for the production of heat, fuel and other energy related activities. As the most traded raw material, it plays a significant role in the world economy, and it links with many economic factors, such as inflation, Gross Domestic Product (GDP), business cycles, etc. Its price is the most volatile in the commodity market.

Gold as a leader in the precious metals has been used as money though history and now is used as an investment tool all over the world. Investors buy gold to hedge the increasing risk in financial markets. On the other hand, the gold market with its associated derivative contracts is referred as speculative. It is also related with inflation, interest rate, exchange rate, especially with the U.S. dollar.

As the most representative commodities, oil and gold are important not only in the valuation of derivatives and hedging strategies, but also in broader financial market and the world economy. With their specific features, the prices of oil and gold depend not only on demand and supply but a lot of other factors. Their prices can impact not only the trends of economy and markets, but also each
other. In addition, as the development of future markets, investors use crude oil and gold to diversify their portfolio or to attempt to achieve abnormal return. Therefore, it is important for investors and financial institutions to know the relationship between crude oil and gold. By using several statistical models, this paper will test whether there is a relationship between the pricing of crude oil and gold futures contracts.

1.2 Background of Study

Gold Futures have a standard contract based on the gold price. They are traded on the New York Mercantile Exchange (NYMEX) and Tokyo Commodity Exchange (TOCOM). NYMEX Gold Futures prices are quoted in U.S. dollars and cents per ounce and the contract size is 100 troy ounces. TOCOM Gold Futures contract prices are quoted in yen per gram and the contract size is 1000 grams (32.15 troy ounces). Investors purchase a gold Futures contract when they believe that gold prices will go up. In contrast, they will short the gold Futures when they think that gold prices will fall. The return is the difference between the price when investors buy and sell.

For crude oil Futures, there are three types: the Light Sweet Crude Oil Futures are traded on the NYMEX, also called West Texas Intermediate (WTI), the Brent Crude Oil Futures are traded on the International Petroleum Exchange (IPE) and
the Dubai Crude Oil Futures are traded on the Singapore Exchange (SGX). A crude oil Future is quoted in U.S. dollar and cents per barrel and the contract size is 1,000 barrels. The trading strategy is as same as for gold Futures.

The previous studies mainly focused on the relationship between oil or gold and macroeconomic factors, such as inflation, interest rate, U.S. dollar exchange rate, etc. Daniel (1997), Hamilton (1996) researched the relationship between oil price and several economic activities. Jones and Kaul (1996) and Brailsford and Faff (1999) did some research on the topic of the relationship between oil price and financial markets. Furthermore, some studies confirmed that the crude oil price is a leading indicator of inflation (e.g. Hunt, 2006; Hooker, 2002).

Most previous research has used the traditional time series model to determine the long-term relationship between crude oil and gold. This assumed that the underlying variables were linear and symmetrical. However, Enders and Granger (1998), and Enders and Siklos (2001) found the traditional cointegration tests were not rational enough because of asymmetric adjustment. This paper will use Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH) models to investigate the accuracy of the long-term relationship between crude oil and gold.
1.3 Outline of the Paper

This paper includes five chapters. This current chapter provides an overall introduction of the paper and it outlines the importance of knowing the relationship between crude oil and gold Futures contracts. The second chapter is the literature review, the theories used in this paper and previous studies are introduced in detail. Chapter 3 provides the data sources, methodology and models. The fourth chapter discusses the empirical results. Chapter 5 provides the conclusion.
Chapter 2: Literature Review

Many existing studies provide research on the relationship between oil and macroeconomic factors, and the same condition in the gold market. However, with the development of world economy, there are significant linkages and interconnection between gold and oil markets. So there is a need to research in this area.

2.1 Studies of Crude Oil

Previous research on crude oil pricing was mainly through two ways: the relationship between oil price and some macroeconomic factors and the relationship between oil price and the stock market.

Gisser and Goodwin (1986) used a Granger Causality test to assess quantitative significance of crude oil price on the U.S. economy. They proved that crude oil prices have had a significant impact on the GNP, consumer price index, unemployment rate and real investment. Furthermore, LeBlanc and Chinn (2004) estimated the effects of oil price changes on inflation in the G-5 countries (United States, United Kingdom, France, Germany, and Japan). They analyzed the linkage of several economic factors, such as GDP, unemployment rate and inflation rate and oil price by using an augmented Phillips curve framework, and found that the weaker inflation effect may be due to the reduction in oil intensity and
several other factors. Furthermore, monetary policy has not provided the basis for a sustained change in the inflation process. The most important suggestion is that an increasing oil price has only a modest influence on inflation in the U.S, Japan, and Europe. However, Cunadoa and Graciab (2005) used cointegration test and Granger Causality test to report that the oil price has a significant impact on inflation in some Asian countries. They also found that this effect was generally more significant than the oil prices – economic activity causality.

Huang, et al (1996) researched the linkage of oil futures and the stock market through two correlation analyses and vector auto regression (VAR). They claimed that the oil Future returns lead the related oil stock return by one day. It was statistically significant. By contract, Geman and Kharoubi (2008) analyzed the diversification benefits brought by crude oil Futures contracts into a portfolio of stocks. They concluded that the crude oil Futures can lead to a diversification both in upward and downward trending equity markets.

### 2.2 Studies of Gold

The studies about the gold market have mainly concentrated on two aspects: the first is whether or not gold is a hedge asset, second the volatility of gold returns.

Kolluri (1987) used two methods to examining the relationship between gold
price and inflation rate in the international market. The first investigated the
gold return and the expected inflation rate. The second method used the return
of different markets, such as U.S. Treasury bills, common stocks, and the return
on gold. It showed that inflation and gold price were positively correlated. In
contrast, Chan and Faff (1998) used monthly data from 1975 to 1994 to point
out “there has been a widespread sensitivity of Australian industry returns to
gold price returns, over and above market return” (Chan and Faff, 1998, p. 242).
They also determined that gold leads the exchange rate, but it has lost its
hedging capacity. However, Capie, et al (2005) used a GARCH process and
several expanded GARCH processes to prove that gold has been a hedge against
the foreign exchange value of the dollar because of its special features. And in
2010, Baur and McDermott also used GARCH processes to examine the data for
30 years. They found gold is a hedging asset in most developed country stock
markets and “gold is a safe haven for increased levels of global uncertainty
proxied by the conditional volatility of a world stock market index” (Baur and

On the other hand, in 2001, Cai, et al (2001) argued that the price of gold Futures
have intraday volatility, which is “the volatilities at market’s open and close are
higher than the volatilities in the middle of the day” (Cai et al, 2001, p. 275). They
also found that the U.S. macroeconomic announcements have a strong influence
on the gold market. Batten and Lucey (2010) also investigated the volatility of


gold futures.

2.3 Studies of the Relationship Between Crude Oil and Gold

Melvin and Sultan (1990) used ARCH and GARCH modes to establish a relationship between oil and gold markets by using the export revenue channel. They found that when the oil prices increase, the revenue of oil exports will rise and it impacts on the gold price level. They also concluded that the conditional variance of gold spot prices significantly determines the South African political unrest. For oil prices changes and the timing vary risk premium in gold Future prices, they found these depend on the conditional variance of recent spot prices forecast errors.

Narayan et al (2010) examined the long-run relationship between gold and oil spot and futures markets with different maturity times by using a structural break cointegration test. They explained that the oil price rise leads to an inflation rate increase, and this translates into higher gold prices. These findings imply that investors use the gold market to hedge inflation and the oil market can be used to predict the gold market prices and vice versa. In addition, Zhang and Wei (2010) found a cointegration relationship and causality between crude oil market and the gold market. Their results indicated that the crude oil price volatility is greater than gold and there is a long-term equilibrium relationship
between crude oil market and gold market. “Specifically, the crude oil price return change linearly Granger causes the movement of the gold price return, but not vice versa” (Zhang and Wei, 2010, p. 176).
Chapter 3: Data and Methodology

3.1 Data

In this paper, the West Texas Intermediate (WTI) crude oil daily price is chosen as the sample price from January 2000 to December 2012. The data were obtained from the U.S. Energy Information Administration. The daily price of COMEX gold from January 2000 to December 2012 was acquired from Bloomberg.

3.2 Data Analysis

3.2.1 Price Analysis

The following table summaries the basic descriptive statistics of the gold price and the crude oil price.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold Price</td>
<td>633.1891</td>
<td>442.8314</td>
<td>196099.6</td>
<td>1.292028</td>
<td>3.412498</td>
</tr>
<tr>
<td>Crude Oil Price</td>
<td>48.69496</td>
<td>30.33739</td>
<td>920.3574</td>
<td>0.7039098</td>
<td>2.375598</td>
</tr>
</tbody>
</table>

Table 3.1 Basic Descriptive Statistics of Daily Prices
From Figure 3.1, the general trend of crude oil price and gold price are the same. Before 2008, the prices of two commodities were increasing followed by a sharp decrease of oil price during 2008 and 2009. But the gold price did not decline dramatically. From 2010 to 2012, the prices of the two commodities fluctuated and at times together.

There is another popular way to quantify the relationship between the crude oil price and gold price, which is the gold-oil ratio. The ratio can be seen in Figure 3.2. When the ratio is high, it implies that the gold price is overvalued compared to the crude oil price, so gold is expensive and the oil is cheap and vice versa.
3.2.2 Return Analysis

In addition, the daily price of gold and crude oil should be calculated and transformed into daily returns. The daily return is defined as:

\[ R_{oil,t} = \frac{P_{oil,t} - P_{oil,t-1}}{P_{oil,t-1}} \]  \hspace{1cm} (3.1)

\( R_{oil,t} \): Return of crude oil during the time period \( t \)

\( P_{oil,t} \): Daily price of crude oil during the time period \( t \)

\( P_{oil,t-1} \): Daily price of crude oil during the time period \( t-1 \)

\[ R_{gold,t} = \frac{P_{gold,t} - P_{gold,t-1}}{P_{gold,t-1}} \]  \hspace{1cm} (3.2)

\( R_{gold,t} \): Return of gold during the time period \( t \)

\( P_{gold,t} \): Daily price of gold during the time period \( t \)
$P_{gold,t-1}$: Daily price of gold during the time period $t-1$

The Table 3.2 summaries the basic descriptive statistics of the daily return of gold and crude oil.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{gold,t}$</td>
<td>0.0346339</td>
<td>1.064923</td>
<td>1.134061</td>
<td>0.1999468</td>
<td>10.55745</td>
</tr>
<tr>
<td>$R_{oil,t}$</td>
<td>0.0675058</td>
<td>2.47905</td>
<td>6.145687</td>
<td>0.2062078</td>
<td>8.462759</td>
</tr>
</tbody>
</table>

Figure 3.3 Crude Oil Daily Returns & Gold Daily Returns

From Figure 3.3, the daily return of crude oil and gold are mainly around zero.

There is not an obvious trend or correlation between the returns of the two
commodities. Furthermore, the volatility of crude oil daily return is higher than the volatilities of gold daily return. For the investment perspective, the high volatility indicated a higher return, but with higher risk at the same time.

After the Kolmogorov-Smirnov (KS) test, which rejects the null hypothesis, the return of gold and the return of crude oil are not normally distributed. Table 3.3 and 3.4 show the test results.

<table>
<thead>
<tr>
<th>Small group</th>
<th>D</th>
<th>P-value</th>
<th>Corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>roil:</td>
<td>0.0422</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Cumulative:</td>
<td>-0.0414</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Combined K-S:</td>
<td>0.0422</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Small group</th>
<th>D</th>
<th>P-value</th>
<th>Corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>rgold:</td>
<td>0.0255</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>Cumulative:</td>
<td>-0.0557</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Combined K-S:</td>
<td>0.0557</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

3.3 Methodology

3.3.1 GARCH Model

Bollerslev (1986) improves the Autoregressive Conditional Heteroskedasticity (ARCH) model to a GARCH model, which can use both the previous error and
previous conditional variance in the model. The GARCH model is preferred, because it provides a more real-world context than other forms when trying to predict the prices and rates of financial instruments.

GARCH \((p, q)\) model process is defined as follow:

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3.3}
\]

\(\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0, i=1,...,q, j=1,...,p\) are model parameters.

When \(i = 1, q = 1, j = 1, p = 1\), GARCH \((1,1)\) is defined as follow:

\[
\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.4}
\]

\(\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0\)

### 3.3.2 TGARCH Model

The TGARCH model was a model developed from GARCH model by Zakoian in 1994. It was similar with GJR GARCH model, which was studied by Glosten et al (1993) and it defined the conditional variance as a linear piecewise function. The TGARCH and GJR GARCH models also relax the linear restriction on the conditional variance dynamics.

TGARCH \((p, q)\) model process is defined as follow:

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \gamma_i \epsilon_{t-i}^2 D_{t-1} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3.5}
\]
\[ D_{t-1} = \begin{cases} 1, & \text{if } \varepsilon_{t-1} < 0 \\ 0, & \text{if } \varepsilon_{t-1} \geq 0 \end{cases} \]

When \( i = 1, q = 1, j = 1, p = 1 \), TGARCH (1,1) is defined as follow:

\[ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 D_{t-1} + \beta_1 \sigma_{t-1}^2 \] \hspace{1cm} \text{(3.6)}

\[ D_{t-1} = \begin{cases} 1, & \text{if } \varepsilon_{t-1} < 0 \\ 0, & \text{if } \varepsilon_{t-1} \geq 0 \end{cases} \]
Chapter 4 Analysis the Empirical Results

4.1 Stationary Test

First, the Akaike's Information Criterion (AIC) is a measure of the relative quality of a statistical model, for a given set of data. It is defined as follows:

\[ \text{AIC} = 2k - 2\ln(L) \] \hspace{1cm} (4.1)

\( k \) is the number of parameters in the statistical model, and \( L \) is the maximized value of the likelihood function for the estimated model. With the minimum AIC value model is the preferred model. By using AIC, the best lag period is determined. For crude oil, the best lag period is nine, for gold the best lag period is eleven. Table 4.1 shows the results.

<table>
<thead>
<tr>
<th>Lag Period</th>
<th>Crude Oil</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.7012</td>
<td>3.14939</td>
</tr>
<tr>
<td>2</td>
<td>4.70167</td>
<td>3.15</td>
</tr>
<tr>
<td>3</td>
<td>4.70168</td>
<td>3.15061</td>
</tr>
<tr>
<td>4</td>
<td>4.7022</td>
<td>3.15121</td>
</tr>
<tr>
<td>5</td>
<td>4.69948</td>
<td>3.15181</td>
</tr>
<tr>
<td>6</td>
<td>4.69655</td>
<td>3.14987</td>
</tr>
<tr>
<td>7</td>
<td>4.69585</td>
<td>3.15019</td>
</tr>
<tr>
<td>8</td>
<td>4.695</td>
<td>3.15066</td>
</tr>
<tr>
<td>9</td>
<td>4.69497*</td>
<td>3.15112</td>
</tr>
<tr>
<td>10</td>
<td>4.6952</td>
<td>3.15124</td>
</tr>
<tr>
<td>11</td>
<td>4.6958</td>
<td>3.14925*</td>
</tr>
</tbody>
</table>
According to the results of the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, the return of gold and the return of crude oil are stationary, because the test statistic values are much smaller than all three levels critical value (10% level, 5% level, 1% level). The data are stationary, which means the GARCH model and TGARCH model can be used. Tables 4.2-4.5 show the test results.

Table 4.2 ADF Test of Crude Oil Returns

<table>
<thead>
<tr>
<th>Test Statistic</th>
<th>1% Critical Value</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z(t)</td>
<td>-20.088</td>
<td>-3.430</td>
<td>-2.860</td>
</tr>
</tbody>
</table>

MacKinnon approximate p-value for Z(t) = 0.0000

Table 4.3 ADF Test of Gold Returns

<table>
<thead>
<tr>
<th>Test Statistic</th>
<th>1% Critical Value</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z(t)</td>
<td>-17.836</td>
<td>-3.430</td>
<td>-2.860</td>
</tr>
</tbody>
</table>

MacKinnon approximate p-value for Z(t) = 0.0000
4.2 Granger Causality Test

The Granger Causality test is a technique for determining whether one time series is useful in forecasting another, Granger (1969).

In this case, the Granger Causality test is used to examine whether the return of gold can forecast the return of oil or the return of oil can forecast the return of gold.

It is defined as follows:

\[
R_{oil,t} = \sum_{i=1}^{n} \alpha_i R_{gold,t-i} + \sum_{j=1}^{n} \beta_j R_{oil,t-j} + \varepsilon_{1t} \quad \text{-------------------------}(4.2)
\]
\[ R_{gold,t} = \sum_{i=1}^{m} \gamma_i R_{oil,t-i} + \sum_{j=1}^{m} \delta_j R_{gold,t-j} + \varepsilon_{2t} \]  

The results are shown in the Tables 4.6 and 4.7.

<table>
<thead>
<tr>
<th>Table 4.6 Granger Causality Test of Crude Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-Value</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>1.47</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Note: * Denotes the hypothesis at the 0.01 level
    ** Denotes the hypothesis at the 0.05 level
    *** Denotes the hypothesis at the 0.10 level

<table>
<thead>
<tr>
<th>Table 4.7 Granger Causality Test of Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-Value</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>2.04</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Note: * Denotes the hypothesis at the 0.01 level
    ** Denotes the hypothesis at the 0.05 level
    *** Denotes the hypothesis at the 0.10 level

From the two tables, F-value of crude oil is smaller than the critical value in three levels. This indicates that the null hypothesis cannot be rejected. The gold return cannot Granger cause the crude oil return. On the other hand, the F-value of gold
is smaller than the critical value of 10% and 5% levels. This implies that the null hypothesis also cannot be rejected at 10% and 5% levels. The return of crude oil cannot Granger cause the gold return.

4.3 GARCH Model Analysis

As the definition of the GRACH model, the GARCH models for crude oil price return and gold price return are set as follows:

Conditional mean equation:

\[ R_{oil,t} = \alpha_{oil,0} + \sum_{i=1}^{9} \alpha_{oil,i} R_{oil,t-i} + \varepsilon_{oil,t} \]  
\[ R_{gold,t} = \alpha_{gold,0} + \sum_{i=1}^{11} \alpha_{gold,i} R_{gold,t-i} + \varepsilon_{gold,t} \]

\( R_{oil,t} \): Returns of the crude oil price at period t

\( R_{oil,t-i} \): Returns of the crude oil price at period t-i

\( R_{gold,t} \): Returns of the gold price at period t

\( R_{gold,t-i} \): Returns of the gold price at period t-i

\( \alpha_{oil,i} \), \( \alpha_{gold,i} \): Estimated coefficients

Conditional variance equation:

\[ \sigma_{oil,t}^2 = \alpha_{oil,0} + \alpha_{oil,1}\varepsilon_{oil,t-1}^2 + \beta_{oil}\sigma_{oil,t-1}^2 + b_{oil,gold}\varepsilon_{gold,t}^2 \]  
\[ \sigma_{gold,t}^2 = \alpha_{gold,0} + \alpha_{gold,1}\varepsilon_{gold,t-1}^2 + \beta_{gold}\sigma_{gold,t-1}^2 + b_{gold,oil}\varepsilon_{oil,t-1}^2 \]

\( \sigma_{oil,t}^2 \): Conditional variance of crude oil price return at period t

\( \sigma_{oil,t-1}^2 \): Conditional variance of crude oil price return at period t-1
\[ \sigma^2_{gold,t} \]: Conditional variance of gold price return at period t

\[ \sigma^2_{gold,t-1} \]: Conditional variance of gold price return at period t-1

\( \alpha_{oil,1}, \beta_{oil,} b_{oil,gold}, \alpha_{gold,1}, \beta_{gold,} b_{gold,oil} \): Estimated coefficients

Table 4.8 and Table 4.9 display the estimated coefficients and other related results of the crude oil and gold conditional mean equations. Table 4.10 and Table 4.11 show the estimated coefficients and other related results of the crude oil and gold conditional variance equations

Table 4.8 Estimated Coefficients of GARCH Conditional Mean Equations about Crude Oil Price Returns

| Coef. | OPG | Z   | P>|Z| | [95% Conf. Interval] |
|-------|-----|-----|------|---------------------|
|       |     |     |      | Std. Err            |
| \( \alpha_{oil,0} \) | 0.1042985 | 0.0390734 | 2.67 | 0.008 | 0.0277161 | 0.1808808 |
| \( \alpha_{oil,1} \) | -0.0238046 | 0.0185834 | -1.28 | 0.200 | -0.0602273 | 0.0126182 |
| \( \alpha_{oil,2} \) | -0.0174745 | 0.0176071 | -0.99 | -0.321 | -0.0519838 | 0.0170347 |
| \( \alpha_{oil,3} \) | -0.0006038 | 0.0188753 | -0.03 | 0.974 | -0.0375987 | 0.0363911 |
| \( \alpha_{oil,4} \) | 0.0144239 | 0.0181377 | 0.0 | 0.426 | -0.0211255 | 0.0499732 |
| \( \alpha_{oil,5} \) | -0.0134316 | 0.01807 | -0.74 | 0.457 | -0.0488481 | 0.0219849 |
| \( \alpha_{oil,6} \) | -0.0401212 | 0.0173276 | -2.32** | 0.021 | -0.0740827 | -0.0061596 |
| \( \alpha_{oil,7} \) | -0.0176876 | 0.0186988 | -0.95 | 0.344 | -0.0543365 | 0.0189613 |
| \( \alpha_{oil,8} \) | -0.0220796 | 0.0184365 | -1.20 | 0.231 | -0.0582144 | 0.0140552 |
| \( \alpha_{oil,9} \) | -0.032517 | 0.0182947 | -1.78 | 0.076 | -0.0683739 | 0.0033399 |

Note: ** 5% significance of confident interval
Table 4.9 Estimated Coefficients of GARCH Conditional Mean Equations about Gold Price Returns

| R_{gold} | Coef.  | OPG   | Z     | P>|Z|  | [95% Conf. Interval] |
|----------|--------|-------|-------|-------|----------------------|
|          |        | Std. Err |
| \( \alpha_{gold,0} \) | 0.0561959 | 0.0165957 | 3.39 | 0.001 | 0.023669 | 0.0887229 |
| \( \alpha_{gold,1} \) | -0.0387111 | 0.0192405 | -2.01 | 0.044 | -0.0764217 | -0.0010004 |
| \( \alpha_{gold,2} \) | 0.0155123 | 0.0187823 | 0.83 | 0.409 | -0.0213003 | 0.0523249 |
| \( \alpha_{gold,3} \) | -0.0006337 | 0.0174982 | -0.04 | 0.971 | -0.0349296 | 0.0336622 |
| \( \alpha_{gold,4} \) | -0.0102981 | 0.0183531 | -0.56 | 0.575 | -0.0462694 | 0.0256733 |
| \( \alpha_{gold,5} \) | -0.0048796 | 0.0186097 | -0.26 | 0.793 | -0.0413539 | 0.0315948 |
| \( \alpha_{gold,6} \) | -0.0533273 | 0.0180176 | -2.96* | 0.003 | -0.0886411 | -0.0180135 |
| \( \alpha_{gold,7} \) | -0.0306657 | 0.0177437 | -1.73 | 0.084 | -0.0654428 | 0.0041113 |
| \( \alpha_{gold,8} \) | 0.0177689 | 0.0179941 | 0.99 | 0.323 | -0.0174989 | 0.0530367 |
| \( \alpha_{gold,9} \) | 0.0124855 | 0.0182327 | 0.68 | 0.493 | -0.0232499 | 0.0482209 |
| \( \alpha_{gold,10} \) | 0.0298283 | 0.0176474 | 1.69 | 0.091 | -0.00476 | 0.0644166 |
| \( \alpha_{gold,11} \) | -0.0368121 | 0.0182988 | -2.01 | 0.044 | -0.072677 | -0.00947 |

Note: * 1% significance of confident interval

From the two tables above, the estimated coefficients of crude oil price return are significant at previous six periods at the 5% level and the estimated coefficients of gold price return are also significant at previous six periods at the 1% level. These indicate that the previous six periods of crude oil price return and gold price return can affect the current price return of the two commodities significantly.
Table 4.10 Estimated Coefficients of GARCH Conditional Variance Equations about Crude Oil Price Returns

| Roi | Coef.  | OPG   | Z   | P>|Z| | [95% Conf. Interval] |
|-----|--------|-------|-----|-----|----------------------|
|     |        | Std. Err |     |     |                      |
| \(\alpha_{oil,0}\) | 4.14898 | .0096002  | 432.17* | 0.000 | 4.130164 - 4.167796  |
| \(\alpha_{oil,1}\) | .0295946 | .0002958  | 100.04* | 0.000 | .0290148 - .0301745  |
| \(\beta_{oil}\) | .8788133 | .0051432  | 192.25* | 0.000 | .8687327 - .9788939  |
| \(b_{oil,gold}\) | .0000816 | .0043544  | 0.02 | 0.985 | -.0084528 - .0086161  |

Note: * 1% significance of confident interval

Table 4.11 Estimated Coefficients of GARCH Conditional Variance Equations about Gold Price Returns

| Rgold | Coef.  | OPG   | Z   | P>|Z| | [95% Conf. Interval] |
|-------|--------|-------|-----|-----|----------------------|
|       |        | Std. Err |     |     |                      |
| \(\alpha_{gold,0}\) | .7640614 | .003467  | 220.38* | 0.000 | .7572662 - .7708566  |
| \(\alpha_{gold,1}\) | .0962811 | .0006669  | 144.38* | 0.000 | .0949741 - .0975882  |
| \(\beta_{gold}\) | .808563 | .0020105  | 501.65* | 0.000 | .804622 - .912503    |
| \(b_{gold,oil}\) | -.0009114 | .0003439  | -2.65* | 0.008 | -.0015855 - -.0002373 |

Note: * 1% significance of confident interval

From the estimation of the crude oil coefficients of conditional variance equations, \(\alpha_{oil,1} > 0\), \(\beta_{oil} > 0\), \(\alpha_{oil,1} + \beta_{oil} < 1\), which means the crude oil price returns is stable through periods. In addition, \(\alpha_{oil,1}\) and \(\beta_{oil}\) are significant at the 1% level. It means the crude oil price return’s previous volatility has influence on the current volatility. \(b_{oil,gold}\) is not significant which indicates
there is no volatility effect from gold price return to crude oil return. On the
other hand, from the estimation of the crude oil coefficients of conditional
variance equations, \( \alpha_{gold,1} > 0, \beta_{gold} > 0, \alpha_{gold,1} + \beta_{gold} < 1 \), which means the
gold price returns are stable through periods. Furthermore, \( \alpha_{gold,1} \) and \( \beta_{gold} \)
are also significant at 1% level. It implies that the previous gold price return
volatility affects the current volatility. \( b_{gold,oil} \) is significant and that the
volatility of crude oil price returns can influence the volatility of gold price
returns.

4.4 TGARCH Model Analysis

The definition of the TGRACH model, the TGARCH models for crude oil price
return and gold price return are set as follows:

For conditional mean equation, it is same as the GARCH model:

\[
R_{oil,t} = \alpha_{oil,o} + \sum_{i=1}^{p} \alpha_{oil,i} R_{oil,t-i} + \varepsilon_{oil,t} \quad \text{----------------------------------------(4.4)}
\]

\[
R_{gold,t} = \alpha_{gold,0} + \sum_{i=1}^{q} \alpha_{gold,i} R_{gold,t-i} + \varepsilon_{gold,t} \quad \text{----------------------------------------(4.5)}
\]

\( R_{oil,t} \): Returns of the crude oil price at period t

\( R_{oil,t-i} \): Returns of the crude oil price at period t-i

\( R_{gold,t} \): Returns of the gold price at period t

\( R_{gold,t-i} \): Returns of the gold price at period t-i

\( \alpha_{oil,i}, \alpha_{gold,i} \): Estimated coefficients
Conditional variance equation:

\[ \sigma^2_{oil,t} = \alpha_{oil,0} + \alpha_{oil,1} \varepsilon^2_{oil,t-1} + \gamma_{oil} \varepsilon^2_{oil,t-1} D_{t-1} + \beta_{oil} \sigma^2_{oil,t-1} \quad (4.8) \]

\[ \sigma^2_{gold,t} = \alpha_{gold,0} + \alpha_{gold,1} \varepsilon^2_{gold,t-1} + \gamma_{gold} \varepsilon^2_{gold,t-1} D_{t-1} + \beta_{gold} \sigma^2_{gold,t-1} \quad (4.9) \]

\( \sigma^2_{oil,t} \): Conditional variance of crude oil price return at period t

\( \sigma^2_{oil,t-1} \): Conditional variance of crude oil price return at period t-1

\( \sigma^2_{gold,t} \): Conditional variance of gold price return at period t

\( \sigma^2_{gold,t-1} \): Conditional variance of gold price return at period t-1

\( \alpha_{oil,1}, \beta_{oil}, \gamma_{oil}, \alpha_{gold,1}, \beta_{gold}, \gamma_{gold} \): Estimated coefficients

For the conditional mean equation, the result is same as the GARCH model, which is shown in Tables 4.8 and 4.9. Tables 4.12 and 4.13 show the estimated coefficients and other related results of the crude oil and gold conditional variance equations.

Table 4.12 Estimated Coefficients of TGARCH Conditional Variance Equations about Crude Oil Price Returns

|        | Coef.  | OPG   | Z     | P>|Z|   | [95% Conf. Interval] |
|--------|--------|-------|-------|-------|----------------------|
|        |        | Std. Err |       |       |                      |
| \( \alpha_{oil,0} \) | 4.52029 | .0106426 | 424.74* | 0.000 | 4.499431 4.541149   |
| \( \alpha_{oil,1} \) | .0293522 | .0002691 | 109.07* | 0.000 | .0288247 .0298796   |
| \( \beta_{oil} \) | 1.021733 | .0002235 | 4571.06* | 0.000 | 1.021295 1.022171   |
| \( \gamma_{oil} \) | .0337673 | .0118089 | 2.86*  | 0.004 | .0106223 .0569122   |

Note: * 1% significance of confident interval
Table 4.13 Estimated Coefficients of TGARCH Conditional Variance Equations about Gold Price Returns

| Rgold    | Coef.  | OPG    | Z      | P>|Z|  | [95% Conf. Interval] |
|----------|--------|--------|--------|------|-----------------------|
|          | Std. Err |        |        |      |                       |
| $\alpha_{gold,0}$ | .7526475 | .0039048 | 192.75*| 0.000| .7449942 - .7603008   |
| $\alpha_{gold,1}$ | .1334844 | .0003482 | 383.36*| 0.000| .1328019 - .1341668   |
| $\beta_{gold}$    | 1.009123  | .0004128 | 2444.45*| 0.000| 1.008314 - 1.009933   |
| $\gamma_{gold}$  | .0350739  | .0039415 | 8.90*  | 0.000| .0273487 - .042799    |

Note: * 1% significance of confident interval

If $\varepsilon_{oil,t-1} < 0$, the regression function is as follow:

$$\sigma_{oil,t}^2 = 4.52 + 0.063\varepsilon_{oil,t-1}^2 + 1.022\sigma_{oil,t-1}^2$$

If $\varepsilon_{oil,t-1} \geq 0$, the regression function is as follow:

$$\sigma_{oil,t}^2 = 4.52 + 0.029\varepsilon_{oil,t-1}^2 + 1.022\sigma_{oil,t-1}^2$$

If $\varepsilon_{gold,t-1} < 0$, the regression function is as follow:

$$\sigma_{gold,t}^2 = 0.753 + 0.168\varepsilon_{gold,t-1}^2 + 1.009\sigma_{gold,t-1}^2$$

If $\varepsilon_{gold,t-1} \geq 0$, the regression function is as follow:

$$\sigma_{gold,t}^2 = 0.753 + 0.133\varepsilon_{gold,t-1}^2 + 1.009\sigma_{gold,t-1}^2$$

The estimated coefficients of $\varepsilon_{oil,t-1}^2$ and $\varepsilon_{gold,t-1}^2$ are different because of the dummy variables. From Tables 4.12 and 4.13, all the estimated coefficients are significant at the 1% level. These indicate that the previous periods volatility of crude oil price return affect the current volatility. The previous periods volatility
of gold price return also affect the current volatility.
Chapter 5 Conclusion

This paper contributes to future market research by examining the long-term relationship between crude oil and gold spot price and price return. For the crude oil and gold price analysis, the prices of two commodities are highly correlated. Because many factors can influence the crude oil price and gold price, and the interconnection between the two commodities are strong, many previous studies have proved that.

On the other hand, for the return analysis, it can be concluded that the previous return of two commodities can affect the current return themselves. Investors should observe and analyze the previous price and return deliberately to determine how to invest in the commodity futures market.

Also, there is no obvious relationship between the crude oil price return and gold price return. However, for the volatility of price return analysis, the volatility of crude oil price return has an effect on the volatility of gold price returns. Investor can monitor the change of gold price returns by watching in advance the volatility of crude oil price returns.
References:


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