

The Fisher Separation Theorem: Finance, Microeconomics and Macroeconomics

This paper is an extension of the Fisher separation theorem of finance and microeconomic theory to macroeconomic theory. This extension highlights the close relationship between the three disciplines and exposes a limitation of the two-period separation theorem in that it ignores income changes suggested by the model itself.

Introduction

This paper is concerned with extending the Fisher separation theorem of basic finance and microeconomic theory to include the Keynesian model of macroeconomics. To be sure it is a leap (somewhat of the nature of the leap of faith attempted by Indiana Jones in the movies) and in more than one step...a fundamental theorem of finance moved to microeconomics and then on to macroeconomics. The journey is not without traps and there are those amongst us who would say that we should not even try to follow this twisty path. We cannot deny the arguments, and we are aware of the quicksand lining the road along the way. In fact, we will even point out some hidden bumps in the asphalt that others may have missed. But the excursion is a worthwhile one, and one that leads the reader to see how closely microeconomics, macroeconomics and finance theory are related.

We begin by describing the separation theorem, then we identify the elements that are similar in standard microeconomics and in the macroeconomics of the Keynesian world. Finally, we explain what is necessary to move from the microeconomic view to the macroeconomic view and how this view leads one to question a conclusion of the separation theorem. This paper is a clarification and elaboration of some issues presented in an earlier paper on the same topic (Hochstein, November, 1993)

The Fisher Separation Theorem

An important theorem presented early on to students of finance theory is known as the Fisher separation theorem. It is described in the very early chapters of Copeland and Weston (1983) and Brealy and Myers (1984), both popular finance textbooks in common reference today. The essence of the theorem is that for any individual, the investment decision can be separated from the consumption decision since they are both dependent upon different criteria. This separation of consumption from investment means that while the consumption basket will likely differ from individual to individual, the investment decision (if the endowment is the same) will be the same for all and as such is independent of differing utility functions. In more simple language, people can trust agents to help with the investment decision, and then use the funds available to consume whatever they want.

The theorem states: given perfect and complete capital markets, the production decision is governed solely by an objective market criterion (represented by maximizing attained wealth) without regard to individuals= subjective preferences which enter into their consumption decisions (Copeland and Weston).

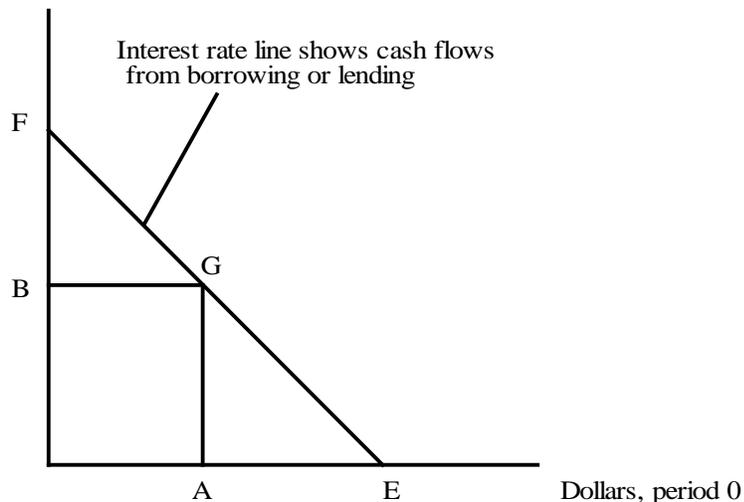
A. The Investment Decision

The separation theorem can be explained by examining the investment decision first. Consider someone with a given endowment of money, E . With the existence of capital markets, this money can be saved and can grow by the interest rate, r , which is assumed here to be both the borrowing rate and the lending rate.

Examine Figure 1 which measures current and future income on the axes. It illustrates an interest rate line. It starts at the horizontal axis at E . The endowment of E , if saved until period 1 will permit a total of F dollars, which is E times $(1+r)$. The slope of this line is then $-(1+r)$. Any point on this line illustrates the income flows that can be used for current and future consumption expenditure, spending which generates utility. For example, suppose, for the moment, that the initial selection of present and future consumption is shown by point G . This indicates that OA of the endowment is to be used immediately for current consumption, and AE is to be saved for the following period permitting OB to be available for consumption then. The higher up along the interest rate line, the more the individual can be described as a miser, (a saver), and the lower along the line, the more the individual can be called a spendthrift since more of the current endowment is being spent immediately.

Figure 1. The Interest Rate Line

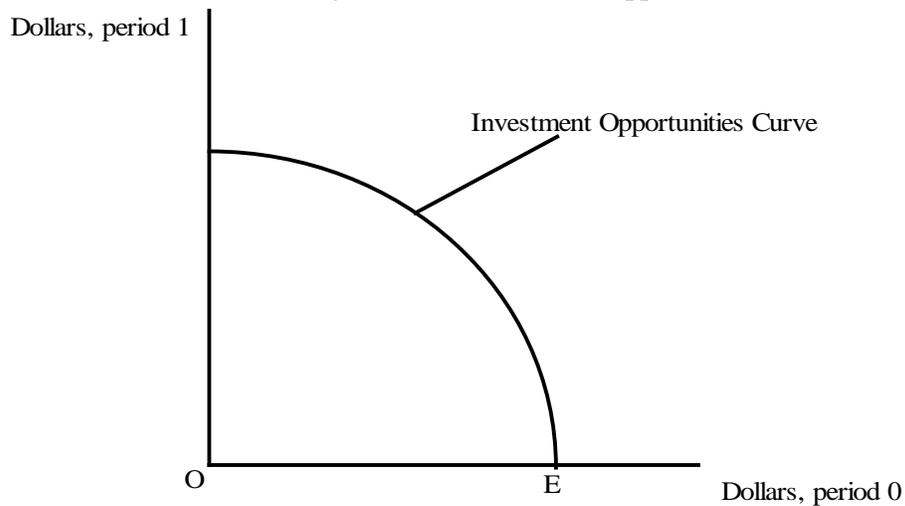
Dollars, period 1



An increase in the endowment, with interest rates constant, is shown by a rightward shift of the interest rate line, parallel to the original curve. Similarly, a decrease in the endowment is shown by a leftward shift of this line.

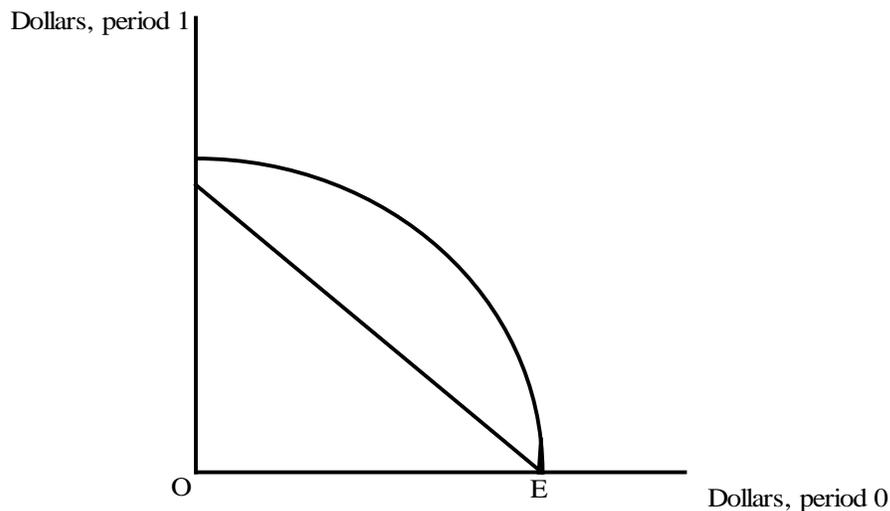
In the separation theorem, individuals may be able to buy plant, machinery or other real assets (investment). In this case, we can plot the returns possible to the investor via an investment-opportunities line, shown in Figure 2. The diagram is derived by asking the individual to rank his or her investment projects from high to low marginal returns. Assuming diminishing returns to the investment projects, the individual beginning with an endowment shown by E will be able to choose investments (assumed to be perfectly divisible) that will lead to returns which, when plotted, will generate a curve convex to the origin.

Figure 2. The Investment Opportunities Curve



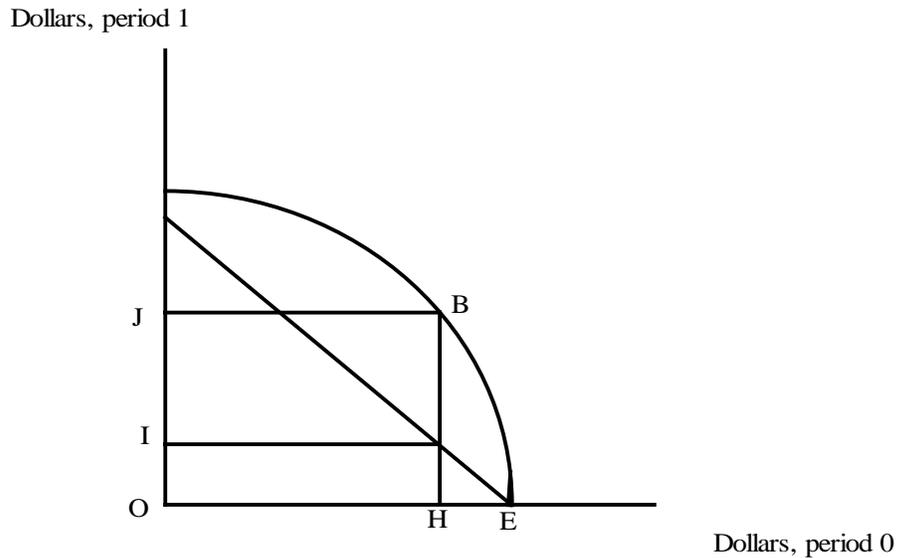
What is our representative participant to do: invest, save or consume, and in which proportions? This is economics, the answer depends upon maximization behaviour. First, consider the investment decision with the help of two diagrams, Figures 3 and 4. Figure 3 simply combines Figures 1 and 2. Figure 4 is this same diagram with decisions being made.

Figure 3. The Interest Rate Line and Investment Opportunities Curve



From Figure 4, the choice for the first use of the funds, say EH, is to invest rather than save because the amount returned from saving, OI, is less than the amount returned through investment, OJ. Again, since, OJ exceeds OI, invest.

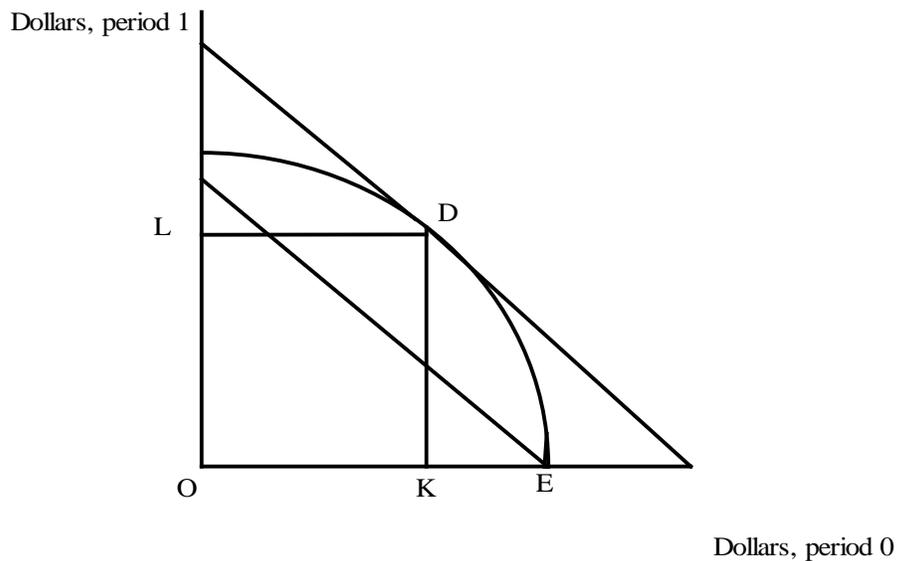
Figure 4. The Choice



Once this investment decision is made, the interest rate line shifts to the right (since the new income value is larger than the endowment), parallel to the initial one, but now it goes through point B (not shown). This process is said to continue again and again until the optimum investment value is found.

Investment continues until the return on the marginal project equals the interest rate since all previous investments would be profitable (McConnell et.al. 1996). This can be illustrated when the interest rate line is tangent to the investment opportunities curve, at point D on Figure 5. Our participant will invest EK and be able to choose any point on the new interest rate line for present and future consumption.

Figure 5. Investment

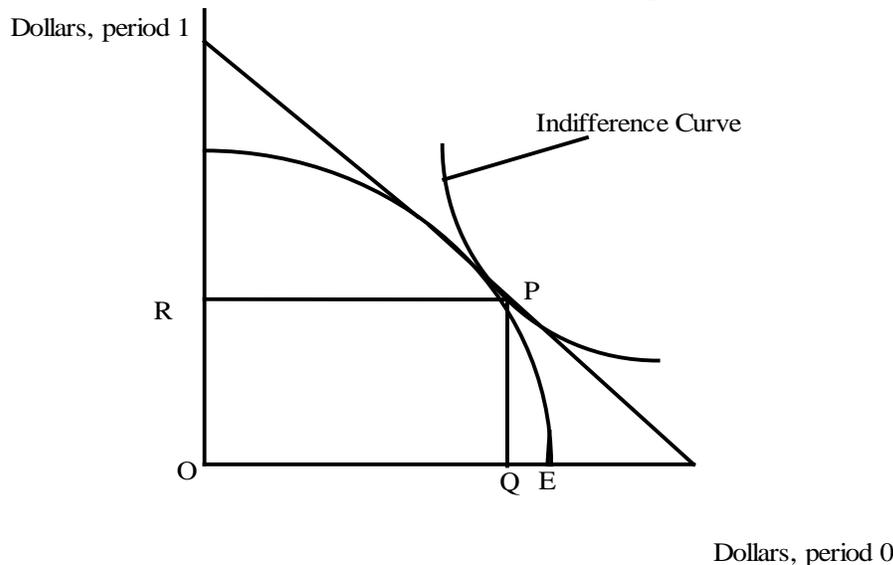


B. The Consumption Decision

The amount of present versus current consumption is the last piece of the puzzle. But again the theoretical solution is quite simple. The traditional procedure found in basic consumer theory is to

permit the household to maximize satisfaction by choosing a point on the constraint which reaches the highest possible indifference curve from the map of curves assumed to exist for all consumers. The tangency solution prevails (Hochstein, 1993). An example is shown in Figure 6. One of the family of indifference curves, the highest one possible, is drawn on the diagram. At point P the marginal rate of substitution between present and future consumption will equal the marginal rate of time preference, or the interest rate. In this particular case, money used for current consumption is OQ, and for future consumption is OR.

Figure 6. Consumption



While other individuals can be assumed to have different tastes, (Point P can be anywhere along the interest rate line drawn) and thus choose different bundles of consumption, the procedure is the same, of course. While not drawn here, it can be easily shown that some will choose to save heavily for the future, period 1, and some will choose to live to the hilt in the present. Chacun son gout.

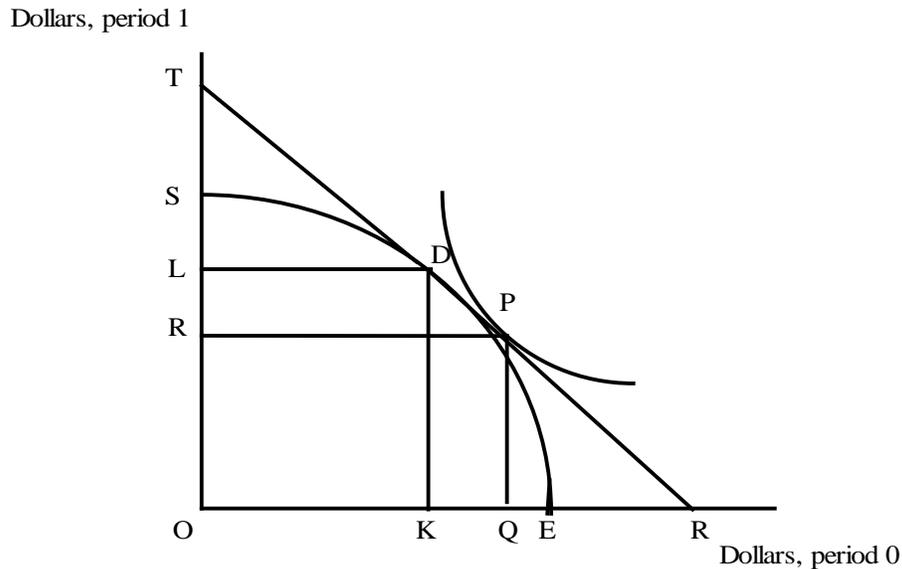
Summary

We can summarize the result of the Fisher separation theorem with the help of Figure 7. The individual under investigation begins with an endowment in the present shown by E dollars. The investment opportunities curve open to this person is ES. Investment is profitable up to EK dollars. This investment in real assets will generate OL dollars in the next period. Now a new income stream of OK + OL dollars exists. The point at which the interest rate line is tangent to the investment opportunities curve (D) indicates the current saving, optimum investment and the current and future income stream for consumption. One can note, as an aside, that the present value of this income stream is OR dollars and since OR exceeds the initial endowment of OE, this indicates the beneficial aspects of investment.

Consumption of goods in the present and the future depends upon time preference or tastes as measured by indifference curves. The indifference curve that is relevant in the case under review here is shown with a tangency at point P, indicating that utility is maximized at point P with OQ income for current consumption, and OR income for future consumption. Notice that the indifference curve going through point P is higher than any one which is constrained by the investment opportunities curve. While point D, the investment decision will be the same for anyone with the same investment

opportunities, P can be anywhere along the drawn interest rate line RT. The investment decision can be separated from the consumption decision.

Figure 7. Summary



The model shown here can also illustrate the importance of capital markets (Copeland and Weston, P. 12). With these efficient markets in existence, funds can be transferred between borrowers and lenders so that individuals with investment opportunities can borrow the funds and use them to purchase real assets whose return exceeds the market interest rate. They might not be able to obtain the funds as easily without these markets. Money can be transferred from those with wealth but few investment opportunities to those with less wealth and more opportunities. Everyone will be made better off without anyone being worse off. Clearly capital markets generate an increase in welfare for the system.

Because of space limitations we have not shown position P in other places along the interest rate line. The procedure is identical, but the conclusion can have saving more than investment, or even saving equal to investment.

An Extension

The separation model is interesting as far as it goes, but it does not go far enough. It is presented as a straightforward consumer theory microeconomic problem, one which requires all the marginal conditions be met...the marginal rate of substitution in consumption must equal the marginal rate of transformation in investment for an optimum solution. And they will both equal the market interest rate. (It is recognized that I have not differentiated between the borrowing and lending rates, but the essence of the argument presented here remains even with these additional caveats). This is very similar to standard micro theory of the consumer maximizing satisfaction from consuming two goods given a budget constraint.

What would Keynes have said about the model? Certainly I suspect that he would have agreed with the standard micro theory approach, but there are critical macroeconomic terms in this model...saving, investment, consumption. And these are not fully explored. In fact, he might have not agreed with an

the separation model. It says there is an endowment that can be spread over two periods, and then it ignores a change in this endowment due from the economic system. But income will change.

Can this move be made

Is our move from microeconomics to macroeconomics a legitimate one? On one level it certainly is. The macro system is developed by summing the activity of participants in the micro world. Consumption is nothing more than the addition of all the purchases of individuals who make their decisions at a micro level. Investment of GDP comes about because of individual purchases of machinery or plant. And there are numerous examples of this move from microeconomics to macroeconomics being made in the literature. For example, the field of welfare economics is famous for its use of microeconomic concepts such as firm marginal cost curves to handle macroeconomic problems such as pollution issues (Solberg, 1992); the theory of international trade uses macroeconomic indifference curves to decide on the optimum amount of imports and exports. The theory of health economics has been known to examine health as a normal good (a micro term) because individuals buy more of it when overall income (a macro term) increases (Feldstein, 1993).

But just because others have done it, doesn't make it legitimate. We all are aware that what may be true in microeconomics is not always true in macroeconomics. Any one individual can increase saving and his or her income can be kept constant, but if all economic agents increased saving, *ceteris paribus*, income would fall. Thus, we must be careful in the move from one view to the other.

In this particular case, the implications of the Fisher separation theorem on the macroeconomic landscape, the jump from finance and microeconomics to macroeconomics brings out some interesting results. Standard micro consumer theory has a consumer with a given income choosing between two goods in the present. Finance has slipped a caveat into the theory and has a consumer choosing to save and invest in the present which has implications for consumption in the present and future periods. Is it proper to permit investment to occur and then ignore the macroeconomic implications of this activity on income? We don't think so.

The critical assumption of the theorem, that a particular endowment can only be increased by using individual investment decisions, has to be reexamined. If the participant studied in this paper is similar to many other participants in the economy, and if indeed overall investment exceeds savings, then the second period's income will not be limited to the result of investment as calculated. There is a macro multiplier effect, and income will rise by more than the amount of investment. The investor from Finance will have to be prepared to have the macro economy force a reappraisal of endowments period by period.

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