Measurement of the partial cross sections $\sigma_{TT}$, $\sigma_{LT}$, and $(\sigma_{T} + \varepsilon\sigma_{L})$ of the $^1\text{H}(e,e'\pi^+)n$ reaction in the $\Delta(1232)$ resonance


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We report precision $^1\text{H}(e,e'\pi^+)n$ measurements in the $\Delta(1232)$ resonance at $Q^2 = 0.127(\text{GeV}/c)^2$ obtained at the MIT-Bates out-of-plane scattering facility. These are the lowest, but nonzero, $Q^2$ measurements in the $\pi^+$ channel. The data offer tests of the theoretical calculations, particularly of the background amplitude contributions. The chiral effective field theory and Sato-Lee model calculations are not in agreement with this experiment.

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Hadrons are composite systems with nonspherical quark-gluon and meson-nucleon interactions, so there is no reason to expect that they will be spherical [1,2]. For the past few decades there has been extensive work to measure and quantify the deviation from spherical symmetry based on the $\gamma^*N \rightarrow \Delta$ reaction. For recent reviews and references to the literature, see Refs. [3,4].

The spectroscopic quadrupole moment provides the most reliable and interpretable measurement of the presence of nonspherical components in the wave function. For the proton it vanishes identically because of its spin-1/2 nature. Instead, the magnitudes of the nonspherical components are measured by the resonant electric quadrupole ($E2$) and Coulomb quadrupole ($C2$) amplitudes $E_{1+}^{3/2}$ and $E_{1+}^{1/2}$ (for the notation, see Ref. [5]) in the predominantly magnetic dipole ($M1$) $M_{1+}^{3/2}$ $\gamma^*N \rightarrow \Delta$ transition [3,4]. Nonvanishing resonant quadrupole amplitudes will signify that either the proton or the $\Delta^+(1232)$ or, more likely, both are characterized by nonspherical components in their wave functions.

In the constituent-quark picture of hadrons, these nonspherical amplitudes are a consequence of the noncentral color-hyperfine interaction among quarks [1,2]. However, it has been shown that this mechanism only provides a small fraction of the observed quadrupole signal at low $Q^2$ [3,4]. At long ranges the dominant contribution originates in the spontaneous breaking of chiral symmetry, which leads to a spherically asymmetric virtual pion cloud [3]. With this in mind our group has focused on the $\gamma^*p \rightarrow \Delta$ reaction at low $Q^2 \leq 0.2(\text{GeV}/c)^2$ [6–12] with precise measurements at Bates and Mainz. Other experiments at low $Q^2$ have also been performed at Mainz and Brookhaven [13–16]. When this experiment started, Jefferson Laboratory had performed a series of experiments primarily at higher $Q^2$ and mostly in the $\pi^0$ channel [17–25]. At the present time much of this gap has been filled by CEBAF large acceptance spectrometer (CLAS) data [26] for $Q^2$ values above 0.16 (GeV/c)$^2$ for which analysis that includes isospin separation has been performed [26]. Our present data represent an accurate addition to this database at a lower $Q^2$ values.

With the existence of nonspherical components in the nucleon wave function well established, recent investigations have focused on testing the reaction calculations and reducing the errors in extracting the resonant multipoles from the data. To do this we have explored all three reaction channels associated with the $\gamma^*N \rightarrow \Delta$ transition: $^1\text{H}(e,e'p)\pi^0$, $^1\text{H}(e,e'p)\pi^0$, and $^1\text{H}(e,e'p)\gamma$. So far at the kinematics of this
work, at $Q^2 = 0.127 \text{(GeV/c)}^2$, only the $^1\text{H}(e,e'p)\pi^0$ channel has been extensively explored.

The exploitation of the other two reaction channels offers us the unique opportunity to explore the physics of interest through different, complementary, reaction channels that can provide additional information with respect to both the resonant and the background amplitude contributions. The background contributions need to be reasonably accurately known for the determination of the weak quadrupole resonant amplitudes. These play a more significant role in the $\pi^0$ channel off resonance or in the $\pi^+$ channel. They also are dominant in the $\text{TL}$ (fifth structure function) observable. This gives three different ways to test these contributions.

Exploring the $^1\text{H}(e,e'\pi^+)n$ reaction channel [complementary to $^1\text{H}(e,e'p)\pi^0$] offers the necessary information in order to separate the two isospin terms ($I = 1/2$ and $I = 3/2$) that both contribute with different weighting in the two channels

$$A(\gamma p \rightarrow n\pi^+) = \sqrt{2}(A^0_1 - \frac{1}{3}A^{(3/2)}_0),$$

$$A(\gamma p \rightarrow pn^0) = A^{1/2}_p + \frac{2}{3}A^{(3/2)}_0,$$

where $A$ is any multipole operator.

The cross section of the $^1\text{H}(e,e'\pi^+)n$ reaction is sensitive to four independent partial cross sections

$$\frac{d\sigma}{d\Omega_e d\Omega_n} = \Gamma(\sigma_T + \varepsilon\sigma_L + \varepsilon\sigma_{TT} \cos 2\phi_{\pi q}$$

$$+ \nu_{LT}\sigma_{LT}\cos \phi_{\pi q}),$$

where $\varepsilon$ is the transverse polarization of the virtual photon, $\Gamma$ is the virtual photon flux, $\phi_{\pi q}$ is the pion azimuthal angle with respect to the momentum transfer direction, and $\nu_{LT} = \sqrt{2\pi(1 + \varepsilon)}$. The $\sigma_T$ and $\sigma_L$ terms can be combined into a single one, $\sigma_0 \equiv \sigma_T + \varepsilon\sigma_L$.

The $E2$ and $C2$ amplitudes manifest themselves mostly through the interference with the dominant dipole ($M1$) amplitude. The longitudinal-transverse (LT) response is sensitive to the $C2$ amplitude through the interference of the $C2$ amplitude with the $M1$, while the transverse-transverse (TT) response is sensitive to the $E2$ amplitude through the interference of the $E2$ amplitude with the $M1$. The $\sigma_0$ partial cross section is dominated by the $M1$ multipole.

The $^1\text{H}(e,e'\pi^+)n$ measurements were performed using the out-of-plane spectrometer (OOPS) system [27] of the MIT-Bates Laboratory. Electrons were detected with the one hundred inch proton spectrometer (OHIPS) spectrometer [28], which employed two vertical drift chambers for the track reconstruction and three scintillator detectors for timing information. A Cherenkov detector and two layers of 18 Pb-glass detectors were used for particle identification. Pions were detected with the OOPS spectrometers, which were instrumented with three horizontal drift chambers for the track reconstruction followed by three scintillator detectors for timing and particle identification. Three identical OOPS modules were placed symmetrically at azimuthal angles $\phi_{\pi q} = 60^\circ$, $90^\circ$, and $180^\circ$ with respect to the momentum transfer direction for the measurement at central kinematics of $\theta_{\pi q}^* = 44.45^\circ$; thus we were able to isolate the $\sigma_{TT}$, $\sigma_{LT}$, and $\sigma_0$ partial cross sections. An OOPS spectrometer was positioned along the momentum transfer direction, thus directly measuring the parallel cross section at $\theta_{\pi q}^* = 0^\circ$. A high duty factor 950-MeV electron beam of 7-$\mu$A average current was scattered from a cryogenic liquid-hydrogen target. Measurements were taken at $W = 1232 \text{MeV}$ and at four-momentum transfer of $Q^2 = 0.127 \text{(GeV/c)}^2$. Point cross sections were derived from the finite acceptances by projecting the measured values to point kinematics using theoretical models [29–32]; the projection to central kinematics had minimal influence on the systematic uncertainty of the results. Two simultaneous redundant measurements were performed at $W = 1232 \text{MeV}, Q^2 = 0.127 \text{(GeV/c)}^2$, and $\theta_{\pi q}^* = 44.45^\circ$ by placing two OOPS spectrometers symmetrically with respect to the scattering plane at $\phi_{\pi q} = 60^\circ$ and $-60^\circ$. The results of both cross sections were in excellent agreement, thus confirming our good understanding of the OOPS system.

FIG. 1. (Color online) The coincidence time-of-flight spectrum.

FIG. 2. Typical missing mass distribution for the reconstructed neutron (with a cut on the time-of-flight peak and without subtraction of accidentals).
Radiative corrections were applied to the data \cite{33} using the code EXCLURAD \cite{34}. The coincidence time-of-flight spectrum and the reconstructed missing mass are presented in Figs. 1 and 2, respectively. Elastic-scattering data for calibration purposes were taken using liquid-hydrogen and carbon targets. The uncertainty in the determination of the central momentum was 0.1% and 0.15% for the pion and electron arms, respectively, while the uncertainty and the spread of the beam energy were 0.3% and 0.03%, respectively. A detailed description of all experimental uncertainties and their resulting effects in the measured partial cross sections is presented in Ref. \cite{33}.

In Fig. 3 we present the experimental results for the measured cross sections as well as for the extracted $\sigma_0$, $\sigma_{LT}$, and $\sigma_{TT}$ partial cross sections. Both the statistical and the total experimental uncertainties are exhibited in Fig. 3. The experimental results are compared with the phenomenological model MAID 2007 \cite{31,32}, the dynamical calculations of Sato-Lee \cite{29} and of Dubna–Mainz–Taipei (DMT) \cite{30}, and the chiral effective field theory (ChEFT) calculation of Pascalutsa and Vanderhaegen \cite{35}. The MAID model, which offers a flexible phenomenology and which provides an overall consistent agreement with the $^1H(e,e'p)\pi^0$ data at the same $Q^2$ \cite{8}, exhibits the best agreement with the experimental results. The DMT and Sato-Lee are dynamical reaction models that include pion-cloud effects, and both calculate the resonant channels from dynamical equations. DMT uses the background amplitudes of MAID with some small modifications, while Sato-Lee calculates all amplitudes consistently within the same framework with only three free parameters. DMT exhibits an overall agreement with the data with the partial exception of the cross-section measurement at $\theta_{\pi_q}^* = 44.45^\circ$, $\phi_{\pi_q}^* = 180^\circ$. On the other hand, the Sato-Lee model exhibits a clear disagreement with the data. The model clearly underestimates the measured cross sections and the extracted $\sigma_0$. For $\theta_{\pi_q}^*$ lower than 30° one can also point out a qualitative disagreement of Sato-Lee both with the data and with the rest of the models; Sato-Lee predicts a plateau below $\theta_{\pi_q}^* = 30^\circ$ for $\sigma_0$, while the data and the rest of the theoretical calculations indicate a $\sigma_0$ increase as we approach $\theta_{\pi_q}^* = 0^\circ$. On the other hand, the calculation is within good agreement with the extracted $\sigma_{LT}$ and $\sigma_{TT}$ cross sections. The magnitude of the disagreement between the Sato-Lee prediction and the
$^1\text{H}(e,e'p)\pi^0n$ results is a bit surprising if we consider the reasonable agreement of the Sato-Lee calculation with the experimental results of the $^1\text{H}(e,e'p)\pi^0\pi^0$ channel at the same $Q^2$ [8].

The ChEFT calculation of Pascualtsa and Vanderhaeghen is a systematic expansion based on QCD [35]. The results of this expansion up to next to leading order are also presented in Fig. 3. Here the contribution of the next-order term in the expansion has been estimated, and the theoretical errors reflect this uncertainty. A significant overestimation of both the $\sigma_0$ and $\sigma_{LT}$ results, even with the relatively large theoretical uncertainty, indicates that the next-order calculation is required.

It is worth pointing out that this calculation is in reasonable agreement with the $\sigma_0$ and $\sigma_{LT}$ results for the $^1\text{H}(e,e'p)\pi^0\pi^0$ channel [9,10].

The pion electroproduction database for the $^1\text{H}(e,e'p)\pi^0$ reaction channel is now rather extensive, resulting in a good understanding of the resonant amplitudes. Because those measurements are twice as sensitive to the resonant $I = 3/2$ multipoles as the present $\pi^+$ measurements, the exploration of the $\pi^+$ channel offers higher sensitivity to the background amplitudes, which can be observed in the dependence of the cross section as a function of $\phi_{\pi q}^*$ in Fig. 3(a). The resonant $I = 3/2$ multipoles dominate the cross section toward $\phi_{\pi q}^* = 0^\circ$, while toward the $\phi_{\pi q}^* = 180^\circ$ region it is the $I = 1/2$ background ones that play the dominant role. One can observe that the MAID, DMT, and Sato-Lee models exhibit an agreement in their description of the cross section at $\phi_{\pi q}^* = 0^\circ$, where the resonant terms dominate, while they deviate as we move toward $\phi_{\pi q}^* = 180^\circ$. This is mainly a consequence of the disagreement of the background amplitudes of the models. These conclusions reinforce those that have been inferred from the background sensitive results previously acquired for the $^1\text{H}(e,e'p)\pi^0\pi^0$ channel at $Q^2 = 0.06$ and 0.20 (GeV/c)$^2$ at the lower wing of the resonance ($W = 1155$ MeV) [11], which exhibit high sensitivity to background amplitude contributions.

This study also shows a relatively better description of the background amplitudes from the MAID and the DMT models compared to the Sato-Lee one, after the resonant amplitudes have been refitted to the resonant data. In contrast to these findings the Sato-Lee dynamical model works well for the CLAS experiment for the $ep \rightarrow e\pi^+n$ reaction at $0.25 \leq Q^2 \leq 0.65$ (GeV/c)$^2$ [24]. In addition the Sato-Lee model is in good agreement with the $^1\text{H}(e,e'p)\pi^0\pi^0$ channel $\sigma_{LT}$ data on resonance [9,10] at $Q^2 = 0.06$ and 0.20 (GeV/c)$^2$, which also provide additional information regarding a part of the background amplitude contributions. In conclusion, we present precise measurements of the $^1\text{H}(e,e'\pi^+)n$ reaction channel in the $\Delta(1232)$ resonance region. This work represents the initial phase of the isospin decomposition of the $\gamma^* p \rightarrow \Delta$ reaction at low $Q^2$. The data, which exhibit a higher sensitivity to the background amplitude terms compared to our previously explored $^1\text{H}(e,e'p)\pi^0\pi^0$ channel studies in the small $Q^2$ region [6–11], provide strong constraints to the most recent theoretical calculations. We find that the DMT [30] and MAID [32] models are in reasonable agreement with the data but that the Sato-Lee model [29] and the chiral effective field theory calculations [35] are not.