Erratum: ‘‘Linear least-squares fits with errors in both coordinates’’ [Am. J. Phys. 5 7, 642–646 (1989)]

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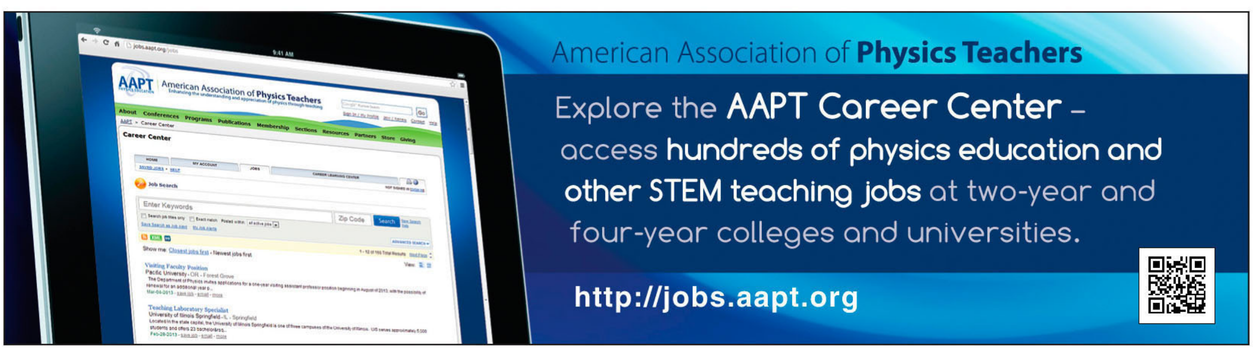

carried out, using the feature that $h_{11} = -h_{00}$, consistent with Eqs. (2) and (3), but not consistent with the correct implication of (1) for this system. In fact, it will be seen that the field equation (1), applied consistently in rectangular coordinates, or its generalization to general coordinate systems, applied to spherical coordinates, implies the unique solution $h_{11} = 0$.

Because most readers are probably not familiar with general tensor analysis, I will only outline briefly the correct formulation of the problem in this form and summarize its consequences. However, I will also indicate how the problem can be solved using (1) in rectangular coordinates, the operations of which can be carried out using ordinary vector analysis. Not surprisingly, the two approaches lead to the same conclusion.

Written in a general coordinate system, Eq. (1) becomes

$$h_{\mu\nu;\alpha}{}^{;\alpha} = \kappa h_{\mu\alpha;\beta} h_{\nu}^{\ \alpha;\beta}, \tag{4}$$

where semicolons indicate covariant differentiation. The metric in spherical coordinates is $g_{\mu\nu} = \mathrm{diag}\,(-1, 1, r^2, r^2 \sin^2\theta]$. For nonzero h_{00} and h_{11}, which are functions only of r, the only nontrivial (other than giving $0=0$) components of (4) are for the two indices μ and ν equal. In the case $\mu = \nu = 0$, (4) reduces to (2). For $\mu = \nu = 1$, (4) reduces to

$$\frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r} - \frac{4\psi}{r^2} = \kappa\left[\left(\frac{\partial\psi}{\partial r}\right)^2 + \frac{2\psi^2}{r^2}\right], \tag{5}$$

where $\psi \equiv h_{11}$. Note that this equation does not agree with (3), even if the vector operators in (3) are expressed in spherical coordinates. However, there are two more nontrivial equations, $\mu = \nu = 2$, and $\mu = \nu = 3$, both of which give the same information, namely,

$$2\psi = \kappa\psi^2. \tag{6}$$

If the nonzero solution of (6) for ψ, $\psi = 2/\kappa$, is substituted into (5), an inconsistency results. Therefore, the unique solution of the pair of equations (5) and (6) is

$$\psi = h_{11} = 0. \tag{7}$$

An alternate way of deriving the same result is to express the tensors in rectangular coordinates. Since the tensor has a nonzero projection only along the radial direction, the spatial components of the tensor h_{ij} in rectangular coordinates are given by

$$h_{ij} = \psi x^i x^j / r^2, \tag{8}$$

where $\psi = \psi(r)$ and $x^i = (x,y,z)$ are the components of the position vector $\mathbf{r}$. This may then be substituted into (1), written out in terms of ordinary differential operators, as

$$\nabla^2 h_{ij} = \kappa\, \partial_k h_{il}\, \partial_k h_{jl}, \tag{9}$$

where Latin indices represent three-dimensional rectangular components. The resulting equation for ψ then has two sets of terms, one set of which is a common factor of the tensor $x^i x^j$, and the other set multiplies the Kronecker delta, δ_{ij}. Since this equation can be satisfied only if the corresponding coefficients of these two tensors are equal, this gives the pair of equations to be satisfied by ψ:

$$\frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r} - \frac{6\psi}{r^2} = \kappa\left[\left(\frac{\partial\psi}{\partial r}\right)^2 + \frac{\psi^2}{r^2}\right], \tag{10}$$

$$2\psi = \kappa\psi^2. \tag{11}$$

The pair of equations (10) and (11) is consistent with the pair (5) and (6), as can be seen by multiplying (11) by $1/r^2$ and adding the result to (10). Again, if the nonzero solution of (11) is substituted into (10), an inconsistency results, indicating that the only consistent solution of (10) and (11) is $\psi = h_{11} = 0$, in agreement with (7).

Therefore, contrary to the claim of Biswas, h_{11} is not the negative of h_{00}, but rather the consistent application of the assumed field equations to the trial form of the potential implies $h_{11} = 0$. As the feature $h_{11} = -h_{00}$ was crucial in obtaining equations of trajectories that agreed with those of general relativity, those equations are not obtained using the correct h_{11}, and the two theories do not predict the same results.

In private correspondence, Dr. Biswas has indicated agreement that there was an error in his article and that the results derived above are the consequences of his stated assumptions. However, he has also indicated that if the trial form for the potential is modified to allow for an extra function, with another adjustable constant, the resulting theory can be made to agree with the general relativistic prediction of the deflection of light and the perihelion shift. Dr. Biswas states that he can provide a revised version of his original article to interested readers.

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Due to an error in programming Eqs. (14) and (15), the uncertainties quoted for the slopes and intercepts of examples I and II in the captions to Figs. 1 and 3 of this paper are in error. For example I these should read $m = 1.167 \pm 0.138$, $c = -0.365 \pm 0.127$, and, for example II, $m = 4.544 \pm 1.600$, $c = -17.483 \pm 11.089$. Also, the last number on the x axis in Fig. 3 should be "16," not "18."

In addition, it must be emphasized that Eqs. (14) and (15) are *approximate* expressions for σ_m and σ_c. More refined treatments make the estimated values of the data points and second-order terms in a series expansion for σ_m. Details are given by Fuller[1]; these refinements can have significant effects in cases where the scatter in the data is large.

I am indebted to Bill Jefferys of the University of Texas at Austin for drawing these matters to my attention.

[1]W. A. Fuller, *Measurement Error Models* (Wiley, New York, 1987), Sec. 1.3.