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Full Name of Author — Nom complet de l'auteur

Colin Calnan

Date of Birth — Date de naissance	Country of Birth — Lieu de naissance
July 17, 1953	Canada

Permanent Address — Résidence fixe

500 Stout Court

Bathurst, N.B.

Title of Thesis — Titre de la thèse

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1979	Dr. Gary A. Welch

Date	Signature
July 7, 1979	*Colin Calnan*

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CORE RADII DETERMINATIONS FOR

FOUR CLUSTERS OF GALAXIES

Colin Calnan

A thesis submitted as partial

requirement for the degree of

Master of Science

from the

Department of Astronomy

at

Saint Mary's University

Saint Mary's University

1979

My thesis now is all complete,

My obligations I did meet.

A new degree

Awarded me,

'Twas not a trifling feat.

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days.

Abstract

Bahcall (1975) has found that the average core radius for a group of 15 clusters of galaxies is 0.25±0.05 Mpc. At the suggestion of Dr. G. Welch it was decided to study four nearby clusters of galaxies (A2052, A2593, A2626, and A154) in order to determine their core radii. If it turned out that the dispersion of core radii at low redshifts is small, then these core radii could be said to be effectively constant. Any variation of the core radius at large redshifts would then be due to the geometry of the universe.

Accordingly, a computer program was written that would find a core radius by fitting ring countsdata from the chosen clusters to an Emden isothermal gas sphere. The ring counts were made to three magnitude limits, one of which approximated that of Bahcall. Also, each magnitude limit was used to find four core radii: one using all the ring count data and a counted background density; one using half the ring count data (only the core region) and a counted background density; one using all the data but solving for a background density (among other parameters); and one using half the data and solving for the background density. These four results were compared in various ways in order to determine which method produced the "best" core radius. Then the "best" core radius for each cluster at the magnitude

limit used by Bahcall was added to her results to obtain a new average and standard deviation.

Several conclusions were drawn from the overall results.

1. In the course of testing the program it was found that different results were found between this and other programs using the same data. This indicates the need of a unique program to be used exclusively.

2. Better results seem to be found when the background density is counted.

3. Better results seem to be found when all data (about out to the Abell radius) is used as opposed to only the core data.

4. Two clusters show evidence of mass segregation (A2052 and A2593).

5. The spread of core radii from the four clusters of this thesis at (or more precisely, "near") Bahcall's magnitude limit is large enough to cast doubt on the idea of using core radii as universal geometry indicators (R_c (average)=0.20±0.13 Mpc for the four clusters of this thesis).

One of the fundamental questions about the uni-
verse concerns its geometry; more specifically, whether
it is open or closed. An indicator of this property is the
deceleration parameter q_o, which is the measure of the
deceleration rate of the expanding universe.

For values of $q_o < \frac{1}{2}$, the universe is expanding too
fast to ever stop and will continue forever; the universe is
open. For $q_o = \frac{1}{2}$, the expansion will stop, but only at an
infinite time in the future. If $q_o > \frac{1}{2}$, the universe will
stop expanding to begin contracting at a definite time in
the future, and the greater the value of q_o the nearer is
this time. With $q_o > \frac{1}{2}$ the universe is said to be closed.

If there were a standard metrestick that could be
placed in space at different distances (as indicated by
recessional speed, or redshift), then the manner in which
its apparent size changed with redshift would depend on q_o.
Therefore a plot of apparent size versus redshift would
enable a user to determine the value of q_o.

As it happens, a standard metrestick may be
available. Studies of clusters of galaxies, principally by
Bahcall (Bahcall 1975, and references therein) have shown
that rich galaxy clusters of low redshift ($z \lesssim 0.14$, where
z=radial velocity/speed of light) have a linear core radius

R_C (to be explained later) which is approximately constant. For fifteen clusters in the redshift range $0.0181 \leqslant z \leqslant 0.134$ the average of R_C is 0.25 ± 0.05 Mpc ($H_O = 50$ km s^{-1}Mpc^{-1}, Bahcall, 1975). If this value is characteristic of clusters of galaxies to within sufficiently narrow limits then R_C may serve as a standard metrestick.

For quite some time it has been known that the radial number density distribution of the members of rich clusters could be closely matched to the radial density distribution of a bounded Emden isothermal gas sphere projected to two dimensions (Zwicky, 1957). To fit observations, the usual model, which is constructed with dimensionless variables, must be scaled in density and size. The core radius is the radius at which the density is about half the central value (see Figure 1).

Actually, by definition $r_C = 3\alpha$, where r_C is the observed core radius in arcmin and α is the structural length (or scale factor) of the cluster in arcmin, a value found during the computer fitting process. (In Figure 1 $\xi = r/\alpha$, and at $r = r_C$, i.e. $\xi = 3$, the actual value of the density is about 0.43.)

Knowing r_C, the redshift of the cluster, and a value for Hubble's constant, the physical core radius R_C in Mpc can be determined. Then a plot of R_C versus z for a large number of clusters can be used to find q_O.

Despite the fact that the physical data are fitted
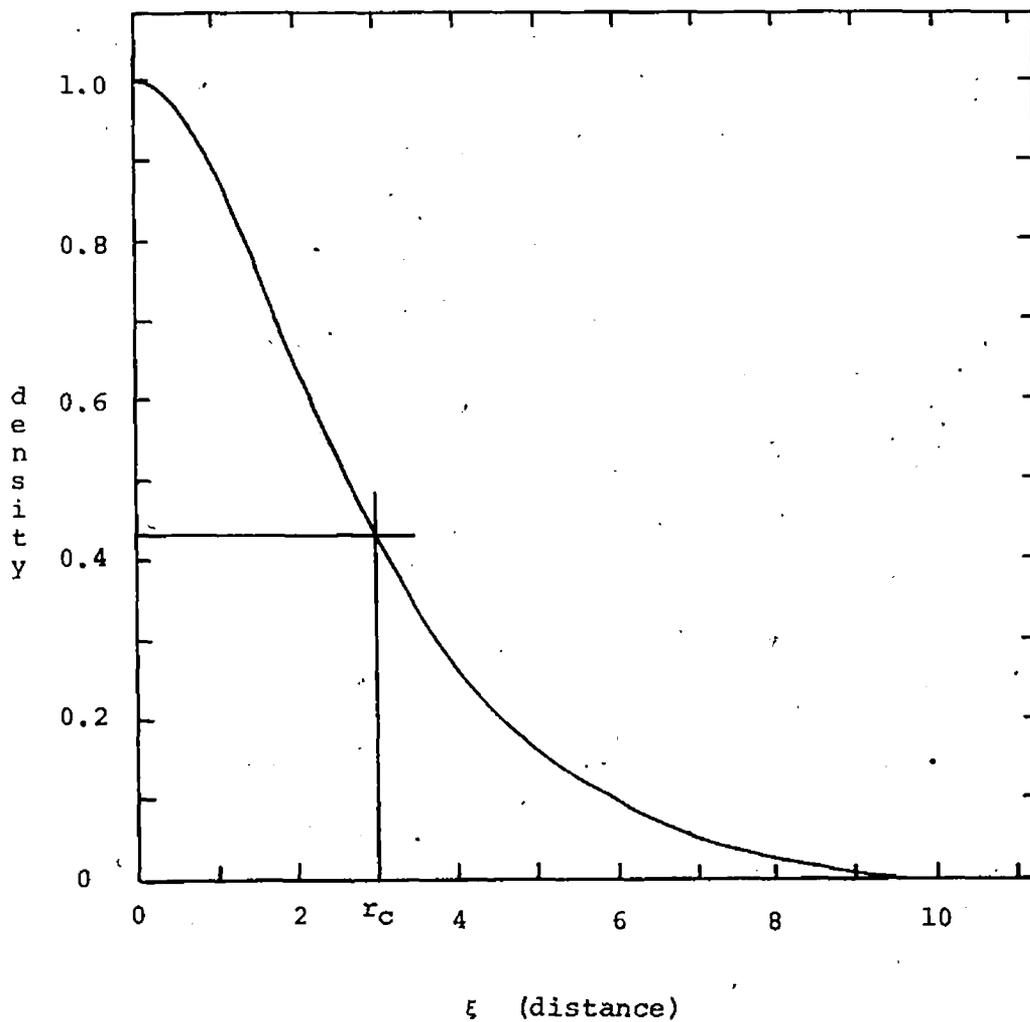
Figure 1   Emden isothermal gas sphere density
profile projected to two dimensions and bounded at 10

to an isothermal gas sphere, it is not necessarily true
that the particles (galaxies) behave as the particles in a
perfect isothermal gas sphere. The only justifications for
using this model are that it fits well to the observed data
and enables definition of a useful parameter, r_c (and so
R_c). Undoubtedly other mathematical relations would do
just as well, and could also (or alternatively) be used.
(Two other relations that also fit well are given by King
1966 and de Vaucouleurs 1960.) The major criteria in
choosing a mathematical relation are its ability to give a
good mathematical fit and a structural size parameter.

The major purpose of this thesis is to obtain
core radii for four rich galaxy clusters at low redshift
(A2052, z=0.0351; A2593, z=0.044; A2626, z=0.055; and A154,
z=0.056). When combined with Bahcall's results these radii
will possibly provide an improved average value and standard
deviation for R_c. If it turns out that the standard
deviation in R_c is small for low redshift clusters then the
assumption can be made that R_c is nearly constant. In that
case, deviations in R_c at high redshifts from this constant
value (assuming the deviations occur in a systematic manner)
can be assumed to be due to the value of q_o, which may then
be determined. Another possibility to explain a changing R_c
with z is that clusters evolve dynamically, and that the
more distant clusters have a different radial distribution.
However, since dynamical cluster evolution is poorly under-

stood it is necessary to neglect it. Since the clusters

being studied in this thesis are all nearby and within a

narrow range of redshifts, then dynamical evolution is not

expected to be of any importance in comparing with the

results of Bahcall.

Computer Program Development

A major task was the writing of a computer program that uses the data (in the form of the numbers of galaxies in rings centred on the cluster centre and the corresponding ring sizes) to find the parameters leading to a best fit with a projected isothermal gas sphere. The basis for most of what follows is a technique suggested by Taff (1975).

Since the isothermal gas sphere model is expressed in terms of particle number density, it is necessary to change the observed counts to number densities:

$$\sigma_{obs}(i) = \frac{N_{obs}(i)}{\pi(r_{i+1}^2 - r_i^2)} \qquad (1)$$

where $\sigma_{obs}(i)$ is the observed number density of galaxies in the ith ring; $N_{obs}(i)$ is the observed number of galaxies in the ith ring; and r_i and r_{i+1} are the inner and outer radii of the ith ring. Note that for the first ring (actually a circle) the inner radius, r_1, is equal to zero. The values $N_{obs}(i)$ and r_i are the input data.

Also needed for the construction of a model are the distance from the centre at which these densities occur. These values, $r_{av}(i)$, are taken to be the radii that divide ring (i) into two rings of equal area. So

$$r_{av}(i) = \{(r_{i+1}^2 + r_i^2)/2\}^{\frac{1}{2}} \qquad (2)$$

The expression for an isothermal gas sphere in terms of the observable quantities is

$$\sigma_{calc}(i) = \sigma_c \sigma_{iso}(r_{av}(i)/\alpha) + \sigma_{bg} \qquad (3)$$

where $\sigma_{calc}(i)$ is the calculated projected number density corresponding to $r_{av}(i)$; σ_c is the projected central number density; and σ_{bg} is the background number density added to the model cluster. The function $\sigma_{iso}(r_{av}(i)/\alpha)$ gives the projected normalized density of the isothermal gas sphere alone at the unitless distance $r_{av}(i)/\alpha$.

If we let

$$x_i = r_{av}(i)/\alpha$$

then the expression for σ_{iso} is given by

$$\sigma_{iso}(x_i) = \frac{\int_{x_i}^{x_o} \sqrt{\xi^2 - x_i^2} \, e^{-\psi} \, \psi' \, d\xi}{\int_0^{x_o} \xi \, e^{-\psi} \, \psi' \, d\xi} \qquad (4)$$

(see Chandrasekhar, 1942). Here $\psi' = d\psi/d\xi$ and the quantity $e^{-\psi}$ is the solution to the equilibrium equation for the three dimensional isothermal sphere, as in:

$$e^{-\psi} = \xi^{-2} \, d/d\xi (\xi^2 d\psi/d\xi) \qquad (5)$$

The upper boundary limit x_o in equation (4) is a

convenient cutoff to integration. For globular clusters, the stellar distribution ceases to approximate an isothermal gas sphere at about $\xi=10$, and so for these clusters the limit is set at about $x_0=10$ (Chandrasekhar, 1942). For galaxy clusters x_0 will be allowed to vary to see which value best fits the cluster.

Equation (4) produces a radial density curve of the type shown in Figure 1, where the projected density becomes zero at x_0.

Numerical values for $e^{-\psi}$, ψ', and ξ can be obtained from various sources. The ones for this thesis were generated by a BASIC program (see Appendix B for a listing) which calculated values at increments of ξ by using the Runga-Kutta method on equation (5). Then $\sigma_{iso}(x_i)$ can be found by numerical integration.

The above equations leave four parameters to be determined for a best fit with the isothermal gas sphere, namely: σ_c, the central density; σ_{bg}, the background density; α, the scale factor; and x_0, the upper limit of integration.

The method of obtaining these is, to some extent, dependant on the procedure used to test the goodness of fit of the model. A common procedure, and the one used here, is the χ^2 test. Besides the fact that the minimum of χ^2 is a well defined indicator of the best fit, there is the advantage that the value of χ^2 can be used to estimate the probability of this specific χ^2 occurring randomly.

The usual way of expressing χ^2 is

$$\chi^2 = \sum (o_i - t_i)^2 / t_i$$

where t_i is the ith theoretical value and o_i is the ith observed value. In the present case we get

$$\chi^2 = \sum \{N_{obs}(i) - N_{calc}(i)\}^2 / N_{calc}(i) \qquad (6)$$

where $N_{calc}(i)$ is the number of galaxies predicted for the ith ring from the equation

$$N_{calc}(i) = \sigma_{calc}(i) \pi (r_{i+1}^2 - r_i^2) \qquad (7)$$

From equations (1) and (3) the equation for χ^2 becomes

$$\chi^2 = \sum \pi (r_{i+1}^2 - r_i^2) \frac{\{\sigma_{obs}(i) - \sigma_c \sigma_{iso}(x_i) - \sigma_{bg}\}^2}{\sigma_c \sigma_{iso}(x_i) + \sigma_{bg}} \qquad (8)$$

This form possesses only three unknown factors: $\sigma_{iso}(x_i)$, σ_c, and σ_{bg}. If the assumption is made, for the moment, that the set of values σ_{iso} is known, then the equation becomes one with two unknown constants, whose values can be found through the minimization of χ^2 with respect to each of them. Since equations of the form of (8) cannot be solved analytically, a numerical method must be used. The method chosen is the Newton-Raphson method, which states

$$\begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} - \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix}^{-1} \begin{pmatrix} f \\ g \end{pmatrix} \qquad (9)$$

when $f(x,y)=0$ and $g(x,y)=0$. In equation (9): $f_x=\partial f/\partial x$; $f_y=\partial f/\partial y$; $g_x=\partial g/\partial x$; $g_y=\partial g/\partial y$; $f=f(x,y)$; and $g=g(x,y)$. All expressions involving f and g and their partial derivatives are evaluated at x_i and y_i.

This method is an iterative one which, given sufficiently accurate initial estimates for the quantities to be found, will quickly converge to the correct value. Since the initial equation (8) has sums of squares over a number of rings, the number of solutions is greater than one. However, if the initial estimates are close to the physically correct solutions, then these solutions will be found.

In this case, to satisfy the conditions for equation (9), and due to the fact that solutions will be found by minimization of χ^2 as expressed in equation (8), the following relations are used. Defining

$$\sigma_c \equiv x_i \text{ (successively)}$$

$$\sigma_{bg} \equiv y_i \text{ (successively)}$$

$$\beta_i \equiv r_{i+1}^2 - r_i^2$$

$$\varepsilon_i \equiv \frac{\sigma_{obs}(i) - \sigma_c \sigma_{iso}(x_i) - \sigma_{bg}}{\sigma_c \sigma_{iso}(x_i) + \sigma_{bg}}$$

and

$$\mu_i \equiv \frac{1}{\sigma_c\sigma_{iso}(x_i)+\sigma_{bg}} + \frac{2\{\sigma_{obs}(i)-\sigma_c\sigma_{iso}(x_i)-\sigma_{bg}\}}{\{\sigma_c\sigma_{iso}(x_i)+\sigma_{bg}\}^2} +$$

$$\frac{\{\sigma_{obs}(i)-\sigma_c\sigma_{iso}(x_i)-\sigma_{bg}\}^2}{\{\sigma_c\sigma_{iso}(x_i)+\sigma_{bg}\}^3}$$

We can further define the terms of equation (9) as:

$$f \equiv \partial\chi^2/\partial\sigma_c$$
$$g \equiv \partial\chi^2/\partial\sigma_{bg}$$
$$f_x \equiv \partial^2\chi^2/\partial\sigma_c^2$$
$$f_y=g_x \equiv \partial^2\chi^2/\partial\sigma_c\partial\sigma_{bg}$$
$$g_y \equiv \partial^2\chi^2/\partial^2\sigma_{bg}^2$$

The terms of equation (9) can now be written in terms related to the isothermal gas sphere:

$$f = \sum\beta_i\sigma_{iso}(x_i)(2\epsilon_i+\epsilon_i^2)=0$$

$$g = \sum\beta_i(2\epsilon_i+\epsilon_i^2)=0$$

$$f_x = 2\sum\beta_i\sigma_{iso}^2(x_i)\mu_i$$

$$f_y=g_x = 2\sum\beta_i\sigma_{iso}(x_i)\mu_i$$

$$g_y = 2\sum\beta_i\mu_i$$

Furthermore, solving equation (9) gives

$$x_{i+1}=x_i - \frac{gf_y - fg_y}{f_x g_y - f_y^2}$$

and
$$y_{i+1}=y_i + \frac{ff_x - gf_x}{f_x g_y - f_y^2}$$

However, to get this far the assumption was made that in equation (8) the set of values σ_{iso} was known, which means that x_0 and α must first be chosen.

Initially x_0 is set to 10 and α to $0.999r_{av}(1)/x_0$, which allows the calculation of σ_{iso}, and so allows σ_c, σ_{bg}, and χ^2 to be found. Then α is incremented by increasing $\log\alpha$ in steps of 0.08. This is continued until either $\log\alpha = \log\alpha_{initial}+4$ or until the results for σ_c and σ_{bg} arising from the $\alpha-x_0$ combination become physically unreasonable. During the process of incrementing α the χ^2 values drop to a minimum and then rise again. The values for α, σ_c, and σ_{bg} that produce the minimum χ^2 are the ones producing the best fitting isothermal gas sphere model for the x_0 used.

A new x_0 is obtained by adding 10 to the previous value, a new α is calculated and incremented as for $x_0=10$, and new σ_c, σ_{bg}, and χ^2 values are found for each α; the usual maximum for x_0 is 200.

What is produced is a set of values of x_0, for each of which exists a set of "chosen" values of α and the

values of σ_c, σ_{bg}, and χ^2 resulting from the Newton-Raphson method. From the set of χ^2_{min} (minimum χ^2 for a specific x_o) the x_o which produces χ^2_{absmin} (the minimum χ^2_{min}) is found. χ^2_{absmin}, therefore, determines the four parameters which produce the best fitting isothermal gas sphere model.

Since values of α are chosen in discrete steps it is probable that the minimum χ^2 for a given x_o will occur somewhere between two α values. However, since χ^2 decreases monotonically to a minimum and then rises again in a similar fashion, a simple approach to look for a "better" α is adopted. The two consecutive values of α giving the lowest χ^2 values are averaged and this average is used to get new σ_c, σ_{bg}, and χ^2. The χ^2 found for the new α is always lower than at least one of the two original χ^2 values. The new α is then averaged with the α giving the smallest of the two original χ^2 values to get another α. This new α value is used to get an even smaller χ^2 value. The procedure of averaging the αs that produce the two smallest χ^2 values is repeated twenty times, at which point successive differences in all other parameters occur only in the fifth or higher significant digit.

A general schematic of what the computer program must be designed to do can be drawn up:

$$N_{obs}(i), r_i \quad \text{-observed galaxy counts and ring sizes}$$

Input

$\xi, e^{-\psi}\psi' \quad$ -model three dimensional gas sphere

$\sigma_c(1), \sigma_{bg}(1) \quad$ -initial estimates for central and background densities

$\sigma_{obs}(i)$ calculated $\qquad$ -equation (1)

$r_{av}(i)$ calculated $\qquad$ -equation (2)

Begin iteration to get minimum χ^2

$x_o=10$ to 200 in steps of 10

$\log\alpha=\log\{0.999r_{av}(1)/x_o\}$ to $4+\log\{0.999r_{av}(1)/x_o\}$ in steps of 0.08

$\sigma_{iso}\{r_{av}(i)/\alpha\}$ calculated from integration subroutine (QSF in IBM's Scientific Subroutine Package) and isothermal sphere densities -equation (4)

For each x_o

for each α

σ_c } calculated by
σ_{bg} } Newton-Raphson method

χ^2 calculated equation (8)

From two smallest χ^2 values, iter-

ative averaging of associated α

values gives a smaller χ^2 and

"better" associated values for α,

σ_c, and σ_{bg}

    The computer program is written in Fortran IV and

a listing is provided in Appendix A, with detailed notes in

Appendix C.

    Another way to approach the problem is to obtain

a value for σ_{bg} by counting galaxies on an area of the

photographic plate removed from the cluster. In this case

only x_o, α, and σ_c are left to be found as free parameters.

If the assumption is again made that σ_{iso} is known (see

equation 8) then only σ_c is left to be found. The Newton-

Raphson method can again be used to find σ_c iteratively,

the form for one unknown is

$$x_{i+1}=x_i-f/f_x$$

when $f(x)=0$. In this equation $x_i=\sigma_c$ (successively);

$f=d\chi^2/d\sigma_c$; $f_x=d^2\chi^2/d\sigma_c^2$; $f=f(x)$; and f and f_x are evaluated

for x_i .

    The only changes this would make in the schematic

is that instead of σ_c and σ_{bg} being calculated, only σ_c is

found, and σ_{bg} is entered as part of the input. A program

was written for each method, total results from both are

presented in Appendix H, partial results (including core radii in Mpc) are presented in Chapter V. Modifications to the original program to get one for the second method are listed in Appendix B and explanations of these changes are in Appendix C.

Program Testing

i) Fits to model isothermal gas sphere

For initial testing of the computer program a data set was fabricated which described a projected isothermal gas sphere with known values of x_0, α, σ_c, and σ_{bg}. It was expected that if the program was working properly the Newton-Raphson method would cause convergence to the correct σ_c and σ_{bg} values and that the iterative dividing method for α would provide a minimum χ^2 for the correct α.

The values chosen to produce the data for this test were: $x_0=10$; $\alpha=2.40$ arcmin; $\sigma_c=0.50$ galaxies/arcmin^2; and $\sigma_{bg}=0.05$ galaxies/arcmin^2.

Figure 2 displays part of the results. The three curves are constructed from the σ_c and σ_{bg} values to which the program converges at the stated α values. These are only three representative cases; many more α values were produced than are displayed in Figure 2 but the trend with changing α is as shown.

As can be seen from this diagram a small value of α tends to produce a compressed gas sphere model and increasingly larger values give increasingly extended models. At large α values, σ_{bg} values are eventually produced which are large and negative and are obviously physically unreasonable.
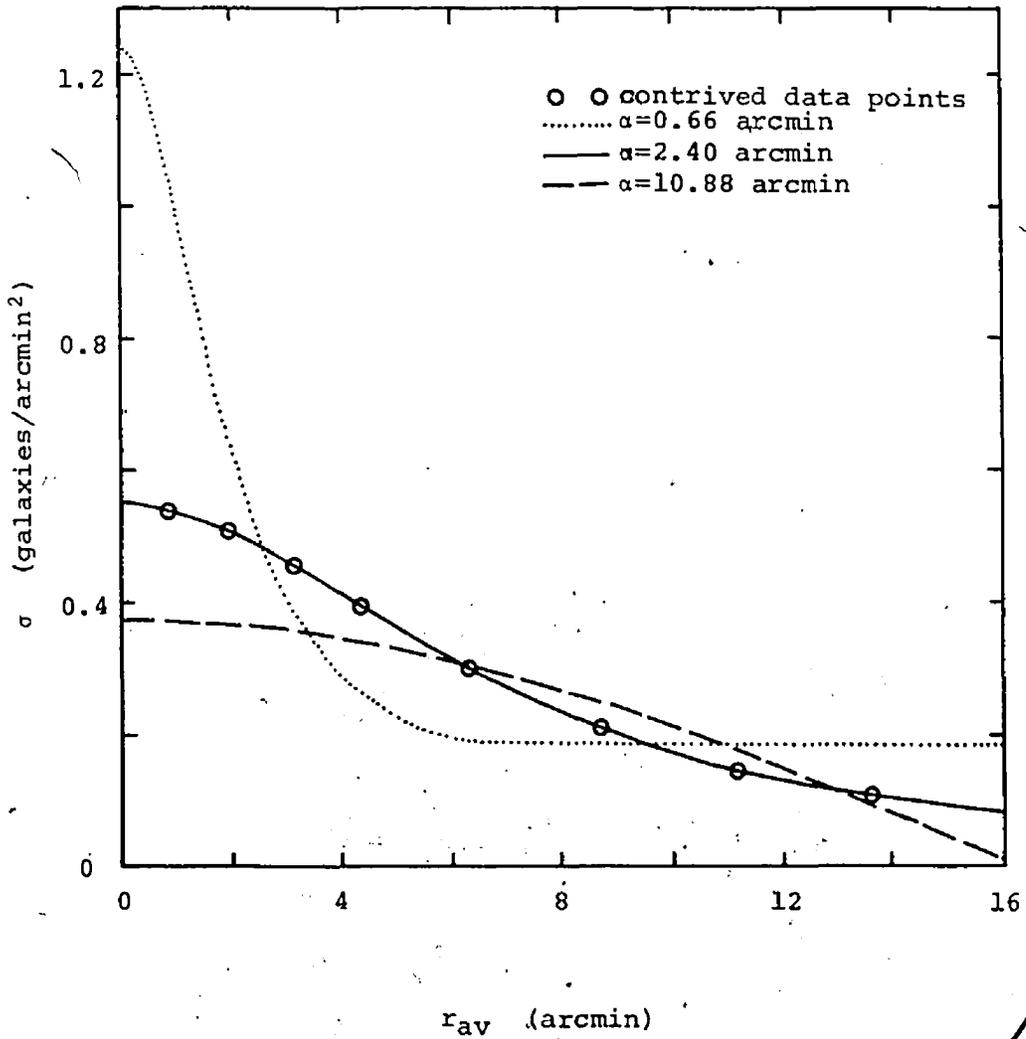
Figure 2 Fits to isothermal gas sphere data

Table 1

Results of fits to isothermal gas sphere data

$(x_O=10)$

α	σ_c	σ_{bg}	χ^2
.1048	17.55	.2241	35.30
.6610	1.055	.1887	19.26
1.149	.7635	.1463	5.772
1.660	.5840	.1067	1.001
2.400	.5000	.05000	3.944(-8)*
4.171	.5398	-.1031	1.356
10.48	1.514	-1.139	4.251

* $3.944(-8)=3.944\times10^{-8}$ This notation
is used elsewhere in this thesis

Units: α-arcmin
σ_c-galaxies/arcmin^2
σ_{bg}-galaxies/arcmin^2

Table 2

χ^2_{min} results for various x_O

x_O	α	σ_c	σ_{bg}	χ^2_{min}
10	2.400	.5000	.5000(-1)	3.944(-8)
40	2.358	.5572	-.5711(-2)	2.160(-4)
70	2.358	.5637	-.1226(-1)	2.218(-4)
100	2.358	.5664	-.1493(-1)	2.203(-4)
130	2.357	.5681	-.1642(-1)	2.278(-4)
160	2.357	.5689	-.1774(-1)	2.202(-4)
190	2.358	.5696	-.1814(-1)	2.209(-4)

Units: as in Table 1

Columns 2 to 4 of Table 1 show the σ_C, σ_{bg}, and χ^2 values that result from the α values in column 1. These particular results are all calculated with $x_O=10$ and are only a small sample of the total but are typical of the values for other integration cutoff limits. Rows 2, 5, and 7 of this table correspond to the three curves of Figure 2. It must be pointed out that to get $\sigma(r_{av}=0)$ the values of σ_C and σ_{bg} must be added.

One of the **first** things obvious from Table 1 is that the program did converge to the correct values, producing the extremely good fit for the $\alpha=2.400$ case. Also as α increases σ_C decreases to a minimum in the neighbourhood of the correct α. A more comprehensive version of Table 1 shows that the minimum σ_C is not reached exactly at $\alpha=2.400$ but at $\alpha=2.886$, the next incremental value of α. For other x_O values the σ_C also reaches a minimum just after the **minimum** χ^2, and rises again as α continues to increase. The background density, however, decreases monotonically with increasing α and eventually becomes negative, a physically unreasonable possibility. The χ^2 is found to decrease monotonically to a minimum at the correct α, σ_C, and σ_{bg} combination, and then rise again monotonically with the rate of increase being less than that of decrease.

Table 2 shows the α, σ_C, and σ_{bg} values producing the χ^2_{min} for the given x_O values. It can be seen that α changes to ~ 2.358 and remains roughly constant as soon as

x_O exceeds 10, and σ_c slowly increases as σ_{bg} slowly decreases at about the same rate. In fact, the sum of σ_c and σ_{bg} produces a minimum of 0.5500 for $x_O=10$ and averages about 0.5515 for the other integration limits, with the sum of 0.5523 for $x_O=100$ being an extreme case. Aside from the absolute minimum for $x_O=10$, χ^2_{min} maintains a fairly constant value as x_O increases. The product $\alpha\sigma_c$, which Bahcall (1972) finds to remain fairly constant for various χ^2_{min}, has a minimum of 1.200 for $x_O=10$ and rises slowly to 1.343 for $x_O=190$, a change of only 12%.

Table 2 indicates that the χ^2_{min} values of α and σ_c arrived at by the fitting process are fairly insensitive to the choice of x_O, even if the choice is far from the one producing χ^2_{absmin}.

Initial testing described above showed that the program successfully converged to the correct parameters when given an artificial data set generated from a projected, bounded isothermal gas sphere.

ii) Comparison with results published by Taff for the
.Perseus cluster and A2199

Secondary testing involved running the program using published data and comparing the results to those

published for these data. Since the program was written

following the technique suggested by Taff (1975) it is

presumably similar to the one used by him, and since the

program and Taff used the same data his results form the

basis for comparison.

Results for the Perseus cluster are found in

Table 3, the data used is from Bahcall (1974). Case (a)

uses counts with galaxies brighter than 16^m0 and case (b)

uses counts with galaxies brighter than 17^m5; in both cases

the number and size of the rings used for counting were the

same. For each case there are three lines of values: the

top line gives Taff's results with the value in parentheses

to the right of the χ^2 column being the χ^2 found by this

program when forced to fit the data to the model made with

Taff's values for x_0, α, σ_c, and σ_{bg}; the second line gives

the χ^2_{absmin} parameters produced by this program; and the

third line consists of the χ^2_{min} parameters produced by this

program using the minimizing value of x_0 found by Taff.

The large discrepancy between Taff's χ^2 and the

one calculated by this program from his parameters may be

due to differences in the fitting procedures. However, if

the value of x_0 is set equal to the best fit value found by

Taff, the values of the three other independant variables

(i.e. α, σ_c, and σ_{bg}) are close to those of Taff, as seen

in row three of each case.

A plot of x_0 versus χ^2_{min} is shown in Figure 3 to

Table 3

Perseus cluster results

	x_0	α	σ_c	σ_{bg}	x^2	
(a)	10	2.89	7.28(-2)	4.24(-3)	1.28	(22.14)
	160	1.95	1.03(-1)	3.19(-3)	6.49	
	10	2.98	7.82(-2)	6.37(-3)	6.78	
(b)	10	3.47	1.33(-1)	1.12(-2)	3.35	(12.55)
	20	2.92	1.65(-1)	1.07(-2)	3.07	
	10	4.28	1.22(-1)	1.14(-2)	4.36	

Units: α-arcmin
σ_c-galaxies/arcmin^2
σ_{bg}-galaxies/arcmin^2
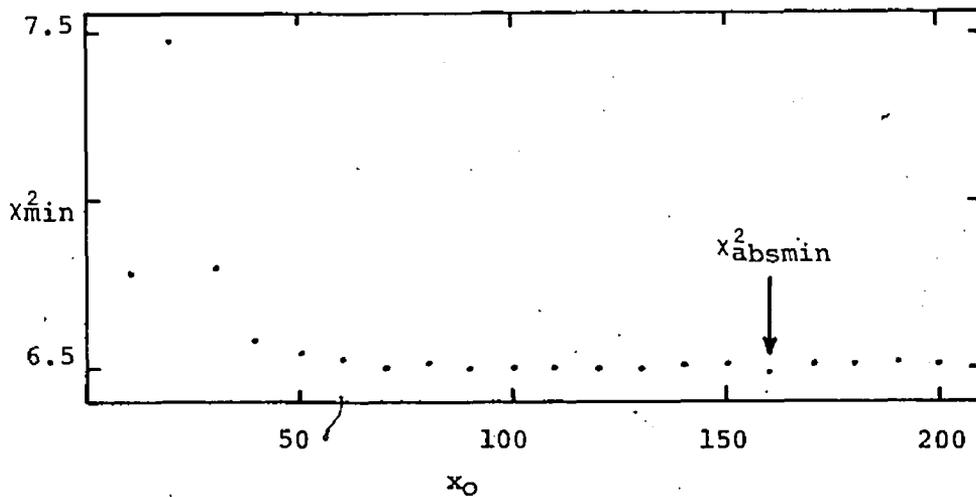
Figure 3

Plot of x^2_{min} to x_0 for case (a) of the Perseus cluster

illustrate the fact that fits can be rather insensitive to the value of x_0. This was also found in the initial testing. Beyond a certain value of x_0 (about 40 in this case), the x^2_{min} values appear to fluctuate randomly, here about a value of -6.5. The fact that x^2_{absmin} occurs at $x_0=160$ is not considered significant, that is, the minimizing value of x_0 does not seem to be well determined. This raises the question of how well determined x^2_{absmin} and x_0 are, a question that will be discussed later.

A graph similar to Figure 3 for case (b) shows the largest x^2_{min} for $x_0=10$, the smallest for $x_0=20$, and increasingly larger x^2_{min} values up to $x_0=80$, beyond which it fluctuates about $x^2_{min}=3.85$.

Another indication that differences exist between the program described in this thesis and the one used by Taff is that this program did not find, for the Perseus cluster, minimum x^2 values using Taff's best fit parameters α, σ_c, and σ_{bg} at the values of x_0 cited by Taff. In case (a) use of Taff's parameters produced x^2_{absmin} of 7.91 at $x_0=40$ (as opposed to $x^2=22.14$ at $x_0=10$) and case (b) $x^2_{absmin}=6.26$ at $x_0=20$ (as opposed to $x^2=12.55$ at $x_0=10$).

It should be pointed out that although this program produces a x^2_{min} that fluctuates about a certain value for large x_0 values (as in Figure 3), the variations of x^2 on x_0 found when the program uses Taff's α, σ_c, and σ_{bg} shows a pronounced minimum, as illustrated in Figure 4.
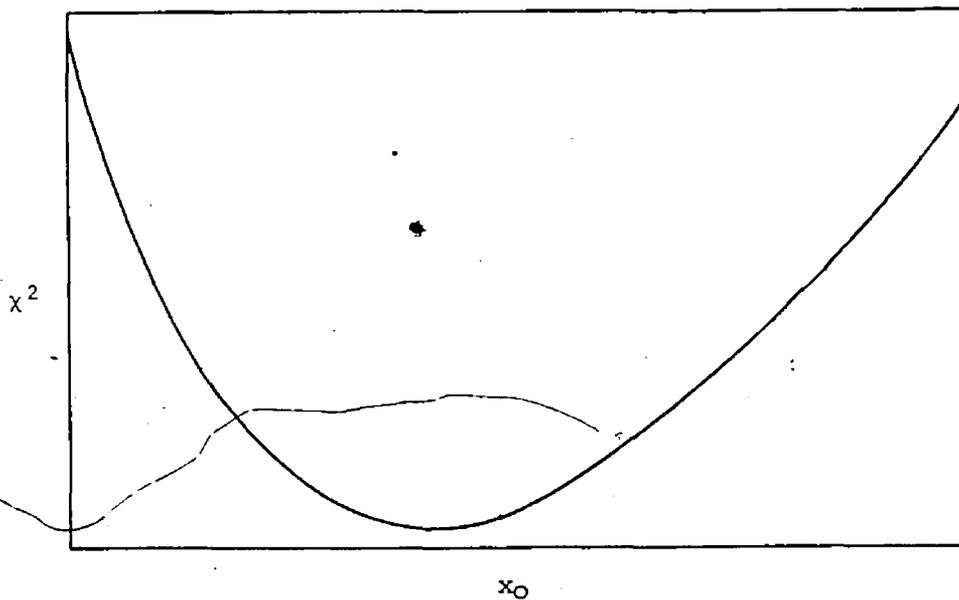
Figure 4 χ^2 versus x_0 for fixed values of α, σ_c, and σ_{bg}.

Table 4

Comparison of α values (in arcmin) for the Perseus cluster

case	mag. limit	Taff	Bahcall	this program
(a)	$16^m.0$	2.89	2.9	1.95
(b)	17.5	3.47	2.7	2.92

This difference in behaviour arises from the fact that in the latter case x_0 is the only remaining variable, whereas normally the parameters α, σ_c, and σ_{bg} are all varied for each x_0.

Table 4 compares the best fit α values found by Taff, Bahcall, and this program. The α values for Taff and this program have been given in Table 3, and Bahcall's results are published with her data (Bahcall, 1974). The case (a) result from this program is significantly different from the other two, but this α value occured when $x_0=160$ (see Table 3). For $x_0=10$, Taff's best fit value, this program finds $\alpha=2.98$. The case (b) result for this program lies between Taff's and Bahcall's values, and so is not significantly different.

Table 5 compares the best fitting models of the cluster A2199. Data for the calculations were obtained from Bahcall (1973). Again, the top line in each case gives Taff's results, the second line those of this program, and the third line this program's results at Taff's best fit x_0. A third line is omitted when this program and Taff agree on the best fit x_0. Cases (a) and (d) use galaxy counts down to $17\overset{m}{.}5$; (b) and (e) use galaxy counts down to $18\overset{m}{.}5$; and (c) and (f) use galaxy counts down to $19\overset{m}{.}0$. Also, cases (a), (b), and (c) use 15 rings out to $30'$ on a 103a-D plate while cases (d), (e), and (f) use 26 rings out to $58\overset{.}{.}24$ from the cluster centre on a IIIa-J plate.

Table 5

A2199 results

	x_O	α	σ_c	σ_{bg}	x^2	f
(a)	30	.897	.525	1.31(-2)	7.22	(18.07)
	20	1.236	.393	2.11(-2)	7.40	
	30	.971	.507	2.01(-2)	7.41	
(b)	20	.931	.721	8.65(-2)	3.29	(6.93)
	30	1.073	.633	8.45(-2)	4.10	
	20	1.233	.558	8.78(-2)	4.15	
(c)	200	.135	9.84	3.63(-1)	50.2	(10.27)
	150	1.261	.768	3.51(-1)	6.33	
	200	1.261	.770	3.50(-1)	6.33	
(d)	200	.262	1.85	6.27(-3)	18.0	(68.53)
	200	.455	1.12	8.35(-3)	24.11	
(e)	200	.262	3.38	1.36(-2)	19.5	(66.48)
	200	.386	2.61	1.58(-2)	24.17	
(f)	200	.262	4.20	2.86(-2)	14.5	(45.39)
	200	.315	4.19	3.25(-2)	19.46	

Table 6
Case (e) extended from Table 5

x_O	α	σ_c	σ_{bg}	x^2
500	.1730	5.994	1.242(-2)	20.47
700	.1564	6.567	9.293(-3)	19.90
1000	.1520	6.722	5.922(-3)	19.71

Units for Tables 5 and 6:　α - arcmin
σ_c - galaxies/arcmin^2
σ_{bg} - galaxies/arcmin^2

For cases (a), (b), and (c) the χ^2_{min} values begin
fluctuating at x_0 equal to about 20, 20, and 80 respectively.
When constrained to Taff's results this program produced
χ^2_{absmin} at $x_0=70$ for case (a) and at $x_0=30$ for case (b).
Case (c) has χ^2_{absmin} for Taff's parameters at 200, where it
was still decreasing. The last three cases show χ^2_{min} for
Taff's parameters and for this program's results still
decreasing at $x_0=200$, which implies that χ^2_{absmin} actually
occurs beyond this limit. In fact, case (e) was extended
out to $x_0=1000$ and was still decreasing, but very slowly.
From $x_0=700$ to $x_0=1000$ χ^2_{min} decreased from 19.90 to 19.71,
a decline of less than 1%. Table 6 gives the χ^2_{min}
parameters for the cases $x_0=500$, 700, and 1000. For these
large x_0 values the parameters are changing very slowly.
Although it is possible to extend the program beyond
$x_0=1000$, for the testing it was not deemed necessary.

Case (c) in Table 5 is anomalous in that Taff's
results differ by almost an order of magnitude from his
results for the first two cases. However, the parameters
for the last three cases found by Taff and this program
change in more or less the same manner from case to case,
as do this program's results for the first three cases. It
appears that Taff's results may be in error for case (c).

For the Perseus cluster the α values found by this
program appear to be somewhat smaller than those of Taff,
but for A2199 Taff's values are consistently smaller. It

was hoped that a study of Taff's computer program could be made to determine the reason for these differences. However it was not possible to obtain a copy of his program.

In general, fairly good agreement is found between results from this program and those published by Taff and Bahcall. However, a possible problem arises because of the fluctuations of χ^2_{min} with changing x_o. Figure 5 illustrates ways in which χ^2_{min} is found to vary with x_o in the tests described above.

In Figure 5 the ordinate represents a possible range of χ^2_{min} values for the range of x_o along the abscissa. In cases 5(a) to 5(d) the choice of χ^2_{absmin} is obvious, but in case 5(e) there are several choices since more than one χ^2_{min} have the same minimum value (within truncation limits). It was decided to take as χ^2_{absmin} the first value arrived at (i.e. that with the lowest x_o value) because i) there may be an indefinite series of χ^2_{absmin} as x_o increases, and since an arbitrary choice must be made, the first will be chosen; and ii) the α values for similar values of χ^2_{min} are nearly identical, as will be seen later.
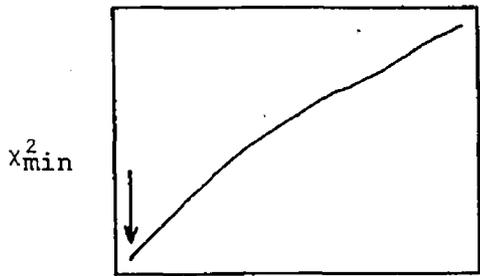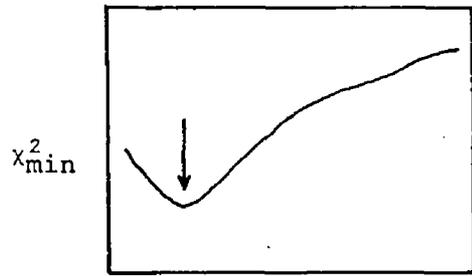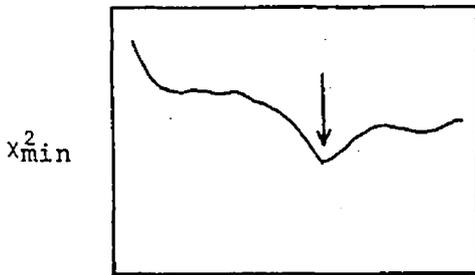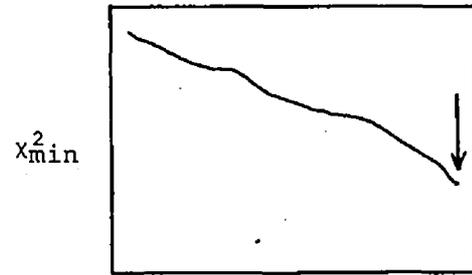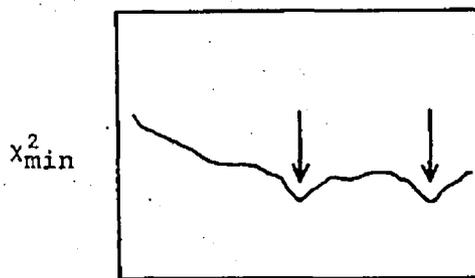
Figure 5 Possible variations of χ^2_{min} with respect to x_0. All arrows point to χ^2_{absmin}.

iii) Comparison with results published by Bahcall for

twelve clusters

A third test of the program was made by comparing results of Bahcall with those of this program using her data. In one of her papers, Bahcall (1975) lists ring counts and ring sizes as well as core radii for twelve clusters. A tabulation of results is shown in Table 7. For this table all α values were converted to core radii in Mpc through the equation

$$R_c (Mpc) = 5.25 \alpha z (1+z)^{-2}.$$

In Table 7 (B) identifies Bahcall's results and (C) identifies the results of this program.

If the two columns of R_c values are compared no systematic differences can be seen. However this program's values are occasionally quite different from those of Bahcall. Two notable examples are A2052 and A2319. These clusters also happen to have, probably not coincidentally, χ^2_{min}-x_0 relations different from the others, which tend to resemble one of the relations shown in Figure 5. The two anomalous relations are shown in Figure 6.

For A2052 the first χ^2_{min} (for $x_0=10$) is much smaller than the rest. The corresponding R_c is 0.43 Mpc. For $x_0=20$ through $x_0=200$ $R_c=0.35$ or 0.36 Mpc, much nearer Bahcall's value. But although the core radius is almost

Table 7

Results of fits to Bahcall's data

Cluster	z	$R_C(B)$	$R_C(C)$	$\chi^2_{absmin}(B)$	$\chi^2_{absmin}(C)$
A194	.0181	.23	.13	1.9	0.50
A1367	.0205	.34	.35	1.9	1.84
A2052	.0351	.28	.43	3.2	1.38
A2319	.0549	.22	.02	2.4	1.43
A2256	~.06	.20	.17	7.5	5.84
A401	.075	.24	.19	2.3	1.43
A1775	.0718	.26	.18	1.3	0.15
A1904	.0719	.24	.24	0.3	0.18
A2065	.0722	.29	.33	11.6	8.63
A2029	.0777	.27	.28	1.3	1.27
A1795	.063	.25	.22	0.5	0.12
A1132	.134	.20	.23	2.3	1.81

Both $R_C(B)$ and $R_C(C)$ are in Mpc

$<R_C(B)> = 0.25 \pm 0.04$ Mpc

$<R_C(C)> = 0.23 \pm 0.11$ Mpc

constant for all x_O values greater than 10, it is this

first value that is used since it produces x^2_{absmin}.

The x^2_{min} values for A2319 decrease out to $x_O=60$,

are nearly constant out to $x_O=160$, and decrease again at

least as far as $x_O=200$, where x^2_{absmin} occurs. For $x_O=60$

and 160, $R_c=0.11$ Mpc, as well as for most cases in between.

However, for the x^2_{absmin} at $x_O=200$, $R_c=0.02$ Mpc. From

Figure 6(b) it is expected that x^2_{absmin}, and possibly R_c,

will decrease even further if x_O is increased beyond 200,

but this expectation has not been tested.

It is also seen in Table 7 that the x^2_{absmin}

values as found by this program are either lower than or

equal to those cited by Bahcall, at least to the accuracy

quoted by her. This means that the parameters found by

this program produce isothermal gas sphere models that fit

the published data better than the parameters found by

Bahcall. However it must be remembered that different

procedures for fitting were used: where this program fit by

changing α, σ_c, and σ_{bg}, the background densities were

fixed as part of the input to Bahcall's program. It is

possible that the background densities found by this

program are vastly different from the actual (counted)

values used by Bahcall, but since she did not publish her

background counts comparison is not possible.
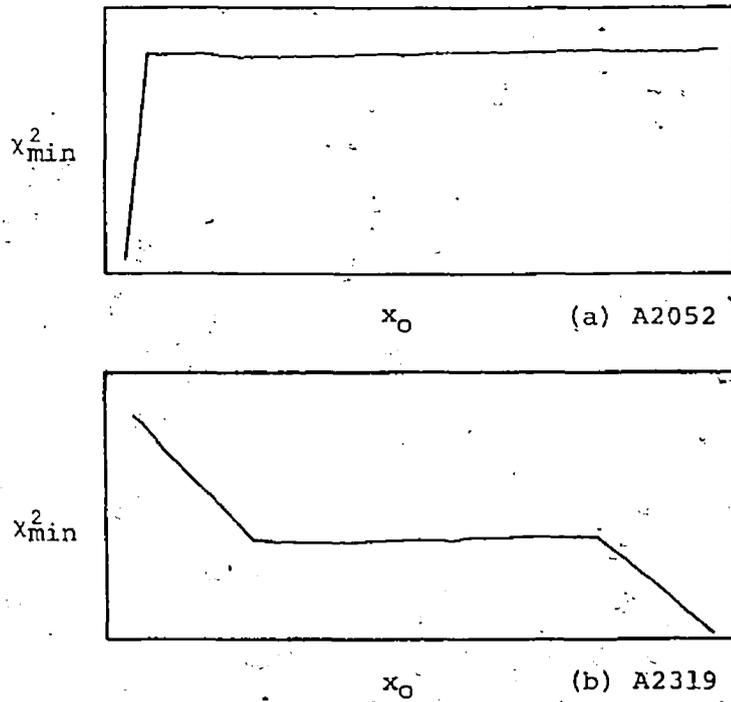
Figure 6 χ^2_{min}-x_0 relations for two clusters

Table 8.

α variations with x_0 for like values of χ^2_{absmin}

Cluster	A1367		A2029	
χ^2_{absmin}	1.835		1.270	
	x_0	α	x_0	α
	60	3.359	110	.8014
	70	3.358	130	.8011
	100	3.357	140	.8007
	150	3.358	150	.8012
	160	3.358	170	.8013
	180	3.357		

Units:
α - arcmin

Two of the clusters of Table 7 showed variations of χ^2_{min} with x_0 which corresponded to the case shown in Figure 5(e). For these two clusters more than one x_0 was found to produce the same value for χ^2_{absmin}. Table 8 below shows how α varies with x_0 for the same values of χ^2_{absmin}.

In each case the R_c values were the same to three decimal places for each value of x_0 shown. This indicates that in cases such as Figure 5(e) the decision to use results from the lowest x_0 producing χ^2_{absmin} will probably not greatly affect the core radius obtained for the cluster.

In summary, although individual values for the core radius may differ from Bahcall's values, the average R_c values are within a standard deviation of each other. It can also be seen that the results of this program show a standard deviation over twice that of Bahcall, even though the models of this program are found in all but one case to yield lower χ^2. This result is of interest because it is the standard deviation of the core radius which measures its usefulness as a cosmological metrestick. Further work should be done to determine whether this difference is produced by the different methods of treating the background or whether it originates within the programs themselves.

Observational Material

i) Program input file

        To be used by the program a data file must
consist of:

        1) a line of 80 characters. This line is
reproduced by the program as entered and is placed in the
data file to ensure that the computer terminal is set at
the proper line width and is operating correctly;

        2) the isothermal gas sphere data. These values,
entered in E8.5 format, correspond to the values of $e^{-\psi}\psi'$
in equation (4) and are obtained for the ξ values 0.0, 0.1,
0.2, 0.3, ..., 9.8, 9.9, 10.0, 11, 12, 13, ..., 98, 99,
100, 110, 120, 130, ..., 980, 990, 1000. These 281 values
were produced by a BASIC program reproduced in Appendix B;

        3) another line of characters which describes the
format of the next three items. This line is skipped by the
program;

        4) the number of rings to be used, entered in I2
format;

        5) the number of galaxies in each ring, from the
centre outwards, in F4.0 format;

        6) the outer radius of each ring in arcmin, from
the centre outwards, in F4.2 format;

        7) the initial estimates of central and background

densities in galaxies/arcmin^2, entered respectively in E5.2, 2X, E5.2 format; and

8) a line of characters describing the format of item 7). Since this line and anything following it are ignored by the program it may be omitted.

N.B. If the program version TAFCHEC is being used a line is inserted between items 7) and 8). This contains Taff's values for α, σ_c, and σ_{bg} for the cluster under study. These values are entered respectively in the format E5.2, 2(2X, E5.2).

The necessary data for each cluster are the number of rings, number of galaxies per ring, ring sizes, and initial estimates of σ_c and σ_{bg}. Furthermore, each cluster was studied to three magnitude limits, two on a IIIa-J plate and one on a 103a-D plate, all plates were taken by Dr. G. Welch on the Hale Observatory's 48 inch Schmidt telescope.

ii) Photographic enlargements

The galaxies were identified on the original plates and their images marked on an enlarged photographic print of the cluster. All further work was performed using the print.

To obtain the prints, contact copies were made of the IIIa-J plates of each of the four clusters studied. The Abell radius of each cluster on the copy was calculated through the equation given by Abell (1958)

$$R_{(Abell)} = 4.6 \times 10^5/cz \text{ mm} = 1.53/z \text{ mm.}$$

The contact copies were used to make prints enlarged so that the Abell diameter was just inside the border of the prints, which were 14 by 14 inches. Two identical prints were made for each cluster, one for marking the location of galaxies identified on the IIIa-J plate and another for the 103a-D plate.

The 10X stereo microscope to be used for identifying galaxies on the original plates was found to have a comfortable viewing area of about 10´ by 10´, so the prints were divided into areas of approximately this angular size. These areas were numbered and a BASIC random number generator was used to determine the order in which they would be examined. It was felt that this process would minimize the systematic effect of any time-dependant errors in identifying galaxies. During the course of examining different areas of the print, some of the first areas checked were re-examined to ensure the consistent use of the chosen limiting magnitude.

iii) Limiting magnitudes

For each of the eight plates the objects identified as galaxies were marked on the prints. In all cases identification was made to the plate limit, where the plate limit is defined as the faintest magnitude at which it is possible to distinguish with certainty stellar from galaxian images.

Since this thesis is attempting to augment the work of Bahcall (1975) who counted galaxies within 3^m of the brightest galaxy of each cluster, and since photometry is unavailable for the clusters being studied here, an approximation to Bahcall's 3^m difference must be made.

From the relation

$$\Delta M = 6.00 \log(x)$$

(from Holmberg, 1975) where ΔM is the magnitude difference between two galaxies whose absolute major axes differ by a factor of x, it is found that an axial ratio of 3 corresponds to a magnitude difference of almost 3. The use of such a relation to approximate a magnitude difference is possible in the present case because the galaxies are assumed to be at the same distance. It must be remembered that Holmberg bases his results on an examination of normal galaxies, whereas the brightest members of A2052, A2593, A2626, and A154 have extended halos characteristic of supergiant

galaxies. It is therefore clear that applying the Holmberg
relation will allow a derived magnitude difference to be
only roughly approximated.

The prints used for IIIa-J counts were examined
under a 4X eyepiece with a graduated reticle and the size
of the major axis of the cluster's brightest galaxy was
estimated. All galaxies on this print that had major axes
greater than or equal to 1/3 this size were identified.
Since the galaxies do not have well defined edges, a cutoff
was chosen arbitrarily where the image density lessened
perceptibly from that of its centre. The use of the print
for this identification was necessary because no means were
available to measure the image size on the original plates
with sufficient accuracy.

Visual examination of the prints showed no evidence
of background density variations which could have arisen
during the production of the prints and might introduce
systematic position-dependent variations in the cutoff
density. Also, as will be seen in the next chapter, the
background number densities computed by the program for this
bright limit agrees well with the background densities
counted at the print corners, which suggests that such
errors are not significant.

This process identifies three magnitude limits: the
faintest being that of the IIIa-J plate; the next being that
of the 103a-D plate; and the brightest corresponding to the

size ratio of 1:3 on the IIIa-J plate. No bright limit was
found for the 103a-D plates because the image resolution
was noticibly poorer than on the IIIa-J plates, making the
establishment of a uniform density cutoff more difficult.

iv) Cluster centres from strip counts

The location of the cluster centre corresponding
to each magnitude limit now had to be obtained. Since
previous work has shown that if a cluster possesses a
dominant galaxy it is usually at or near the cluster centre,
it was assumed that such was the case for the clusters
being studied here. Of the four, three have a dominant,
probably cD, galaxy and the other (A154) has a dominant
binary galaxy.

A square grid of strips 1.5 cm by 18 cm was
centred over the dominant galaxy (or between the pair of
A154) and strip counts were taken of all galaxies to the
limit being studied. Counts were made on the print in four
orientations: N-S; E-W; NE-SW; and SE-NW. The estimated
cluster centre for each orientation was the point having
equal numbers of galaxies on either side. The cluster
centre for each magnitude limit was found by averaging the
estimates of each orientation.

Table 9 is a partial result of strip counting. In it is presented the maximum difference between the cluster centres determined from the three limits, both in arcmin and as a fraction of the width of the rings used to tabulate the radial density distribution.

Table 9

Separation of magnitude limit centres

cluster	A2052	A2593	A2626	A154
distance (arcmin)	0.79	0.88	2.08	0.84
dist/(ring width)	0.35	0.42	1.28	0.54

Three of the clusters show all three estimates to lie much closer together than the resolution of the ring counts, but the IIIa-J bright limit estimate for A2626 differs significantly from the other two (the IIIa-J faint and the 103a-D limits for A2626 are 0.31 ring widths apart). The difference is assumed to be real and so the centres for each limit will be taken as those found from the strip counts. The small differences among centre positions is not considered likely to introduce significant differences

in the ring counts and eventually core radii. Complete
results of strip counting are presented in Appendix E.

v) Ring counts

After the centre was chosen for each magnitude
limit a grid of 20 concentric rings, having radii differing
by 7.9 mm, was laid over the centre and ring counts were made.
These counts were performed on one quadrant at a time to
check for major azimuthal density variations that might
suggest a mislocation of the cluster's centre. No such
variations were found. The ring count results are presented
in Appendix F.

Table 10 gives, in arcmin, strip widths and
lengths, the width of each ring, the overall ring radius
(i.e. the radius of the 20th ring), and the Abell radius for
each cluster studied.

The ring sizes and number of galaxies per ring
for each magnitude limit were converted into densities and
average radii (r_{av}, see equation 2), and a plot of density
versus r_{av} was made. A smoothed curve was drawn by eye to
obtain an initial estimate for σ_c.

The initial estimate for σ_{bg} was obtained in a
different manner. Since the actual background density is

Table 10

Strip width and length, ring size, and total and Abell radii

cluster	A2052	A2593	A2626	A154
strip width	4.28	4.04	3.12	2.97
strip length	51.36	48.48	37.44	35.64
ring width	2.24	2.12	1.63	1.55
outer ring radius	44.80	42.40	32.66	31.09
Abell radius	48.96	38.95	31.16	30.60

Units: all values are in arcmin

Table 11

Areas involved in background counts

(see Figure 7)

cluster	A2052	A2593	A2626	A154
one corner area	620.0	587.0	350.4	317.6

Units: all values are in arcmin^2

needed for the program BGIN, which more closely approximates Bahcall's program, it was decided that the estimate should be the background density as obtained from counts.

An area about 9 cm by 9 cm was marked off at each corner of each print (NE, NW, SE, and SW, the prints being so aligned) and the galaxies in each corner were counted to the limits previously discussed. This provided the estimate for σ_{bg}, the counts for these are in Appendix G.

The diagram on the next page (Figure 7) is a scale drawing of the "working features" of the prints used. The concentric circles indicate the 10 and 20 ring sizes and the four corner squares represent the areas used for background counting. It can be seen that the background areas overlap rings out to about fifteen. This is not considered a matter of concern since the density profiles usually reach background levels by the 10th and almost always by the 14th ring.

Of the original list of required data all values are fixed but the number of rings. Since the background was usually just reached by the 10th ring and because data were obtained for all 20 rings, it was decided to run the programs twice for each set of data, once with all 20 rings and once with only the inner 10. The 20 ring case gives higher weight to the background and the 10 ring case emphasizes the cluster but loses information regarding background. It is
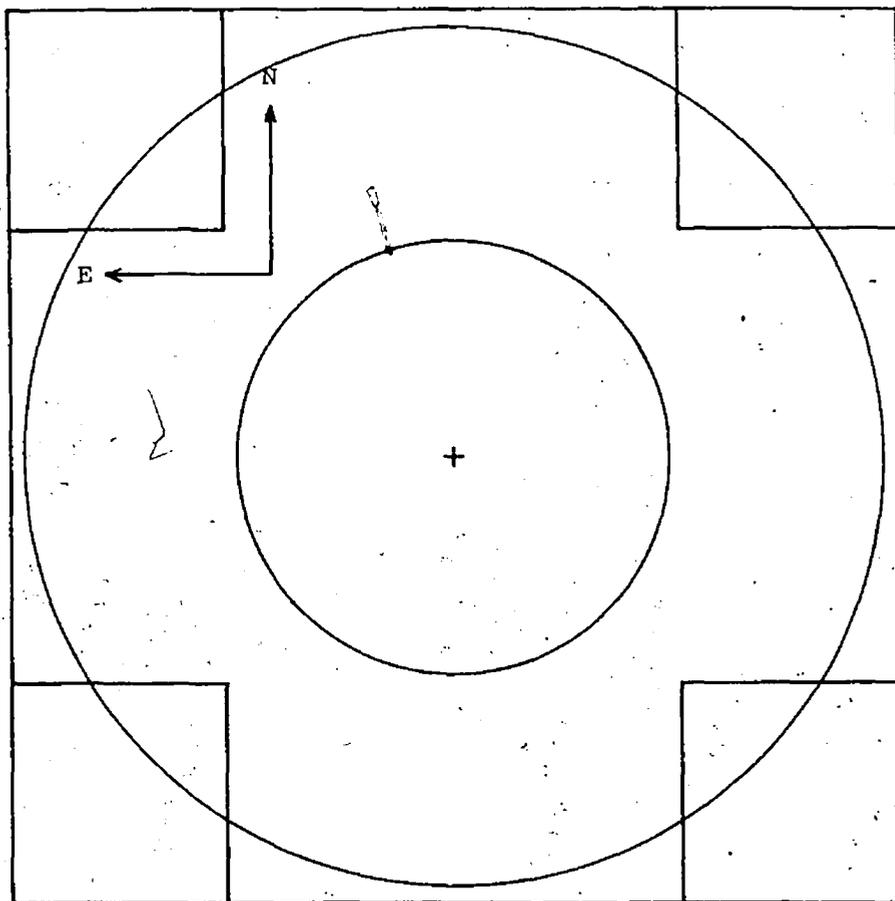
Figure 7 Diagram of the major "working features"
    of the photographic prints drawn to scale

expected that if the program produces realistic fits and the background is uniform throughout the cluster then results from both runs should be similar. For the purpose of this thesis nothing was done with the rings but use either all 20 or just the inner 10. At no time were rings combined in any fashion (e.g. as done by Bahcall, 1975).

Results

i) Results tabulated

A table containing core radii and background densities is presented on the next page, Appendix H contains the complete results. The computer programs used to produce the values for the tables in this chapter do not output linear core radii but give structural lengths, α, in arcminutes. Conversion to linear core radii is done through the relation

$$R_c = 5.25\alpha z(1+z)^{-2} \text{ Mpc.}$$

In Table 12, for each cluster there are three double rows of numbers. The top pair corresponds to the IIIa-J bright magnitude limit (the brightest limit), the second pair to the 103a-D limit, and the third to the IIIa-J faint limit (the faintest limit). Henceforth these limits are to be referred to as the "b", "D", and "f" limits respectively. The top line of each pair presents results obtained when the counts from all 20 rings are used and the bottom line gives the results when the counts from the inner 10 rings are used.

The columns show, from left to right: the emulsion; ML - the magnitude limit; ΔM - the approximate magnitude difference between the b limit and the D and f limits;

Table 12

Final results

Cluster/Emulsion	ML	ΔM	NR	NG	R_C	R_C^*	σ_{bg}	σ_{bg}^*
A2052:								
IIIa-J	b	–	20	58	.037	.031	2.42(-3)	4.44(-3)
			10	40	.023	.030	8.52(-3)	"
103a-D	D	1.6	20	341	.495	.508	4.31(-2)	4.15(-2)
			10	140	.499	.458	8.20(-3)	"
IIIa-J	f	2.3	20	848	.467	.477	1.13(-1)	1.12(-1)
			10	315	.413	.467	1.28(-1)	"
A2593:								
IIIa-J	b	–	20	78	.303	.253	1.19(-2)	6.39(-3)
			10	36	.311	.318	3.15(-3)	"
103a-D	D	1.8	20	678	.554	.481	4.52(-2)	7.75(-2)
			10	312	.656	.672	5.74(-3)	"
IIIa-J	f	2.1	20	882	.829	.847	6.34(-2)	1.13(-1)
			10	359	.794	.780	4.55(-2)	"
A2626:								
IIIa-J	b	–	20	91	.336	.330	1.19(-2)	1.85(-2)
			10	40	.365	.334	9.67(-3)	"
103a-D	D	1.8	20	697	.408	.299	1.67(-1)	2.14(-1)
			10	228	.268	.251	2.02(-1)	"
IIIa-J	f	2.3	20	1528	.378	.313	3.79(-1)	4.44(-1)
			10	498	.280	.250	5.36(-1)	"
A154 :								
IIIa-J	b	–	20	95	.167	.178	1.79(-2)	1.65(-2)
			10	40	.014	.178	3.77(-2)	"
103a-D	D	1.3	20	498	.133	.144	1.05(-1)	1.06(-1)
			10	187	.016	.146	1.70(-1)	"
IIIa-J	f	1.5	20	634	.051	.070	1.51(-1)	1.37(-1)
			10	232	.027	.038	1.71(-1)	"

Units: ΔM – magnitudes

R_C and R_C^* – Mpc

σ_{bg} and σ_{bg}^* – galaxies/arcmin^2

NR – the munber of rings used; NG – the number of galaxies included; R_c – the core radius in Mpc obtained when σ_{bg} is treated as a free parameter; R_c^* – the core radius in Mpc obtained using the observed background density as a fixed value; σ_{bg} – the background density in galaxies/arcmin^2 obtained by treating this density as a free parameter; and σ_{bg}^* – the observed background density in galaxies/arcmin^2, a value that is the same for the 20 and 10 ring cases for a given magnitude limit. In addition, the cluster to which each set of figures pertains is listed at the upper left. The values of ΔM are obtained from the relation

$$\Delta M = 1.6667 \log (N_2/N_1)$$

where N_2 is either the D or f background count and N_1 is the b background count. The only major assumption incorporated into this relation is that the galaxies counted are uniformly distributed in space. For a derivation of this relation see Mihalas (1968).

Table 12 shows that in cases where the same number of rings are used the greatest number of galaxies is included in the f limit and the smallest number in the b limit. This reflects the different magnitude limits to which galaxies are counted.

The core radii alone, in Mpc, are presented in Table 13 in the same format as in Table 12.

Table 13

Core radii only

Emulsion	ML	NR	A2052		A2593		A2626		A154	
			R_C	R_C^*	R_C	R_C^*	R_C	R_C^*	R_C	R_C^*
IIIa-J	b	20	.037	.031	.303	.253	.336	.330	.167	.178
		10	.023	.030	.311	.318	.365	.334	.014	.178
103a-D	D	20	.495	.508	.534	.481	.408	.299	.133	.144
		10	.499	.458	.656	.672	.268	.251	.016	.146
IIIa-J	f	20	.467	.477	.829	.847	.378	.313	.051	.070
		10	.413	.467	.794	.780	.280	.250	.027	.038

Units for R_C and R_C^* are Mpc

ii) Comparison of σ_{bg} and σ_{bg}^{*}

When the computer generated σ_{bg} values are
compared to the corresponding observed σ_{bg}^{*} values, it is
found that the computer program generally produces a
realistic background density. Only a few computed values
are significantly different from the observed ones.
However, whenever the relative difference is greatest for
each cluster (A2052-D, A2593-D, A2626-b, and A154-b) the
computed value is obtained for the 10 ring case. This is
probably because the counts reach background by about the
10th ring, allowing a more accurate background fit to the
20 ring counts. The 10 ring counts, therefore, refer mainly
to cluster galaxies and the background is given little
weight. As Table 10 shows, the diameter of the 20th ring
is almost coincident with the Abell diameter.

These facts suggest that if a computer program
similar to the one used here is to consistently obtain a
realistic background density as part of the fitting process,
galaxy counts should be made out to the Abell radius. This
further suggests that core radii obtained from 10 ring
counts with σ_{bg} treated as a free parameter may also be
unrealistic, a possibility that will be checked in the next
section.

iii) Comparison of 10 ring core radii and 20 ring core radii

If core radii obtained from 10 ring counts when the background density is calculated ($R_C(10)$) are occasionally unrealistic because of poorly determined background densities, then these radii should be significantly different from those obtained using 10 ring counts with the observed, fixed, background densities ($R_C^*(10)$). These differences would be expected to be larger than the differences between $R_C(20)$ and $R_C^*(20)$, because using 20 rings presumably allows a more realistic background density to be determined. The results of Table 13 were used to compute the percent difference between the 10 ring and 20 ring core radii using the expressions

$$\frac{|R_C(10)-R_C^*(10)|\times100}{0.5\times\{R_C(10)+R_C^*(10)\}} \quad \text{and} \quad \frac{|R_C(20)-R_C^*(20)|\times100}{0.5\times\{R_C(20)+R_C^*(20)\}}$$

These values were calculated for each magnitude limit and each cluster. Then the b(10), D(10), and f(10) differences and the b(20), D(20), and f(20) differences were averaged for each cluster to see if there was a significant discrepancy between the 10 and 20 ring cases.

For two clusters the 20 ring cases produced smaller differences than the 10 ring cases by factors of 2 and 10. However, for the other two clusters the 10 ring cases produced differences smaller by factors of 5 and 2.

The 20 ring and 10 ring average differences for all four clusters were respectively 12%±11% and 37%±61%. The large standard deviation in the 10 ring difference is due mainly to the A154 results, which have large internal inconsistencies. If A154 is omitted the 20 and 10 ring averages become 12%±10% and 9%±8% respectively.

For individual clusters the percentage differences show that one or the other of the 10 or 20 ring core radii are probably better representatives for the cluster. The overall averages, however, do not suggest that either 10 or 20 ring counts consistently give better results. If A154 is omitted, these results indicate that the occasional inability of the program to produce realistic background densities using 10 ring counts does not significantly affect the value of the core radii.

iv) Comparison of R_c and R_c^*

Since core radii depend to some extent on background densities it is appropriate to consider the effects of differences in background densities on these radii. Specifically, consider for each magnitude limit the $R_c(10)$ and $R_c(20)$ percentage differences (where each value has a characteristic background density) to the $R_c^*(10)$ and

$R_C^*(20)$ percentage differences (where each value has the same background density). If the observed background provides a better base for calculating core radii then the percentage differences between $R_C^*(10)$ and $R_C^*(20)$ should be smaller on the average than those between $R_C(10)$ and $R_C(20)$.

For each magnitude limit and each cluster the percentage differences were found through

$$\frac{|R_C(10)-R_C(20)|\times100}{0.5\times\{R_C(10)+R_C(20)\}} \quad \text{and} \quad \frac{|R_C^*(10)-R_C^*(20)|\times100}{0.5\times\{R_C^*(10)+R_C^*(20)\}}$$

The $b(R_C)$, $D(R_C)$, and $f(R_C)$ differences and the $b(R_C^*)$, $D(R_C^*)$, and $f(R_C^*)$ differences were averaged for each cluster and compared.

For three clusters the differences between the R_C^* values were less than the R_C differences by factors of 2, 4, and 6. In the other case the differences between R_C values were smaller by a factor of 3.

The overall average of all four clusters including the three magnitude limits showed that R_C^* differences were 15%±17% while the R_C differences were 46%±58%. Most of the spread in the R_C difference is again due to A154, which has a wide range of core radii. Without A154, the R_C^* percentage difference is still smaller (13%±11% compared to 18%±17% for R_C), but the discrepancy between the two has shrunk considerably. These percentage differences suggest that R_C^* values may be slightly more consistent than R_C values (in

that there is closer agreement, generally, between $R_C^*(10)$
and $R_C^*(20)$ than between $R_C(10)$ and $R_C(20)$). It appears
that, again neglecting A154, only marginal differences in
core radii result when the background density is either left
constant or calculated with the other parameters, a result
consistent with the findings of the previous section.

There are seven cases where the differences are
greater than 30% of the average: the R_C-b case of A2052
(47%); the R_C^*-D case of A2593 (33%); the R_C-D case of A2626
(41%); the three R_C cases of A154 (b-169%, D-159%, and
f-62%); and the R_C^*-f case of A154 (59%). Five of these
seven have density profiles with a first ring density
significantly higher than the rest of the ring densities,
the A2593 and A2626 cases are the exceptions. If the high
first ring densities are the cause of the discrepancies, it
is probably because of the higher weight these points have
in the 10 ring case. The effect on the model is to produce
a higher central density and a correspondingly smaller core
radius, particularly in the cases where the background density
is found as part of the fitting procedure. Table 13 supports
this conclusion, showing that for the cases where the
discrepancy is in the R_C columns, it is indeed the 10 ring
core radius that is smaller.

To see how significant the central data point is,
the data for A2052-b were run with the inner ring omitted.
The core radius obtained for the outer 19 rings was $R_C^* = 0.602$

Mpc, as opposed to 0.031 Mpc with all 20 rings. (For

comparison, this core radius occurs at 10.5 on the first

graph of Appendix I.)

v) Individual cluster abnormalities

The Rood-Sastry (1971) classification for each

cluster is listed below.

Table 14

Rood-Sastry classifications

cluster	R-S class
A2052	cD
A2593	--
A2626	cDp
A154	Bb

The R-S type for A2626 indicates that the

supergiant galaxy is a multiple or has some other sort of

peculiarity. What this may be is not discussed by Rood and

Sastry, but visual inspection through a 10X stereo

microscope shows that the densest part of the image is not
located at the centre of symmetry. It is not known if this
is the peculiarity referred to, or even if this observed
oddity is inherent in the galaxy or due to a superimposed
stellar or galaxian image.

The type assigned A154 indicates a cluster with a
central binary galaxy whose components are connected by a
luminous bridge. It is not known whether binary clusters
are basically different in structure, which might explain
the anomalous (i.e. very small) f radii of this cluster.

A2593 is not classified by Rood and Sastry since
they feel it to be either an I cluster superimposed on a cD
cluster or a single peculiar I cluster. The possibility
that we view two superimposed clusters is supported both by
inspecting the prints with galaxies identified to D and f
limits and by strip count histograms. These suggest the
presence of a small group of relatively faint galaxies about
15 south of the dominant elliptical. If this is the case,
then the strip count centre would possibly be chosen further
south than otherwise, resulting in a larger core radius.

To test this possibility ring counts were made of
only the north half of this cluster, with rings centred on
the brightest galaxy. These numbers were doubled to
simulate a cluster with north-south symmetry and treated as
a cluster by the program. The resulting core radii are
presented in Table 15 with a format and notations identical

to those of Table 13.

These results show that the smallest changes in core radius are those of the b limit. This is consistent with the existance of a background cluster which would not have been included in these counts.

The D limit core radii have changed drastically. This is mainly because 10 of the 13 galaxies in the first ring are located in the northern half of the cluster. The assumption of north-south symmetry about the brightest galaxy thus leads to a central ring containing 20 galaxies, an increase of 54%.

The f limit values also decreased, as would be expected if a background cluster were present to the south. However, an inspection of ring counts indicates that this may be due more to an increase of galaxies in the first ring than to a decrease in numbers in the outer rings, a supposition supported by the new f core radii.

The evidence is that the background cluster, if it exists, becomes apparent between the b and D magnitude limits, and affects the corresponding core radii. But it is also apparent that asymmetries in the distribution of fainter galaxies within the foreground cluster itself can significantly change the core radius, depending upon the choice of centre for the ring counts. Fortunately, neither this asymmetry nor the possible background cluster affect the b limit core radii.

Table 15

Core radii for A2593

Emulsion	ML	NR	From Table 13		N only	
			R_C	R_C^*	R_C	R_C^*
IIIa-J	b	20	.303	.253	.212	.250
		10	.311	.318	.318	.349
103a-D	D	20	.554	.481	.044	.043
		10	.656	.672	.032	.044
IIIa-J	f	20	.829	.847	.659	.459
		10	.794	.780	.381	.381

Units for R_C and R_C^* are Mpc

vi) The mass segregation question

It has been decided to use the $R_C^*(20)$ values
hereafter as the values representing the core radii for the
magnitude limits. These were chosen because the R_C^* values
have been found to be possibly more consistent than the R_C
values and because 10 ring counts generally do not extend
appreciably into the background and so possibly give
unrealistic radii at times (the results of section iii
notwithstanding). These core radii are displayed in
Table 16.

Table 16

Adopted core radii in Mpc

ML	A2052	A2593	A2626	A154
b	.031	.253	.330	.178
D	.508	.481	.299	.144
f	.477	.847	.313	.070

As clusters of galaxies evolve the tendency
towards equipartition of energy results in the more massive
galaxies losing kinetic energy to the less massive ones.
As a result the less massive galaxies will move farther out

in the cluster and the more massive ones will fall to the centre. Equipartition of energy will therefore result in a radial segregation of mass. The more massive galaxies, as a group, would thus define a smaller core radius than a group of less massive galaxies, with a mixed group having an intermediate core radius.

Oemler (1974) has found evidence of mass segregation in all six cD clusters he studied (of a total of 15). Quintana (1979) has recently found evidence for mass segregation by fitting isothermal gas sphere models to galaxy counts of the Coma and CA0340-538 clusters. Dressler (1978), on the other hand, has studied 15 rich clusters, including five cD clusters, and has found evidence of mass segregation in only three cases, only one of which is a cD cluster. The other four cD clusters display somewhat larger core radii for the brighter (more massive) galaxies than for fainter ones. This phenonenom is attributed to a further stage of cluster evolution in which bright galaxies near the cluster centre are accreted by the central cD galaxy. The resulting lower central number density is reflected in a larger core radius.

From the values in Table 16, only A2052 and A2593 give evidence of mass segregation. Both of these clusters have possible anomalies, however. The extremely low core radius for A2052-b seems unlikely (as do other core radii near this size) despite the goodness of fit, especially when

the core radius obtained through omission of the inner ring
is considered (see section iv of this chapter). Also the
A2593-D and f core radii may be too large because of the
presence of a superimposed cluster.

A2626 shows no evidence for mass segregation and
the A154 values suggest that the fainter galaxies are more
centrally concentrated than the brighter ones, a situation
which is consistent with the accretion process described by
Dressler. The A154-f result is probably due mainly to a
grouping of faint galaxies observed around the central
binary, a phenomenon not believed to be associated with
accretion but merely a chance positioning on galaxies.

If the four values for each magnitude limit are
averaged, the ratio $R_C^*(b):R_C^*(D):R_C^*(f)$ is .20:.36:.43, which
taken at face value implies an overall tendency towards mass
segregation. However the scatter is so large that these
averages are probably not significant. Although a general
trend can not be cited, two of the clusters studied do show
signs of mass segregation.

vii) Comparison with published core radii

Core radii have been published by Bahcall (1975)
for A2052 and by Dressler (1978) for A154, allowing
comparisons with the values found here.

Bahcall's data for A2052 indicate that her counts
covered 12 of the 20 rings used here. Furthermore, instead
of using rings of uniform width, Bahcall used four central
rings 2.24 wide and four outer rings 4.48 wide while this
investigation used rings 2.24 wide. Therefore, the data
found here were combined to simulate wider rings in order
to determine if results similar to those of Bahcall would
be obtained.

Table 17 shows in column 1 the data source of the
ring counts. Column 2 indicates the magnitude limit of the
present data, the b limit was chosen because it is closest
to Bahcall's magnitude limit. Column 3 gives the number of
rings used, either the inner 10, the inner 12 (B), or all
20. The last two columns give, respectively, the number of
galaxies used in the calculations and the core radii
obtained in Mpc.

The top three rows present results using the data
from this thesis. The bottom rows give results produced by
Bahcall and by this program using Bahcall's data.

The results show that analysing the present data
using rings having the same width as those used by Bahcall

Table 17

Core radii for A2052

	ML	NR	NG	R_c
Thesis data:	b	10	40	.023
		B	46	.025
		20	58	.037
Bahcall's data:				
her results	–	B	73	.2B
this program	–	B	73	.430

produces no appreciable difference in the core radius

compared to others found with this study's data. The

bottom lines show that using Bahcall's data this program

produces a much larger core radius than that found by

Bahcall, a result which has been discussed in Chapter III.

Comparison with Table 13 shows that this core radius is

similar to the value found with the f limit data for this

cluster. This result is probably at least partly due to

mass segregation. This possibility presents itself because

Bahcall counts 60% more galaxies and so presumably reaches

a fainter magnitude limit than the b limit used here. Mass

segregation in this cluster is strongly suggested by the

values in Table 16. The possibility exists that the

disparate results are due to the two sets of data (Bahcall's

and those of this study) being centred differently. This

seems unlikely to give the large differences, however, as the

centres are the same to within $0^m.1$ and 1^s.

Dressler used a method similar to that of Bahcall

to obtain the core radius of A154. He found a core radius

of 0.19 Mpc with counts out to about the eighth ring used

here. This area was apparently divided into 11 rings of

the same width. Unfortunately, neither ring counts nor

limiting magnitudes are given by Dressler so a detailed

comparison is precluded. However, his result is close to the

A154 core radius obtained from the b counts (see Table 16).

viii) Combination of present core radii with those of Bahcall

Bahcall (1975) has published core radii of 15

Abell clusters and has obtained an average value of

0.25±0.05 Mpc (the 0.05 value is the standard deviation of

scatter, and has no bearing on errors inherent in the

individual core radii). The b limit core radii of Table 16

will be combined with Bahcall's results to obtain a new

average and standard deviation. Bahcall's results may have

been influenced by the fact that she only once counted out

to near the Abell radius; on the average the distance to which

she counted from the cluster centre is only 53% of the

Abell radius, as opposed to an average of 102% for this

thesis.

Using the core radii specified above from Table 16 and Bahcall's results, the combined average is

$$<R_c>=0.24\pm0.07 \text{ Mpc} \quad (H_O=50 \text{ km s}^{-1}\text{Mpc}^{-1})$$

Bahcall's value is not changed much, mainly because of her larger sample. For comparison, the average of the four core radii from this thesis is 0.20 ± 0.13 Mpc. (If the core radius of A2052 is neglected the average core radius for the remaining three clusters is 0.25 ± 0.08 Mpc.)

To see if the number of core radii averaged is a significant factor, four random groups of four core radii were taken from Bahcall's results and averaged. The averages found were 0.28 ± 0.07 Mpc, 0.23 ± 0.02 Mpc, 0.24 ± 0.07 Mpc, and 0.24 ± 0.06 Mpc. This implies that the large standard deviation for the average of the four core radii of this thesis has little to do with the number of values averaged.

The fact that the clusters studied here tend to be cD clusters as opposed to spiral rich or spiral poor clusters is not an influential factor. An inspection of the clusters used in Bahcall's paper show six cD, four B, two L, one F, one C, and an unclassified cluster. No type shows a significantly larger or smaller mean core radius.

If the spread of core radii is as large as Table 16 (or worse yet, Table 13) implies then the validity of using this radius as a cosmological metrestick may be

questioned. Calculations can be made to determine how far away in z clusters would have to be before the spread of standard deviation is overcome by the changes in radius caused by the value of q_O. For example, in universes with deceleration parameters 0 and +1, a 0.25 Mpc object would differ in size by 0.05 Mpc (Bahcall's standard deviation) at $z = 0.43$, and the same object would differ in size by 0.13 Mpc (the standard deviation for all four clusters from this thesis) at $z = 1.17$. It is obvious that if the value of the standard deviation is near the value found for the four clusters of this thesis the probability of determining q_O from core radii is low.

Also, if there is mass segregation present in some clusters of galaxies further problems arise, namely that the core radius will be a function of the limiting magnitude.

ix) Conclusions

1.) Results in the bottom two rows of Table 17, as well as those in Chapter III, indicate that the same data can produce widely differing results depending on their treatment. Even when the general method of analysing the data is supposedly the same (see Chapter III, section iii)

different results are obtained by different programs for individual clusters.

This suggests that a study should be made of all programs used by researchers to determine which is best, however that may turn out to be defined, for a given method of finding core radii. This program should then be used by everyone in this line of study to ensure consistent results. Or, if it happens that no one program is any better than another, to maintain consistency one program should be chosen to be used exclusively. Then the comparison of results would acquire a greater significance. (This obviously does not exclude further work at attempts to devise an improved program for core radius determination.)

2.) More consistent results seem to be found in the present investigation when the background density is counted, rather than calculated as a free parameter in the fitting process.

3.) Care should be taken to include a large background sample in the data by counting out sufficiently far from the cluster centre. The Abell radius seems to contain a large enough area for this purpose. This procedure seems to be of greater importance when the background density is to be calculated rather than counted directly.

4.) Two clusters show evidence of mass segregation but a general trend is not evident in the small sample studied here. In at least one of these clusters,

A2593, this effect may be caused by the presence of a second cluster in the field.

5.) The spread of core radii appears to be larger than that in the sample studied by Bahcall. This raises new questions concerning the use of these radii as standard metresticks in attempts to determine the value of the deceleration parameter.

If the spread of core radii is as great as is suggested by this thesis, then only very large z (greater than about 1) clusters will be usable in determinations of q_o. The lack of rich clusters at these distances could prevent the determination of the deceleration parameter.

Part of this spread may be due to mass segregation. If this is so, then establishment of a sufficiently accurate magnitude limit would be required before a large sample of such clusters could be used for this purpose.

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Appendix A

Program listing

A listing is provided of the program version
(named YAHOO) that finds the best fit values for x_o, α, σ_c,
and σ_{bg}. The listing should contain enough comment cards
to enable a user to follow procedings; if these are
insufficient an extensive explanation is presented in
Appendix C. These, combined with the description of the
type and format of required input data provided at the
beginning of Chapter IV, fully explain the workings of this
program. The first page of a typical output run of YAHOO
is presented in Appendix J with sample outputs of two other
programs.

```
C
C
C      INPUT
C        DATID(I)  PRINTER CHECK
C        EPSI(I)   THE SERIES OF VALUES (EXP(-PSI)*PSI')
C        J         TOTAL NUMBER OF RINGS (NOT NECESSARILY ALL OF SAME WIDTH)
C        NOBS(I)   NUMBER OF OBSERVED GALAXIES IN RING I
C        ROUT(I)   OUTER RADIUS OF RING I
C        SC1       INITIAL ESTIMATE OF CENTRAL DENSITY
C        SBG1      INITIAL ESTIMATE OF BACKGROUND DENSITY
C
C      MAJOR ARRAYS AND VARIABLES
C        XI(I)     THE SERIES OF XI VALUES CORRESPONDING TO EPSI(I)
C        SIGOB(I)  GALAXY DENSITY IN RING I
C        SIGISO(I) CALCULATED GALAXY DENSITY AT SAME DISTANCE FROM CLUSTER
C                  CENTRE AS SIGOB(I)
C        NCALC(I)  THEORETICAL VALUE FOR NOBS(I), CALCULATED FROM SIGISO(I)
C        INTDI(I)  STORAGE PLACE OF INTEGRATION RESULTS, REQUIRED BY
C                  INTEGRATION SUBROUTINE
C        RAV(I)    AVERAGE (I.E. EQUAL AREA) RADIUS BETWEEN ROUT(I) AND
C                  ROUT(I+1)
C        AXXA(I)   STORAGE SPACE FOR ALPHA, SIGC, SIGBG, AND CHISQ BEFORE
C                  PRINTING
C        XIEPS(I)  ARRAYS TO BE INTEGRATED; FED INTO SUBROUTINES
C        SC(I)     SERIES OF CENTRAL DENSITIES FOUND ITERATIVELY BY THE
C                  NEWTON-RAPHSON METHOD
C        SBG(I)    BACKGROUND DENSITY ANALOGUE TO SC(I)
C        XO        UPPER LIMIT TO ISOTHERMAL GAS SPHERE INTEGRATION
C        LOGA1     MAXIMUM ALPHA FOR A GIVEN XO
C        LOGA1     LOG(MAXIMUM ALPHA FOR A GIVEN ALPHA)
C        LOGAL     INCREMENTAL BASE FOR LOG(ALPHA)
C        DIVN      ISOTHERMAL GAS SPHERE VALUE; TOTAL INTEGRAL FROM 0 TO XO
C        SIGC      NEWTON-RAPHSON VALUE FOR CENTRAL DENSITY FOR A SPECIFIC
C                  XO-ALPHA COMBINATION
C        SIGBG     BACKGROUND DENSITY ANALOGUE TO SIGC
C        CHISQ     CHI-SQUARE VALUE FROM COMPARING CURRENT MODEL TO DATA
C        IAM       ERROR MESSAGE MARKER
C        G, F, FC, FBG, GBG
C                  SUMS USED BY NEWTON-RAPHSON METHOD TO GET SIGC AND SIGBG
C        C2        MINIMUM CHI-SQUARE FOR AN XO
C        A2, S2, SB2
C                  ALPHA, SIGC, AND SIGBG ASSOCIATED WITH C2
C        IND       MARKER: IND=5 ALPHA BEING INCREMENTED; IND=70 LOWEST
C                  CHISQ ALPHAS BEING-AVERAGED
C        IDD       COUNTER: NO. OF TIMES ALPHA AVERAGING HAS OCCURRED
C
C      SUBROUTINES
C        SIGI      CALCULATES AN INTEGRAL FROM X TO XO FOR AN ISOTHERMAL
C                  DENSITY AT RADIAL DISTANCE X
C        DIVIN     CALCULATES AN INTEGRAL FROM 0 TO XO FOR ISOTHERMAL DENSITIES
C                  USING THAT SPECIFIC XO
C        IQSF      CALCULATES INTEGRALS OVER RANGES WITH EQUALLY SPACED
C                  FUNCTION VALUES
C
```

```
          C
          C
  0001          REAL NOBS(40),ROUT(41),RAV(40)
  0002          DOUBLE PRECISION BI(40),SIGISO(40),NCALC(40),SIGOB(40)
  0003          REAL EPSI(281),XI(281),LOGAL,LOGA1,AXXA(12)
  0004          DOUBLE PRECISION INTDI(281),XIEPS(281),CHISO,DIVN,SIGC,SIGBG,
                 $SC(20),SBG(20),XX1,XX2,G,F,FC,FBG,GBG,DNOM,Z1,Z2,E1,E2
  0005          INTEGER XO,DATID(40)
          C
          C          DATA READ IN: SC1, SBG1, AND J PRINTED
          C
  0006          READ(1,888)(DATID(I),I=1,40)
  0007      888 FORMAT(40A2)
  0008          WRITE(6,889)(DATID(I),I=1,40)
  0009      889 FORMAT(5X,40A2)
  0010          READ(1,225)(EPSI(I),I=1,281)
  0011      225 FORMAT(10(E8.5))
  0012          READ(1,222)J
  0013      222 FORMAT(/,I2)
  0014          READ(1,223)(NOBS(I),I=1,J)
  0015      223 FORMAT(20(F4.0))
  0016          IJK=J+1
  0017          READ(1,224)(ROUT(I),I=2,IJK)
  0018      224 FORMAT(20(F4.2))
  0019          READ(1,230)SC1,SBG1
  0020          SC2=2.*SC1
  0021          SBG2=2.*SBG1
  0022          WRITE(6,880)SC1,SBG1
  0023      880 FORMAT(/,5X,'INITIAL CENT DENS=',1PE11.2,/,5X,'INITIAL  BG  DENS',
                 $'=',1PE11.2)
  0024      230 FORMAT(E5.2,2X,E5.2)
  0025          IF(J.LE.40)GO TO 15
  0026          WRITE(6,803)J
  0028      803 FORMAT(//,10X,'CHANGE LINE 1 AND 2 ARRAY SIZES FROM 40 TO',I4,//)
  0029          GO TO 999
  0030       15 IJK=0
  0031          WRITE(6,883)J
  0032      883 FORMAT(/,5X,'THERE ARE',I3,' RINGS',/)
          C
          C          VALUES OF XI CALCULATED IN THREE RANGES: BY 0.1 FROM 0 TO 10,
          C          BY 1 FROM 10 TO 100, AND BY 10 FROM 100 TO 1000
          C
  0033          DO 50 I=1,100
  0034          X=I-1.
  0035          X=X/10.
  0036       50 XI(I)=X
  0037          JX=10
  0038          DO 51 I=101,190
  0039          XI(I)=JX
  0040       51 JX=JX+1
  0041          JX=100
  0042          DO 52 I=191,281
  0043          XI(I)=JX
  0044       52 JX=JX+10
```

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```
          C          RAV(I) AND SIGOB(I)-CALCULATED
          C          NOBS, ROUT, SIGOB, AND RAV PRINTED
          C
0045            WRITE(6,885)
0046      885   FORMAT(//,8X,'NOBS',9X,'ROUT',8X,'SIGOB',9X,'RAV',/)
0047            ROUT(1)=0.0
0048            DO 2 I=1,J
0049            BI(I)=(ROUT(I+1))-(ROUT(I))*ROUT(I))
0050            SIGOB(I)=NOBS(I)/(3.141593*BI(I))
0051            RAV(I)=SQRT(0.5*(ROUT(I)*ROUT(I)+ROUT(I+1)*ROUT(I+1)))
0052      2     WRITE(6,886)NOBS(I),ROUT(I),SIGOB(I),RAV(I)
0053      886   FORMAT(4(5X,F8.4))
0054            WRITE(6,884)ROUT(J+1)
0055      884   FORMAT(18X,F8.4)
          C
          C          MAIN LOOP FOR CHANGING XO-STARTS
          C          INITIAL AND FINAL ALPHA, SC(1), AND SBG(1) VALUES SET
          C          XO AND RESULT HEADINGS PRINTED
0056            DO 100 XO=10,200,10
0057            IND=5
0058            C=9.0E-15
0059            A2=75
0060            SC(1)=SC1
0061            SBG(1)=SBG1
0062            ALPHA=0.999*RAV(1)/XO
0063            LOGAL=ALOG10(ALPHA)
0064            LOGAL=LOGAL+4.0
0065            IZ=1
0066            WRITE(6,24)XO
0067      24    FORMAT(//,' XO-IS-EQUAL TO ',I4)
0068            WRITE(6,25)
0069      25    FORMAT(3(4X,'ALPHA       CENT DENS    BG DENS    CHI-SQ '))
          C
          C          AN INTEGRAL NEEDED FOR SIGISO IS CALCULATED
0070            DO 1 I=1,281
0071      1     XIEPS(I)=EPSI(I)*XI(I)
0072            CALL DIVN(XIEPS,XO,INTDI,XI,IJK)
0073            IF(IJK.EQ.0)GO TO 40
0075            WRITE(6,41)
0076      41    FORMAT(//,'********** TROUBLES AT DIVN **********')
0077            GO TO 100
0078            DIVN=INTDI(1)
          C
          C          LOOP TO CALCULATE SIGISO AND INCREMENT ALPHA-START
0079      10    DO 8 I=1,J
0080            X=RAV(I)/ALPHA
0081            IF (X.GE.XO)GO TO 12
0082            XSQ=X*X
0083            DO 33 M=1,281
0084            IF(XI(M).GE.X)GO TO 34
```

```
0087          WRITE(6,35)X,M,XI(M)
0088     35   FORMAT(//,12X,'X=',F7.2,' AND XI(',I3,')=',F7.2,/,12X,'SEE SIGISO',
             $' LOOP')
0089          GO TO 999
0090     34   DO 36 K=M,281
0091          XYZ=XI(K)*XI(K)-XSQ
0092     36   XIEPS(K)=EPSI(K)*SQRT(XYZ)
0093          M=M-1
0094          DO 37 K=1,M
0095     37   XIEPS(K)=0.0
0096          CALL SIGI(XIEPS,X0,X,XI,INTDI,IJK)
0097          IF(IJK.EQ.0)GO TO 8
0099          WRITE(6,42)
0100     42   FORMAT(//,'**********  TROUBLES AT SIGI  **********')
0101          GO TO 100
0102     8    SIGISO(I)=INTDI(1)/DIVN
0103          GO TO 9
0104     12   IF(I.EQ.1)GO TO 29
0106          DO 13 II=I,J
0107     13   SIGISO(II)=0.0
        C
        C    END OF SIGISO LOOP; LOOP TO CALCULATE SIGC AND SIGBG BY NEWTON-
        C    RAPHSON METHOD STARTED -- ITERATION IS DONE 20 TIMES
        C    II IS THE NUMBER OF TIMES THE LOOP RUNS, MAX IIM4
0108     9    II=1
0109          IXY=0
0110     211  DO 202 IP=II,19
0111          F=0.0
0112          G=0.0
0113          FC=0.0
0114          FBG=0.0
0115          GBG=0.0
0116          DO 203 L=1,J
0117          Z1=SC(IP)*SIGISO(L)+SBG(IP)
0118          IF(Z1.NE.0.0)GO TO 804
0120          IAM=1
0121          GO TO 444
0122     804  Z2=SIGOB(L)-Z1
0123          E1=Z2/Z1
0124          E1=2.*E1+E1*E1
0125          E2=(1./Z1)+((2.*Z2/Z1)/Z1)+(((Z2/Z1)*Z2/Z1)/Z1)
0126          F=F-BI(L)*SIGISO(L)*E1
0127          G=G-BI(L)*E1
0128          FC=FC+2.*BI(L)*SIGISO(L)*SIGISO(L)*E2
0129          FBG=FBG+2.*BI(L)*SIGISO(L)*E2
0130     203  GBG=GBG+2.*BI(L)*E2
0131          DNOM=FC*GBG-FBG*FBG
0132          IF(ABS(DNOM).GT.1.0E-12)GO TO 204
0134          SC(IP+1)=SC(IP)
0135          SBG(IP+1)=SBG(IP)
0136          IXY=IXY+1
0137          IF((IXY.GE.5).AND.(IP.GE.19))GO TO 202
0139          IAM=2
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0140            GO TO 444
0141     204    IXY=0
0142            SC(IP+1)=SC(IP)+(G*FBG-F*GBG)/DNOM
0143            SBG(IP+1)=SBG(IP)+(F*FBG-FC*G)/DNOM
0144     202    CONTINUE
C
C          END OF SIGC-SIGBG LOOP; LOOP TO CHECK FOR CONVERGENCE OVER THE
C          LAST SIX VALUES OF SC AND SBG--NO-CONVERGENCE; MESSAGE
C          AND GOES TO END OF XO LOOP
C
0145            DO 207 L=16,19
0146            XX1=ABS(SC(L)-SC(L-1))+ABS(SC(L))*0.005
0147            XX2=ABS(SC(L+1)-SC(L))
0148            IF(XX2.LE.XX1)GO TO 807
0150            IAM=3
0151            GO TO 444
0152     807    XX1=ABS(SBG(L)-SBG(L-1))+ABS(SBG(L))*0.005
0153            XX2=ABS(SBG(L+1)-SBG(L))
0154            IF(XX2.LE.XX1)GO TO 207
0155            IAM=4
0156            GO TO 444
0157     207    CONTINUE
C
C          END OF CONVERGENCE LOOP; CHECK TO SEE IF LAST TWO VALUES OF SC
C          AND SBG ARE WITHIN 0.01% OF EACH OTHER. IF NOT, THE LAST SC AND
C          SBG VALUES ARE TAKEN TO BE THE INITIAL ESTIMATES AND NEWTON-
C          RAPHSON CONVERGENCE IS AGAIN USED; TO A MAXIMUM OF FOUR TIMES.
C          INSUFFICIENT AGREEMENT PRINTS A MESSAGE AND GOES TO END OF XO LOOP
C
0159            XX1=ABS(SC(19))/ABS(SC(20))
0160            XX2=ABS(SBG(20)-SBG(19))/ABS(SBG(20))
0161            IF((XX1.LE.0.0001).AND.(XX2.LE.0.0001))GO TO 209
0163            IF(II.LE.4)GO TO 810
0165            IAM=5
0166            GO TO 444
0167     810    II=II+1
0168            SC(II)=SC(20)
0169            SBG(II)=SBG(20)
0170            GO TO 211
C
C          IF CONVERGENCE AND AGREEMENT TESTS ARE PASSED SC(20) AND SBG(20)
C          ARE ACCEPTED AS SIGC AND SIGBG. IF THESE ARE LESS THAN TWICE
C          INITIAL ESTIMATES THESE ARE THE INITIAL ESTIMATES FOR THE
C          NEXT ALPHA.
C
0171     209    SIGC=SC(20)
0172            SIGBG=SBG(20)
0173            IF(SIGC.GT.SC2)GO TO 700
0175            SC(1)=SC(20)
0176            GO TO 701
0177     700    SC(1)=SC1
0178     701    IF(SIGBG.GT.SBG2)GO TO 702
0180            SBG(1)=SBG(20)
0181            GO TO 703
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0182        702 SBG(1)=SBG1

       C
       C         5C AND SBG 2-20 SET TO 0 AS A SAFETY PRECAUTION
       C
0183        703 DO 310 IR=2,20
0184            SC(IR)=0.0
0185        310 SBG(IR)=0.0

       C
       C         CHISQ CALCULATED
       C
0186            CHISQ=0.0
0187            DO 200 I=1,J
0188            NCALC(I)=3.14159*BI(I)*(SIGC*SIGISO(I)+SIGBG)
0189        200 CHISQ=CHISQ+(NOBS(I)-NCALC(I)**2)/NCALC(I)

       C         IF ALPHA IS BEING AVERAGED OPERATIONS ARE PASSED ON
       C         IF CHISQ<0 IT IS NOT COMPARED TO C2 BECAUSE SIGBG<<0
       C         IF THE PREVIOUS CHISQ IS C2 (KK=1) THEN CHISQ AND ALPHA ARE SAVED

0190            IF(IND.GT.20)GO TO 316
0192            IF(CHISQ.LT.0.0)GO TO 317
0194            IF(CHISQ.LT.C2)GO TO 315
0196            KK=KK+1
0197            IF(KK.GT.1)GO TO 317
0199            C3=CHISQ
0200            A3=ALPHA
0201            GO TO 317

       C         IF CHISQ IS THE SMALLEST SO FAR IT AND ALPHA ARE SAVED, AND THE
       C         PREVIOUS C2 AND A2 ARE RENAMED
       C
0202        315 C1=C2
0203            C2=CHISQ
0204            A1=A2
0205            A2=ALPHA
0206            S2=SIGC
0207            SB2=SIGBG
0208            KK=0

       C         RESULTS ARE STORED AS GROUPS OF FOUR. IF AXXA IS FULL THE 12
       C         VALUES ARE PRINTED AND AXXA IS INDEX IS RESET
       C
0209        317 AXXA(IZ)=ALPHA
0210            AXXA(IZ+1)=SIGC
0211            AXXA(IZ+2)=SIGBG
0212            AXXA(IZ+3)=CHISQ
0213            IZ=IZ+4
0214            IF(IZ.LE.9)GO TO 29
0216            IZ=IZ-1
0217            WRITE(6,28)(AXXA(IX),IX=1,IZ)
0218         28 FORMAT(I2(1PE11.3))
0219            IZ=1

       C
       C         ALPHA IS INCREMENTED, IF IT IS SUFFICIENTLY SMALL GO
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        C
        C          BACK TO START OF SIGISO LOOP
0220    29   LOGAL=LOGAL+.08
0221         ALPHA=10.**LOGAL
0222         IF(LOGAL.LT.LOGA1)GO TO 10
0224         GO TO 445
0225    444  IF(IND.LT.20)GO TO 445
        C
        C          VARIOUS ERROR MESSAGES PRINTED (CONVERGENCE, ETC.)
0227         IF(IAM.EQ.1)WRITE(6,91)SC(IP),SIGISO(L),SBG(IP),IP
0229    91   FORMAT(10X,'SC(IP)=',1PE11.3,4X,'SIGISO(L)=',1PE11.3,' SB',
            $'G(IP)=',1PE11.3,'     IP=',I3,/,10X,'PROBLEMS',
            $' IN THE 20) LOOP')
0230         IF(IAM.EQ.2)WRITE(6,806)DNOM,SC(IP),SBG(IP)
0232    806  FORMAT(10X,'DNOM=',1PE11.3,'   AND NON-ITERATION OCCURRED TOO O',
            $'FTEN   SC=',1PE11.3,'   SBG=',1PE11.3)
0233         IF(IAM.EQ.3)WRITE(6,808)L,SC(L-1),SC(L),SC(L+1)
0235    808  FORMAT(10X,'L=',I3,'     SC(L-1,L,L+1)=',3(1PE12.3),5X,'NOT ',
            $'CONVERGING')
0236         IF(IAM.EQ.4)WRITE(6,809)L,SBG(L-1),SBG(L),SBG(L+1)
0238    809  FORMAT(10X,'L=',I3,'     SBG(L-1,L,L+1)=',3(1PE12.3),5X,
            $'NOT CONVERGING')
0239         IF(IAM.EQ.5)WRITE(6,811)
0241    811  FORMAT(10X,'NO CONVERGENCE WITHIN 0.0001 AFTER 80 ITERATIONS')
0242         WRITE(6,324)X1,X2,X3,X4,IDD
0243    324  FORMAT(32X,'BOMB OUT:     ',4(1PE11.3),10X,'IDD=',I4,
            $/,10X,'INITIAL SIGC AND/OR SIGBG PROBABLY BAD')
0244         GO TO 100
        C
        C          IF A NEW XO IS TO BE USED AND AXXA VALUES REMAIN UNPRINTED
        C          THESE VALUES AND THE MINIMUM CHISQ PARAMETERS ARE PRINTED
0245    445  IF(IZ.LT.2)GO TO 318
0247         IZ=IZ-1
0248         WRITE(6,28)(AXXA(IX),IX=1,IZ)
0249         IZ=1
0250    318  WRITE(6,322)A2,B2,SB2,C2
0251    322  FORMAT(24X,'MIN FROM PROGRAM:   ',4(1PE11.3))
        C
        C          THE SECOND SMALLEST CHISQ AND ITS ALPHA ARE PUT IN C3 AND A3
0252         IF(C1.GT.C3)GO TO 320
0254         A3=A1
0255         C3=C1
        C
        C          IND IS RESET, ALPHA AVERAGING STARTS
        C          ALPHA AVERAGING IS DONE BEFORE A NEW XO IS CHOSEN
0256    320  IND=70
0257         SC(1)=S2
0258         SBG(1)=SB2
0259         ALPHA=0.5*(A3+A2)
0260         IDD=1
```
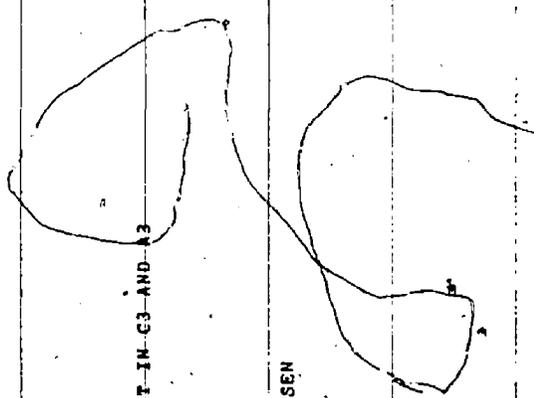
```
0261        GO TO 10
     C
     C        AFTER CHISQ IS FOUND AND IF IND=70 OPERATIONS ARE SENT HERE
     C        IF NEW CHISQ<C2 APPROPRIATE VALUES ARE RESET
     C
0262    316 IF(CHISQ.GE.C2)GO TO 340
0264        C3=C2
0265        A3=A2
0266        C2=CHISQ
0267        A2=ALPHA
0268        S2=SIGC
0269        SB2=SIGBG
0270        GO TO 341
     C
     C        IF NEW CHISQ>C2 BUT <C3 APPROPRIATE VALUES ARE RESET
     C
0271    340 IF(C3.LT.CHISQ)GO TO 341
0273        C3=CHISQ
0274        A3=ALPHA
     C
     C        IF AVERAGING HAS NOT OCCURRED 20 TIMES IT CONTINUES AND
     C        PROGRAM RETURNS TO START OF SIGISO LOOP
     C
0275    341 IF(IDD.GT.20)GO TO 342
0277        ALPHA=0.5*(A3+A2)
0278        IDD=IDD+1
0279        GO TO 10
     C
     C        IF 20 AVERAGES FOUND, MINIMUM CHISQ PARAMETERS (C2, A2, ETC.) PRINTED
     C
0280    342 WRITE(6,343)A2,S2,SB2,C2
0281    343 FORMAT(25X,'MINIMUM METHOD : ',4(1PE11.3))
0282    100 CONTINUE
     C
     C        END OF XO LOOP
     C
0283    999 WRITE(6,555)
0284    555 FORMAT(///)
0285        CALL EXIT
0286        END
```

```
0001        SUBROUTINE SIGI(EPSI,IXO,X,XI,INTDI,IJK)
       C
       C       CALCULATES INTEGRAL FROM X TO IXO
       C          EPSI   THE RANGE OF VALUES TO BE INTEGRATED OVER
       C          IXO    UPPER LIMIT OF INTEGRATION
       C          X      LOWER LIMIT OF INTEGRATION
       C          XI     SET OF HORIZONTAL AXIS VALUES CORRESPONDING TO EPSI
       C          INTDI  STORAGE SPACE OF INTEGRATION RESULTS
       C          IJK    ERROR MARKER
       C
0002        DOUBLE PRECISION EPSI(1),INTDI(1),SUM
0003        DIMENSION XI(1)
0004        IJK=0
0005        IKK=0
       C
       C       X IS CHECKED TO ENSURE IT IS BETWEEN 0 AND IXO
       C
0006        IF(X.LT.0.0)GO TO 100
0008        IF(X.LT.IXO)GO TO 101
0010        WRITE(6,200)X,IXO
0011   200 FORMAT(//,5X,'TROUBLES... X=',F7.2,'  AND IXO=',I6)
0012        GO TO 198
       C
       C       STORAGE SPACE FOR INTEGRATION RESULTS SET TO 0
       C       POSITION OF XI VALUE EQUAL TO OR GREATER THAN X FOUND
       C       IF X>XI(281) ERROR MESSAGE PRINTED
       C
0013   101 SUM=0.0
0014        DO 102 I=1,281
0015        IF(XI(I)-X)102,103,104
0016   102 CONTINUE
0017   100 WRITE(6,201)X
0018   201 FORMAT(//,5X,'X IS ODD, IT IS',F7.2)
0019        GO TO 198
       C
       C       IF NO XI VALUE=X MARKER SET; POSITION OF XI VALUE EQUAL TO OR
       C       GREATER THAN IXO FOUND; IF IXO>XI(281) ERROR MESSAGE OUTPUT
       C
0020   104 IJK=25
0021   103 DO 105 K=1,281
0022        IF(XI(K)-IXO)105,106,107
0023   105 CONTINUE
0024        WRITE(6,202)IXO
0025   202 FORMAT(//,5X,'IXO IS ODD, IT IS',I6)
0026        GO TO 198
       C
       C       IF NO VALUE OF XI=IXO, MARKER SET
       C
0027   107 IKK=30
0028        GO TO 106
       C
       C       IF IKK=30 TRAPEZOIDAL AREA BETWEEN IXO, EPSI(IXO), THE
       C       CLOSEST XI, AND ITS EPSI IS FOUND AND ADDED TO SUM. EPSI(IXO)
       C       IS FOUND BY LINEAR INTERPOLATION
```

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        C
 0029    147 XAV=(XI(K)+XI(K-1))*0.5
 0030        IF(XAV.LT.IXO)GO TO 108
 0032        XFRAC=(IXO-XI(K-1))/(XI(K)-XI(K-1))
 0033        XX=XFRAC*(EPSI(K)-EPSI(K-1))
 0034        XX=EPSI(K-1)+XX*0.5
 0035        SUM=SUM+(XX*(IXO-XI(K-1)))
 0036        K=K-1
 0037        GO TO 106
 0038    108 XFRAC=(XI(K)-IXO)/(XI(K)-XI(K-1))
 0039        XX=XFRAC*(EPSI(K)-EPSI(K-1))
 0040        XX=EPSI(K)-XX*0.5
 0041        SUM=SUM-(XX*(XI(K)-IXO))
 0042        GO TO 141
        C
        C       THE NUMBER OF XI VALUES BETWEEN X AND IXO IS FOUND.  IF IT IS
        C       ZERO, SUM IS FOUND USING A TRIANGULAR AREA
        C
 0043    106 NDIM=K-I+1
 0044        IF(NDIM-1)109,140,111
 0045    109 XX=(IXO-X)/(XI(I)-X)
 0046        SUM=0.5*XX*EPSI(I)*(IXO-X)
 0047        GO TO 110
        C
        C       IF NDIM=0, SUM IS FOUND USING A TRIANGULAR AREA
        C
 0048    140 IF(SUM.LT.0.0)SUM=0.5*(((IXO-X)/(XI(I)-X))*EPSI(I))*(IXO-X)
 0050        IF(SUM.GT.0.0)SUM=SUM+0.5*(XI(I)-X)*EPSI(I)
 0052        GO TO 110
        C
        C       IF NO XI=X, AREA FROM X TO NEAREST LARGER XI IS FOUND TRIANGULARLY
        C       IF NO XI=IXO, AREA FROM IXO TO NEAREST XI IS FOUND (SEE 29-42 ABOVE)
        C
 0053    111 IF(IJK.EQ.25)SUM=0.5*(XI(I)-X)*EPSI(I)
 0055        IF(IKK.EQ.30)GO TO 147
        C
        C       CHECK TO SEE WHICH RANGES OF XI HOLD X AND XI, IF NOTHING
        C       MATCHES, ERROR MESSAGE PRINTED
        C
 0057    141 IF((I.LE.101).AND.(K.LE.101))GO TO 112
 0059        IF(I.LE.100)GO TO 113
 0061        IF((I.LE.191).AND.(K.LE.191))GO TO 114
 0063        IF(I.LE.190)GO TO 115
 0065        IF(I.LE.201)GO TO 116
 0067        WRITE(6,205)
 0068    205 FORMAT(//,5X,'TROUBLE IN THE ''IF'' SECTION')
 0069        GO TO 197
        C
        C       IF X AND IXO ARE IN THE FIRST RANGE SUM INCREMENT AND TOTAL
        C       ARE FOUND. IF NDIM=2 INTEGRATION SUBROUTINE WON'T WORK SO
        C       SUM INCREMENT CALCULATED AS A TRAPEZOIDAL AREA.
        C       IF INTDI=2 OR THE DISTANCE FROM X OR IXO TO THE END OF A RANGE
        C       IS LESS THAN 3 STEPS ANYWHERE IN THIS SUBROUTINE OR THE NEXT
        C       ONE, THE SUM INCREMENT IS FOUND TRAPEZOIDALLY (FOR IXO) OR
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          C          TRIANGULARLY (FOR X), FOR NDIM>2 OR MORE THAN THREE STEPS
          C          ARE AVAILABLE, THE SUM INCREMENT IS FOUND BY THE
          C          SUBROUTINE IQSF.
          C
0070      112 IF(NDIM.GT.2)GO TO 117
0072          SUM=SUM+0.1*(EPSI(I)+EPSI(K))*0.5
0073          GO TO 110
0074      117 IJ=1
0075          DO 118 J=I,281
0076          EPSI(IJ)=EPSI(J)
0077      118 IJ=IJ+1
0078          CALL IQSF(0.1,EPSI,INTDI,NDIM)
0079          SUM=SUM+INTDI(NDIM)
0080          GO TO 110
          C
          C          IF X IS IN THE FIRST XI RANGE AND IXO ISN'T, THE AREA (INTEGRAL)
          C          FROM X TO THE RANGE'S END IS FOUND
          C
0081      113 IF(I.LT.100)GO TO 119
0083          SUM=SUM+0.1*(EPSI(100)+EPSI(101))*0.5
0084          NDIM=101
0085          GO TO 120
0086      119 NDIM=102-I
0087          IJ=1
0088          DO 121 J=I,281
0089          EPSI(IJ)=EPSI(J)
0090      121 IJ=IJ+1
0091          CALL IQSF(0.1,EPSI,INTDI,NDIM)
0092          SUM=SUM+INTDI(NDIM)
0093      120 IF(K.GE.191)GO TO 122
          C
          C          IF X IS IN THE FIRST RANGE AND IXO IN THE SECOND, THE AREA FROM
          C          THE SECOND RANGE'S START TO IXO IS FOUND AND THE TOTAL INTEGRAL
          C          FOUND
          C
0095          IF(K.GT.102)GO TO 123
0097          SUM=SUM+0.5*(EPSI(NDIM)+EPSI(NDIM+1))
0098          GO TO 110
0099      123 IJ=1
0100          DO 124 J=NDIM,281
0101          EPSI(IJ)=EPSI(J)
0102      124 IJ=IJ+1
0103          NDIM=K-100
0104          CALL IQSF(1.0,EPSI,INTDI,NDIM)
0105          SUM=SUM+INTDI(NDIM)
0106          GO TO 110
          C
          C          IF X IS IN THE FIRST RANGE AND IXO IN THE THIRD THE SUM INCREMENT
          C          OF THE INTEGRAL OVER THE SECOND RANGE IS FOUND
          C
0107      122 IJ=1
0108          DO 125 J=NDIM,281
0109          EPSI(IJ)=EPSI(J)
0110      125 IJ=IJ+1
```

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```
0111        NDIM=91
0112        CALL IQSF(1.0,EPSI,INTDI,NDIM)
0113        SUM=SUM+INTDI(NDIM)
0114        IF(K-192)110,126,127
       C
       C    IF X IS IN THE FIRST RANGE AND IXO IN THE THIRD, THE LAST SUM
       C    INCREMENT, FROM THE THIRD'S START TO THE XI NEAREST IXO, IS
       C    FOUND AND THE FINAL INTEGRAL TOTALED.
0115   126  SUM=SUM+(EPSI(NDIM)+EPSI(NDIM+1))*5.0
0116        GO TO 110
0117   127  IJ=1
0118        DO 128 J=91,281
0119        EPSI(IJ)=EPSI(J)
0120   128  IJ=IJ+1
0121        NDIM=K-190
0122        CALL IQSF(10.0,EPSI,INTDI,NDIM)
0123        SUM=SUM+INTDI(NDIM)
0124        GO TO 110
       C
       C    IF X AND IXO ARE IN THE SECOND RANGE THE TOTAL INTEGRAL IS FOUND
       C
0125   114  IF(NDIM.GT.2)GO TO 129
0127        SUM=SUM+0.5*(EPSI(I)+EPSI(K))
0128        GO TO 110
0129   129  IJ=1
0130        DO 130 J=I,281
0131        EPSI(IJ)=EPSI(J)
0132   130  IJ=IJ+1
0133        CALL IQSF(1.0,EPSI,INTDI,NDIM)
0134        SUM=SUM+INTDI(NDIM)
0135        GO TO 110
       C
       C    IF X IS IN THE SECOND RANGE AND IXO IN THE THIRD THE INTEGRAL
       C    INCREMENTS FROM X TO THE END OF THE SECOND RANGE AND FROM THE
       C    END OF THE SECOND TO THE XI VALUE NEAREST IXO ARE FOUND
       C    AND THE TOTAL INTEGRAL IS ADDED UP
0136   115  IF(I.LT.190)GO TO 131
0138        SUM=SUM+0.5*(EPSI(190)+EPSI(191))
0139        NDIM=191
0140        GO TO 132
0141   131  NDIM=192-I
0142        IJ=I
0143        DO 133 J=I,281
0144        EPSI(IJ)=EPSI(J)
0145   133  IJ=IJ+1
0146        CALL IQSF(1.0,EPSI,INTDI,NDIM)
0147        SUM=SUM+INTDI(NDIM)
0148   132  IF(K.GT.192)GO TO 134
0150        SUM=SUM+5.0*(EPSI(NDIM)+EPSI(NDIM+1))
0151        GO TO 110
0152   134  IJ=1
0153        DO 135 J=NDIM,281
```

```
0154          EPSI(IJ)=EPSI(J)
0155    135 IJ=IJ+1
0156          NDIM=K-190
0157          CALL IOSF(10,0,EPSI,INTDI,NDIM)
0158          SUM=SUM+INTDI(NDIM)
0159          GO TO 110
       C
       C       IF X AND IXO ARE IN THE THIRD RANGE THE REMAINING INTEGRAL
       C       INCREMENT (FROM X TO THE XI VALUE NEAREST IXO) IS
       C       OBTAINED AND THE INTEGRAL TOTAL CALCULATED
       C
0160    116 IF(NDIM.GT.2)GO TO 136
0162          SUM=SUM+5.0*(EPSI(I)+EPSI(K))
0163          GO TO 110
0164    136 IJ=1
0165          DO 137 J=I,281
0166          EPSI(IJ)=EPSI(J)
0167    137 IJ=IJ+1
0168          CALL IOSF(10.0,EPSI,INTDI,NDIM)
0169          SUM=SUM+INTDI(NDIM)
       C
       C       INTEGRAL TOTAL IS REASSIGNED, ERROR MARKER IS RESET, AND IF
       C       INTEGRAL<0 ERROR MARKER IS RESET AND ERROR MESSAGE PRINTED
       C       RETURN TO MAIN PROGRAM WITH INTEGRAL
       C
0170    110 INTDI(1)=SUM
0171          IJK=0
0172          IF(SUM.GE.0.0)GO TO 199
0174    197 WRITE(6,204)I,K,X,IXO,XI(I),XI(K),EPSI(I),EPSI(K),SUM
       $,NDIM,EPSI(K-1)
0175    204 FORMAT(///,'   CONFUSION IN SIGI AT 110: I=',I6,/,29X,
       $'K=',I6,/,29X,'X=',1PE15.6,/,27X,'IXO=',I7,/,25X,'XI(I)=',
       $1PE15.6,/,25X,'XI(K)=',1PE15.6,/,23X,'EPSI(I)=',1PE15.6,/,
       $23X,'EPSI(K)=',1PE15.6,/,27X,'SUM=',1PE15.6,//,
       $26X,'NDIM=',I6,/,21X,'EPSI(K-1)=',1PE15.6,//)
0176    198 IJK=111
0177    199 RETURN
0178          END
```

```
0001        SUBROUTINE DIVIN(XIEPS,IXO,INTDI,XI,IJK)
      C
      C       CALCULATES INTEGRAL FROM 0 TO IXO. INTEGRATIONS IN THIS SUBROUTINE
      C       ARE SUBJECT TO THE SAME LIMITATIONS AS IN THE PREVIOUS SUBROUTINE
      C       AND ARE CIRCUMVENTED IN THE SAME WAY
      C          XIEPS   THE RANGE OF VALUES TO BE INTEGRATED OVER
      C          IXO     UPPER LIMIT OF INTEGRATION
      C          INTDI   STORAGE SPACE OF INTEGRATION RESULTS
      C          XI      SET OF HORIZONTAL AXIS VALUES CORRESPONDING TO XIEPS
      C          IJK     ERROR MARKER
      C
      C
0002        REAL XI(1)
0003        DOUBLE PRECISION INTDI(1),SUM,XIEPS(1)
      C
      C       INTEGRAL STORAGE SPACE SET TO 0 AND POSITION OF XI VALUE
      C       NEAREST BUT NOT SMALLER THAN IXO FOUND
      C
0004        SUM=0.0
0005        DO 60 I=1,281
0006        IF(XI(I)-IXO)60,61,62
0007     60 CONTINUE
      C
      C       IF IXO<XI(281) ERROR MESSAGE PRINTED
      C
0008        WRITE(6,65)IXO
0009     65 FORMAT(///,'   IN DIVIN 60 LOOP IXO=',I7)
0010        IJK=111
0011        GO TO 999
      C
      C       IF IXO DOES NOT EQUAL A VALUE OF XI THE AREA BETWEEN IXO AND
      C       THE NEAREST XI VALUE IS FOUND TRAPEZOIDALLY AND ADDED TO SUM
      C
0012     62 XAV=(XI(I)+XI(I-1))/2.0
0013        IF((XAV.GT.IXO).AND.(I.GT.2))GO TO 64
0015        XFRAC=(XI(I)-IXO)/(XI(I)-XI(I-1))
0016        XX=(XIEPS(I)-XIEPS(I-1))*XFRAC
0017        XX=(XIEPS(I)+XIEPS(I)-XX)/2.0
0018        SUM=SUM-(XX*(XI(I)-IXO))
0019        GO TO 61
0020     64 XFRAC=(IXO-XI(I-1))/(XI(I)-XI(I-1))
0021        XX=(XIEPS(I)-XIEPS(I-1))*XFRAC
0022        XX=(XIEPS(I-1)+XIEPS(I-1)+XX)/2.0
0023        SUM=SUM+((IXO-XI(I-1))*XX)
0024        I=I-1
      C
      C       IF IXO IS IN THE FIRST XI RANGE THE TOTAL INTEGRAL IS FOUND,
      C       IF IT IS NOT THE INTEGRAL PART FROM 0 TO THE END OF THE FIRST
      C       RANGE IS FOUND AND ADDED TO SUM
      C
0025     61 IF(I.GE.101)GO TO 3
0027        NDIM=I
0028        IF(NDIM.GT.2)GO TO 5
0030        SUM=SUM+(0.1*(XIEPS(1)+XIEPS(2))/2.0)
```

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```
0031        GO TO 998
0032   3    NDIM=101
0033   5    CALL IQSF(0.1,XIEPS,INTDI,NDIM)
0034        SUM=SUM+INTDI(NDIM)
0035        IF(I.LE.101)GO TO 998
       C
       C        IF IXO IS IN THE SECOND XI RANGE THE INTEGRAL PART
       C        FROM THE START OF THE SECOND RANGE TO IXO IS FOUND AND ADDED
       C        TO SUM FOR THE TOTAL INTEGRAL. IF IT IS IN THE THIRD RANGE
       C        THE INTEGRAL PART COVERING THE SECOND RANGE IS FOUND AND
       C        ADDED TO SUM
       C
0037        IF(I.GE.191)GO TO 4
0039        NDIM=I-100
0040        IF(NDIM.GT.2)GO TO 8
0042        SUM=SUM+((XIEPS(101)+XIEPS(102))/2.0)
0043        GO TO 998
0044   4    NDIM=91
0045   8    IJ=101
0046        DO 9 J=1,181
0047        XIEPS(J)=XIEPS(IJ)
0048   9    IJ=IJ+1
0049        CALL IQSF(1.0,XIEPS,INTDI,NDIM)
0050        SUM=SUM+INTDI(NDIM)
0051        IF(I.LE.191)GO TO 998
       C
       C        FOR IXO IN THE THIRD XI RANGE THE REMAINDER OF THE INTEGRAL
       C        (FROM THE BEGINNING OF THE THIRD RANGE TO IXO) IS FOUND
       C        AND ADDED TO SUM FOR THE TOTAL INTEGRAL
       C
0053        NDIM=I-190
0054        IF(NDIM.GT.2)GO TO 12
0056        SUM=SUM+(10.0*(XIEPS(91)+XIEPS(92))/2.0)
0057        GO TO 998
0058   12   IJ=91
0059        DO 13 J=1,91
0060        XIEPS(J)=XIEPS(IJ)
0061   13   IJ=IJ+1
0062        CALL IQSF(10.0,XIEPS,INTDI,NDIM)
0063        SUM=SUM+INTDI(NDIM)
0064   998  INTDI(1)=SUM
0065   999  RETURN
0066        END
```

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```
0001          SUBROUTINE IQSF(H,Y,Z,NDIM)
      C
      C          THIS SUBROUTINE WAS LIFTED BODILY FROM IBM'S SSP WHERE IT WAS
      C          CALLED "QSF", HERE THE INTEGRAL IS FOUND BY SIMPSON'S RULE
      C          H    THE INCREMENT OF ARGUEMENT VALUES (I.E. A CONSTANT)
      C          Y         THE INPUT FUNCTION VALUES, SEPARATED BY H
      C          Z         THE RESULTING INTEGRAL VALUES
      C          NDIM   THE NUMBER OF VALUES TO BE INTEGRATED OVER
      C
0002          DOUBLE PRECISION Y(1),Z(1),SUM1,SUM2,AUX,AUX1,AUX2
0003          HT=.33333333*H
0004          L1=1
0005          L2=2
0006          L3=3
0007          L4=4
0008          L5=5
0009          L6=6
0010          IF (NDIM-5)7,8,1
      C
      C          NDIM IS GREATER THAN 5, PREPARATIONS OF INTEGRATION LOOP
      C
0011     1    SUM1=Y(L2)+Y(L2)
0012          SUM1=SUM1+SUM1
0013          SUM1=HT*(Y(L1)+SUM1+Y(L3))
0014          AUX1=Y(L4)+Y(L4)
0015          AUX1=AUX1+AUX1
0016          AUX1=SUM1+HT*(Y(L3)+AUX1+Y(L5))
0017          AUX2=HT*(Y(L1)+3.875*(Y(L2)+Y(L5))+2.625*(Y(L3)+Y(L4))+Y(L6))
0018          SUM2=Y(L5)+Y(L5)
0019          SUM2=SUM2+SUM2
0020          SUM2=AUX2-HT*(Y(L4)+SUM2+Y(L6))
0021          Z(L1)=0.
0022          AUX=Y(L3)+Y(L3)
0023          AUX=AUX+AUX
0024          Z(L2)=SUM2-HT*(Y(L2)+AUX+Y(L4))
0025          Z(L3)=SUM1
0026          Z(L4)=SUM2
0027          IF (NDIM-6)5,5,2
      C
      C          INTEGRATION LOOP
      C
0028     2    DO 4 I=7,NDIM,2
0029          SUM1=AUX1
0030          SUM2=AUX2
0031          AUX1=Y(I-1)+Y(I-1)
0032          AUX1=AUX1+AUX1
0033          AUX1=SUM1+HT*(Y(I-2)+AUX1+Y(I))
0034          Z(I-2)=SUM1
0035          IF(I-NDIM)3,6,6
0036     3    AUX2=Y(I)+Y(I)
0037          AUX2=AUX2+AUX2
0038          AUX2=SUM2+HT*(Y(I-1)+AUX2+Y(I+1))
0039     4    Z(I-1)=SUM2
0040     5    Z(NDIM-1)=AUX1
```

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```
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0041          Z(NDIM)=AUX2
0042          RETURN
0043    6     Z(NDIM-1)=SUM2
0044          Z(NDIM)=AUX1
0045          RETURN
        C
        C     END OF INTEGRATION LOOP
0046    7     IF (NDIM-3)12,11,8
        C
        C     NDIM IS EQUAL TO 4 OR 5
0047    8     SUM2=1.125*HT*(Y(L1)+Y(L2)+Y(L2)+Y(L3)+Y(L3)+Y(L4))
0048          SUM1=Y(L2)+Y(L2)
0049          SUM1=SUM1+SUM1
0050          SUM1=HT*(Y(L1)+SUM1+Y(L3))
0051          Z(L1)=0.
0052          AUX1=Y(L3)+Y(L3)
0053          AUX1=AUX1+AUX1
0054          Z(L2)=SUM2-HT*(Y(L2)+AUX1+Y(L4))
0055          IF(NDIM-5)10,9,9
0056    9     AUX1=Y(L4)+Y(L4)
0057          AUX1=AUX1+AUX1
0058          Z(L5)=SUM1+HT*(Y(L3)+AUX1+Y(L5))
0059    10    Z(L3)=SUM1
0060          Z(L4)=SUM2
0061          RETURN
        C
        C     NDIM IS EQUAL TO 3
0062    11    SUM1=HT*(1.25*Y(L1)+Y(L2)+Y(L2)-.25*Y(L3))
0063          SUM2=Y(L2)+Y(L2)
0064          SUM2=SUM2+SUM2
0065          Z(L3)=HT*(Y(L1)+SUM2+Y(L3))
0066          Z(L1)=0.
0067          Z(L2)=SUM1
0068    12    RETURN
0069          END
```

Appendix B

Auxiliary listings

A listing is provided of the BASIC program that produces values of ξ, $e^{-\psi}$, ψ', and $e^{-\psi}\psi'$ (see equation 4, page 7). The last set of values are those entered as isothermal gas sphere data for the appropriate ξ values as specified on page 36.

To obtain values used in this thesis for the first range of ξ (0.0 to 9.9 in increments of 0.1), the step size to enter is 0.001 and the number of steps is 100; for the second range of ξ (10 to 99 in increments of 1) the step size is 0.01 and the number of steps is 100; and for the third ξ range (100 to 1000 in increments of 10) the step size is 0.1 and the number of steps is 100.

As can be seen for the portion of an output run included in Appendix J, the values of ξ are not exact, but the small difference is not considered significant and so is ignored. Also, when the program is run for the second ξ range, values for $e^{-\psi}$, ψ', and $e^{-\psi}\psi'$ are calculated for $\xi=1, 2, 3, \ldots, 9$. These $e^{-\psi}\psi'$ values were not used since values corresponding to the first nine ξ values were provided by calculations for the first ξ range. Similarly, the first nine sets of numbers produced in third range calculations were ignored since second range calculations had included them.

The BASIC program does not give a value of $e^{-\psi}\psi'$ for $\xi=0$, a necessary value, but since for this ξ the product $e^{-\psi}\psi'=0$ this is not a problem.

Also included in this Appendix are the modifications performed on YAHOO to get the programs BGIN and TAFCHEC. BGIN is the variant that reads the counted background density as a constant and only makes best fit determinations for x_o, α, and σ_c. The first page of a typical output of BGIN is also provided in Appendix J.

TAFCHEC was used for the program testing in Chapter III, section (ii). The only difference between TAFCHEC and YAHOO is that the former also calculates the χ^2 found for each x_o when comparing the data to the model made using the current x_o and Taff's values of α, σ_c, and σ_{bg}. This provides the bracketed numbers of Tables 3 and 5 (pages 23 and 27 respectively).

Detailed explanations of the modifications done to YAHOO to obtain BGIN and TAFCHEC are given in Appendix C.

```
10 DIM Y(2),Z(2),F(2),A(2),B(2),C(2),D(2)
20 X=0
30 Y(1)=0
40 Y(2)=0
50 PRINT 'ENTER STEP SIZE, NUMBER OF STEPS'
60 INPUT H,N
70 PRINT H,N
71 PRINT
72 PRINT 'XI','EXP(-PSI)','PSI''','EXP(-PSI)*PSI'''
73 PRINT
75 FOR K=1 TO 100
80 GOSUB 110
85 X3=EXP(-Y(1))
86 X4=X3*Y(2)
90 PRINT X,X3,Y(2),X4
95 NEXT K
100 STOP
110 FOR J=1 TO N
120 Z(1)=Y(1)
130 Z(2)=Y(2)
140 GOSUB 360
150 A(1)=F(1)
160 A(2)=F(2)
170 Z(1)=Y(1)+H*A(1)/2
180 Z(2)=Y(2)+H*A(2)/2
190 X=X+H/2
200 GOSUB 360
210 B(1)=F(1)
220 B(2)=F(2)
230 Z(1)=Y(1)+H*B(1)/2
240 Z(2)=Y(2)+H*B(2)/2
250 GOSUB 360
260 C(1)=F(1)
270 C(2)=F(2)
280 Z(1)=Y(1)+H*C(1)
290 Z(2)=Y(2)+H*C(2)
300 X=X+H/2
310 GOSUB 360
312 D(1)=F(1)
315 D(2)=F(2)
320 Y(1)=Y(1)+(A(1)+2*B(1)+2*C(1)+D(1))*H/6
330 Y(2)=Y(2)+(A(2)+2*B(2)+2*C(2)+D(2))*H/6
340 NEXT J
350 RETURN
360 F(1)=Z(2)
370 IF X>0 THEN 400
380 F(2)=1/3
390 RETURN
400 F(2)=EXP(-Z(1))-2*Z(2)/X
410 RETURN
420 END
```

Ready

Modifications to YAHOO to get BGIN

The line numbers referred to are those of YAHOO
as it is found in Appendix A. All changes are in the MAIN
program; the subroutines SIGI, DIVIN, and IQSF are left
unchanged.

Change line 4 to:

        DOUBLE PRECISION INTDI(281), XIEPS(281), CHISQ, DIVN,

            SIGC, SC(20), XX1, XX2, SUM1, SUM2, Z1, Z2, EI, UI

Change line 19 to:

        READ(1,230) SC1, SBG

Delete line 21.

Change line 22 to:

        WRITE(6,880) SC1, SBG

Change line 23 to:

    880 FORMAT(/,5X,'INITIAL CENT DENS=',1PE11.2,/,14X,

            'BG DENS=',1PE11.2)

Delete line 61.

Change line 69 to:

```
   25  FORMAT(4(4X,'ALPHA    CENT DENS    CHI-SQ  '))
```

Replace lines 111 through 115 inclusive by:

```
     SUM1=0.0

     SUM2=0.0
```

Change line 117 to:

```
     Z1=SC(IP)*SIGISO(L)+SBG
```

Replace lines 123 through 132 inclusive by:

```
     EI=Z2/Z1

     UI=(1./Z1)+((2.*Z2/Z1)/Z1)+(((Z2/Z1)*Z2/Z1)/Z1)

     SUM1=SUM1+BI(L)*SIGISO(L)*(2.*EI+EI*EI)

 203 SUM2=SUM2+BI(L)*SIGISO(L)*SIGISO(L)*UI

     IF(ABS(SUM2).GT.1.0E-12)GO TO 204
```

Delete line 135.

Change line 142 to:

```
     SC(IP+1)=SC(IP)+SUM1/(2.*SUM2)
```

Delete line 143.

Change line 148 to:

```
     IF(XX2.LE.XX1)GO TO 207
```

Delete lines 152 through 157 inclusive.

Delete line 160.

Change line 161 to:

    IF(XX1.LE.0.0001)GO TO 209

Delete line 169.

Delete line 172.

Change line 176 to:

    GO TO 703

Delete lines 178 through 182 inclusive.

Change line 184 to:

    310 SC(IR)=0.0

Delete line 185.

Change line 188 to:

    NCALC(I)=3.141593*BI(I)*(SIGC*SIGISO(I)+SBG)

Delete line 207.

Replace lines 211 through 214 inclusive by:

```
     AXXA(IZ+2)=CHISQ
     IZ=IZ+3
     IF(IZ.LE.11)GO TO 29
```

Replace lines 227 through 232 inclusive by:

```
     IF(IAM.EQ.1)WRITE(6,805)SC(IP), SIGISO(L), IP, L
 805 FORMAT(10X,'SC(IP)=',1PE11.3,4X,'SIGISO(L)=',1PE11.3,
    '     IP=',I3,'    L=',I3,/,10X,'PROBLEMS IN THE
     203 LOOP')
     IF(IAM.EQ.2)WRITE(6,806)DNOM, SC(IP)
 806 FORMAT(10X,'DNOM=',1PE11.3,'    AND NON-ITERATION
     OCCURRED TOO OFTEN    SC=',1PE11.3)
```

Delete lines 236 and 238.

Change lines 250 and 251 to:

```
 318 WRITE(6,322)A2, S2, C2
 322 FORMAT(13X,'MIN FROM PROGRAM:    ',3(1PE11.3))
```

Delete line 258.

Delete line 269.

Insert between lines 277 and 278:

```
     X1=ALPHA
```

```
     X2=SIGC

     X3=CHISQ
```

Change lines 280 and 281 to:

```
  342 WRITE(6,343)A2, S2, C2

  343 FORMAT(14X,'MINIMUM METHOD:    ',3(1PE11.3))
```

Modifications to YAHOO to get TAFCHEC

Again line numbers refer to those of YAHOO as it appears in Appendix A and all subroutines are unchanged.

Insert between lines 24 and 25:

```
     READ(1,235)TAL, TSC, TSBG

  235 FORMAT(E5.2,2(2X,E5.2))

     WRITE(6,236)TAL, TSC, TSBG

  236 FORMAT(/,'  TAFF''S VALUES: ALPHA=',1PE11.2,/,18X,
          'SIGC=',1PE11.2,/,17X,'SIGBG=',1PE11.2)
```

Insert between lines 56 and 57:

```
     ITAFF=5
```

Insert between lines 102 and 103:

```
     IF(ITAFF.GT.100)GO TO 820
```

Insert between lines 107 and 108:

```
      IF(ITAFF.GT.100)GO TO 820
```

Insert between lines 281 and 282:

```
      ITAFF=500
      ALPHA=TAL
      GO TO 10
  820 CHISQ=0.0
      DO 821 I=1,J
      NCALC(I)=3.141593*BI(I)*(TSC*SIGISO(I)+TSBG)
  821 CHISQ=CHISQ+((NOBS(I)-NCALC(I))**2)/NCALC(I)
      WRITE(6,882)ALPHA, CHISQ
  882 FORMAT(/,26X,'TAFF''S VALUES :     ',1PE11.3,22X,
     1PE11.3)
```

Appendix C

Detailed program explanations

A detailed explanation of the program YAHOO is provided. Also given are explanations of the modifications of YAHOO needed to obtain BGIN and TAFCHEC.

In all cases in this Appendix, line numbers refer to the line numbers of YAHOO as they occur in Appendix A. The insertions, deletions, changes, and replacements referred to in explanations of the modifications for BGIN and TAFCHEC are those listed in Appendix B.

*YAHOO

MAIN Program

Lines	Function and/or relation to theory
1-5	Declaration statements
6-9	Read in and rewrite a message at the beginning of the data deck. This checks to make sure that the proper type of data is being used and to make sure the printer is on the 132 line width mode (item 1 on page 36).
10-11	Isothermal gas sphere density data is entered (data is from the BASIC program; item 2 on page 36).

12-19   J, NOBS, ROUT, and initial estimates for σ_c (SC1)
and σ_{bg} (SBG1) are read in (items 4 to 7 on page
36). The command to skip a line in line 13 allows
the program to jump over the line describing the
format of items 4 to 6 (item 3 on page 36).

20-21   Values two times those of the initial estimates
for σ_c and σ_{bg} are put aside for future use (lines
173 and 178).

22-23   The initial estimates for σ_c and σ_{bg} are printed
out.

24   Format for line 20.

25-29   If there are more rings than the necessary array
sizes permit, a proper notice is printed and the
program stops.

30   IJK, an error marker needed later on, is set to
zero.

31-31   The number of rings being used is printed out.

33-44   The series XI (ξ in equation 4, page 7), the
unitless radius of the isothermal gas sphere, is
calculated for the corresponding densities entered
in line 10.

45-46   Table titles are printed.

47   ROUT(1)$\equiv r_1 = 0$, see explanation of equation (1),
page 6.

48-55   A series of values is calculated and printed.
BI(I)$\equiv \beta_i = (r_{i+1}^2 - r_i^2)$, see equation (1), page 6,

and series of equations defining the Newton-Raphson terms, pages 10 and 11. NOBSand ROUT are reprinted to ensure proper entry; SIGOB and RAV are printed to enable the drawing of a radial density diagram for the cluster.

56   Start of the XO loop. $XO \equiv x_0$ in equation (4), page 7, and is the upper limit to integration of the isothermal gas sphere. As can be seen from XI, data are sufficient to allow a maximum XO of 1000 (lines 33-44).

57   IND is a marker used to determine whether the program is calculating values by (I) increasing α or by (II) averaging the α values producing the two smallest χ^2.

$$IND=5 \rightarrow (I)$$
$$IND=70 \rightarrow (II)$$

For case (II) storage spaces are needed for the smallest χ^2 values and their associated α values, they are C1, C2, A1, and A2. Intermediate values are stored in C3 and A3. For the current minimum χ^2 in case (II), the associated values obtained for σ_c and σ_{bg} are stored in S2 and SB2 respectively.

58-59   C2 and A2 are set abnormally high so as to allow the first value for χ^2 obtained to become the current minimum. It is necessary to set them high because finding minimum values works by comparison.

(An alternate method would be to assign Cl and C2 (and Al and A2) the first two χ^2 (and α) values calculated in the loop starting at line 79, but counters, etc. would have to be added making this method more cumbersome.)

60-61   Since σ_c and σ_{bg} are solved iteratively, the iterated solutions are stored in arrays for later testing. These arrays, SC(I) and SBG(I) respectively, start with the estimated values that were entered as data. -

62   An initial α is calculated.

63   $\log(\alpha)$ is stored. Since α is incremented in steps of $\log(\alpha)+0.08$ this value is necessary.

64   $LOGA1 \equiv \log(\alpha_{initial})+4$. When (or if) α reaches this value, the program moves to case (II).

65   IZ is a counter needed for storing a triple row of results before printing; see lines 209-219.

66-67   The current value of XO is printed.

68-69   Headings for the results are printed.

70-71   The values $\xi e^{-\psi}\psi'$ are calculated (see equation 4, page 7) as XIEPS(I).

72   The integral

$$\int_0^{x_0} \xi\, e^{-\psi}\, \psi'\, d\xi$$

is calculated in subroutine DIVIN.

73-77   If there is an error somewhere in the subroutine,

IJK=111. This causes the printing of the message and the choosing of the next XO.

78   The integral is returned to the MAIN program as INTDI(1). Since the array INTDI is needed later, the integral is stored as DIVN.

79   An important loop is started. This one calculates the density of an isothermal gas sphere at the distances at which observed densities are found.

80      $X \equiv x_i$ in equation (4). It is the unitless distance of the observed density.

81   If X is greater than the upper limit of integration this loop is exited. Go to line 104; calculation is impossible under these conditions.

83   X^2 is calculated for later usage.

84-85   The array XI is searched to find the position of the value $\geqslant$ X.

87-89   If no value of XI suits, a message is printed and the program stops.

90-92   The values $e^{-\psi} \psi' \sqrt{\xi^2 - x^2}$ are calculated as XIEPS (see equation 4, page 7).

93   The position of the first XI $\geqslant$ X is decreased by one and

94-95   all XIEPS values up to the first XI $\geqslant$ X are set to zero.

96   The integral

$$\int_{x}^{x_0} \sqrt{\xi^2 - x^2}\ e^{-\psi}\psi'\ d\xi$$

is calculated in the subroutine SIGI; the result is stored in INTDI(1).

97-101 Check for errors in the subroutine, if there is one the message is printed and the next XO is chosen. If there are no errors, continue.

102 End of the isothermal density loop. The isothermal gas sphere density value for this value of RAV is calculated. The operations return to line 80 to calculate the σ_{iso} for the next RAV. -

103 After σ_{iso} is calculated for each RAV the program goes to line 108.

104 Operations go here if conditions in line 81 are met. If I=1 (i.e. the initial X > XO) then the operations are sent to line 220 to increase α and so decrease X.

106-107 If I > 1, then the values for σ_{iso} not yet calculated, and so unable to be calculated, are set to zero.

108-109 After σ_{iso} is calculated two counters are set. The loop to follow runs 20 times, but can go to 40, 60, 80, or 100. The counter II indicates how many groups of 20 times the loop has run. The counter IXY will be explained later.

110 Start of the iteration loop for the Newton-Raphson

method. The iteration runs 20 times, which has been found through tests to be sufficient to get convergence to within 10^{-3}. Tests are done later to check for 10^{-4} interior convergence and the 20 step iterative procedure can be repeated up to four times if necessary.

111-115   Summation terms to be used in the iteration are set to zero initially.

116   An interior loop which is used to perform the necessary summations is started.

117   $Z1 \equiv \sigma_c \sigma_{iso}(x_i) + \sigma_{bg}$. This is the density of an isothermal gas sphere model at the distance x_i using the current density values of σ_c and σ_{bg}.

118-121 $Z1$ is tested to see if it equals zero. If so, IAM, a printing command, is set accordingly and the operations move to line 225.

122-125   If $Z1 \neq 0$, continue setting up sub-components of the summations. Referring to the equations of pages 10 and 11,

$$E1 = 2\varepsilon_i - \varepsilon_i^2.$$

and

$$E2 = \mu_i$$

126-130 $F \equiv f$; $FC \equiv f_x$; $GBG \equiv g_y$;

$G \equiv g$; $FBG \equiv f_y g_x$;   see pages 10 and 11

Line 130 is the end of the summation loop.

131   The denominator term from the equations of page 12 (i.e. $f_x g_y - f_y^2$) is calculated.

132   The size of DNOM is checked. If it is too small,

the size of the calculated value using DNOM would probably exceed the computer's capacity, and so no iteration is performed. If DNOM is not too small operations proceed to line 141.

134-135   Instead of iterating, the next values of σ_c and σ_{bg} are set to the previous values.

136   Also, counter IXY is increased by 1.

137   If this has happened less than 6 times in a row (i.e. IXY$\leqslant$5) and the loop is at less than the 19th iterative step (i.e. IP<19), then operations return to the beginning of the iteration loop, line 110, for the next iterative step.

139-140   If this has happened 6 or more times and IP=19, then an error message counter is set and the operations go to line 225.

141   If the value of DNOM is sufficiently large iteration can be performed and the value of IXY is reset to zero.

142-143   The iterative steps are performed (see the equations on page 12).

144   After the steps are performed, return to the start of the Newton-Raphson loop, line 110.

145-147   After 20 iterations a loop is started to check the last 6 σ_c and σ_{bg} values, in groups of 3 consecutive values, for convergence, with some leeway for slight nonconvergence. This is done by

comparing the size differences between the L and
L-1 terms and the L and L+1 terms. If the
difference between L and L-1 is greater than the
difference between L and L+1, then the series is
converging. Leeway is built in by adding 0.005 of
the L term to the L-(L-1) difference. XX1 is the
L-(L-1) difference with the leeway term, and XX2
is the (L+1)-L difference.

148   If convergence occurs for σ_c, the same test is
used for σ_{bg}.

150-151   If σ_c convergence does not occur, the error
message counter is set and operations go to line
225.

152-158   Convergence for σ_{bg} is tested. If it is found,
operations return to the loop's start, line 145,
and if convergence is not found the error message
counter is set and operations move to line 225.

159-161   After convergence for the last 6 iterations of σ_c
and σ_{bg} has been confirmed, the degree of
convergence is tested. If the difference between
the last and second last iterated values for both
σ_c and σ_{bg} is less than or equal to 10^{-4} of the
last iterated value, the values are satisfactory
and operations proceed to line 171. In this check
XX1 refers to σ_c and XX2 to σ_{bg}.

163   If the degree of convergence is not sufficient and

the iterative loop has been run less than 5 times, operations proceed to line 167.

165-166 If the degree of convergence is insufficient and the loop has run 5 times an error message counter is set and operations move to line 225.

167 From line 163. The counter which keeps track of the number of times the iteration loop is run is incremented by 1.

168-169 The last iterated value for σ_c and σ_{bg} from the last run through the iteration loop is moved to a lower place in the SC and SBG arrays and will be the initial value for the next run through of the iteration loop.

170 The program is sent to the start of the Newton-Raphson iteration loop, line 110.

171-172 From line 161. If convergence standards are met, the final values obtained from the iteration loop are accepted as the best σ_c and σ_{bg} for the $x_0-\alpha$ combination used.

173-182 This section checks to see if the accepted values are greater than twice the initial estimates which were fed in. If they are, the initial estimates are used as the first values in the Newton-Raphson iteration loop for the next α value. If the accepted values are less than twice the initial estimates then the accepted values will be used.

This is done because tests have found that if initial values in the iteration loop are too large by several times, the loop usually converges to the wrong root of the set of equations. However, if the initial estimates are too small, but still positive, this will not occur.

183-185   After the initial values for the next run of the iteration loop have been set, the rest of the σ_c and σ_{bg} arrays are set to zero as a safety measure.

186   Since the χ^2 test involves a summation, the space used to store the χ^2 value is initially set to 0.

187   The loop to calculate χ^2 is started and runs once for each ring.

188   The theoretical number of galaxies for the specific ring is calculated from the ring area and the density in galaxies/arcmin^2. This density is calculated from the σ_c and σ_{bg} values (obtained from the iteration loop) and from the isothermal gas sphere densities previously obtained for these rings.

189   The χ^2 is calculated.

190   If IND=5 (see line 57) the program is still operating in method (I). If IND=70 the program is operating by method (II). If the program is in method (II) operations proceed to line 262.

192   The value of χ^2 is checked. If $\chi^2<0$, then at

least one of the values found for σ_c and σ_{bg} is large and negative and so physically unreasonable. In this case the present χ^2 is not to be compared to the current minimum χ^2 and operations go to line 209.

194-208   This section preserves the minimum χ^2 and associated α, σ_c, and σ_{bg} as C2, A2, S2, and SB2. The χ^2 and α values preceding and following the minimum χ^2 are saved, respectively as C1, A1 and C3, A3. This is done so the minimum χ^2 and associated values are isolated from the rest of the results found for a given XO and may be printed separately and also so that the three smallest χ^2 and associated α values are available for method (II).

209-212   The current χ^2 and associated values are stored in part of an array, AXXA, in groups of 4.

213   The array index, IZ, is incremented by 4 to allow the next group of 4 values to be stored the next time operations reach line 209. This means that consecutive groups of results, with 4 numbers per group, are stored linearly in larger collections of 3 groups. This is due to the results being printed in the same manner in which they are stored and paper width only allows the printing of 12 numbers. The output, to be read sequentially,

must be read as 3 groups of 4 numbers from left to
right across the page before proceeding to the
next line.

214   Since AXXA has only 12 spaces, when they are
filled a line of results must be printed before
more can be stored. If it is filled by now IZ=13,
so this line checks to see if AXXA is filled. If
it is not, proceed to line 220.

216-219   Because AXXA is filled, its contents are printed
and IZ is reset to allow values to be stored in the
array again.

220-221   From lines 214 or 104. α is increased by
incrementing $\log(\alpha)$.

222    $\log(\alpha)$ is checked to see if it is too large (see
lines 62-64). If it is not too large a best σ_c
and σ_{bg} will be found for the new $x_0-\alpha$ combination
starting at line 79.

224   Since α is now too large, go to line 245.

225   From lines 121, 140, 151, 157, or 166. If the
program is in method (I) it proceeds to line 245.

227-243   Since the program is in method (II) the proper
message is printed to explain why the program
cannot operate properly as indicated by the error
message counter IAM.

244   The program goes to line 282 to choose a new XO,
and reverts to method (I).

245-249   From line 225. Since the program cannot operate any further in method (I), the last of the results stored in AXXA are printed and IZ is reset for the next run of method (I).

250-251   The minimum χ^2 and associated values as found from method (I) for the current XO are printed.

252-255   The program is about to commence operating in method (II). In this method the smallest χ^2 is still called C2, but the second smallest χ^2 is called C3. These lines check the χ^2 preceding and following C2 as found in method (I) to see which is smaller. If C3 is already smaller than C1 the program proceeds to line 256. If C1 is smaller, C3 is assigned its value and A3 is assigned the value of A1.

256   IND is reset to indicate the usage of method (II).

257-258   The initial σ_c and σ_{bg} to be used in the Newton-Raphson iteration loop are set to be the values producing the minimum χ^2.

259   A new α is found by averaging the α values producing the two smallest χ^2 values.

260   A counter to indicate how often method (II) has run for this XO is set.

261   With the new α, new values of σ_c, σ_{bg}, and χ^2 are to be found. Proceed to line 79.

262   From line 190. The χ^2 found from the new α as

obtained by method (II) is compared to the previous minimum x^2. If it is not smaller, go to line 271.

264-270 Since the new x^2 is smaller than C2, values are reassigned accordingly, with the new x^2 becoming C2 and the old C2 becoming C3, the α values being reassigned similarly, and the σ_c and σ_{bg} producing this new minimum x^2 being stored.

271 From line 262. Even though the new x^2 is greater than C2, it is checked to see if it smaller than the second smallest x^2. If not, operations go to line 275.

273-274 Since the new x^2 is smaller than the previous second smallest x^2, C3 and A3 are reassigned accordingly.

275 From lines 270, 271, or 274. IDD is checked to see if method (II) has averaged α values the required number of times. If it has, go to line 280.

277 A new α is obtained from those associated with the two smallest x^2 values.

278 The method (II) frequency counter is incremented.

279 With the new α, proceed to line 79.

280-282 From line 275. Since method (II) has been run the appropriate number of times the results obtained are printed. Then the program chooses a new XO and returns to begin method (I) again.

283-286   From lines 29, 89, or 282. Either a fatal error

has occurred or the program has operated over the

required range of XO values. Several lines are

skipped on the output and the program ends.

Subroutine SIGI

This subroutine is called from line 96 in the MAIN.
program and is used to calculate the integral

$$\int_{x}^{x_O} \sqrt{\xi^2 - x^2}\ e^{-\psi}\ \psi'\ d\xi$$

The factors transferred to this subroutine from
the MAIN program are the values x and x_O, the series of
values of ξ from 0 to 1000, and the series of values for
$\sqrt{\xi^2 - x^2}\ e^{-\psi}\ \psi'$ corresponding to the ξ values. These factors
are represented in this subroutine as X, IXO, XI, and EPSI
respectively, with the last two being arrays. The values of
EPSI from XI=0 (i.e. XI(1)) to the value of XI nearest but
still smaller than X are all equal to zero (see MAIN, lines
84-95).

The major facet complicating this subroutine is
that while XI increases in three ranges with different
incremental step sizes in each range (i.e. in the first

range XI increases in steps of 0.1 from 0 to 9.9; in the second XI range the increment size is 1 from 10 to 99; and in the third range XI increases in steps of 10 from 100 to 1000) and X and IXO can be in any of these ranges, the integration subroutine IQSF can only integrate over an interval using identical incremental steps. Therefore, unless X and IXO occur in the same range, the different ranges must be integrated separately and the results summed.

Lines	Function and/or relation to theory
1	Subroutine declaration statement and transfer of necessary data. INTDI is an array needed by the secondary integration subroutine IQSF to store results as the integration procedes and IJK is the error marker mentioned in the MAIN program (see MAIN lines 73-77 and 97-101) and is also used to indicate whether or not X equals a specific value of XI.
2-3	Declaration of arrays and double precision. SUM is the space in which results of separate integrations are added.
4	The error marker is set to zero.
5	A marker used to indicate whether or not IXO is equal to a specific value of XI is set to zero.
6	X is checked again to see if it has a negative value. If so operations proceed to line 17.

8	X is compared to IXO, if X<IXO the integration can be performed and so operations proceed to line 13.
10-12	Since X > IXO, an error message is printed and the program is sent to line 176.
13	Initial value of SUM is set.
14-16	A loop is used to determine the position (I) of the value of XI equal to or immediately greater than X. If a value of XI equals X the program is sent to line 21, and if a value is not equal but larger than X, operations go to line 20.
17-19	From line 6 or if X is larger than all values of XI. In this case an error message is printed and the program is sent to line 176.
20	From the loop in lines 14-16. The marker is set to 25 to indicate that no value of XI equals X.
21-23	From line 20 or the loop in lines 14-16. A loop is used to determine the position (K) of the value of XI equal to or greater than IXO. If a value of XI equals IXO the program is sent to line 43, and if a value is not equal but larger than IXO the program is sent to line 27.
24-26	If all values of XI are smaller than IXO the error message is printed and operations are sent to line 176.
27	From the loop in lines 21-23. The marker is set to 30 to indicate that no value of XI equals IXO.

28   Procedd to line 43.

29   From line 55. If operations reach this line then

        IXO is between two adjacent values of XI, namely

        XI(K-1) and XI(K). Since IQSF can only integrate

        up to XI(K-1) or XI(K) and not between them, the

        area under the EPSI curve between XI(K-1) and IXO

        must be calculated another way. Accordingly, the

        average position between XI(K-1) and XI(K) is

        found: if IXO is greater or equal to this average,

        the area between IXO and XI(K) is found and

        subtracted from the SUM and the curve is integrated

        out to XI(K); if IXO is less than the average, the

        area between XI(K-1) and IXO is found and added to

        the SUM and the curve is integrated out to XI(K-1).

        To obtain the area between IXO and the required XI

        value (to be called XI(R)), the values of EPSI at

        XI(K-1) and XI(K) were first interpolated linearly

        to obtain an EPSI value at IXO. Then with ÉPSI

        for IXO and XI(R) and with the difference between

        IXO and XI(R) the area was calculated as a trapezoid.

        In line 29 the average position between XI(K-1) and

        XI(K) is found.

30   If XAV<IXO the program is sent to line 38.

32-35   The area under the EPSI curve between XI(K-1) and

        IXO is calculated and added to the SUM.

36   Since the integration is to procedd to the $(K-1)^{th}$

position, but is told to integrate to position "K", the value of K is decreased by 1.

37   Go to line 43.

38-41 From line 30. The area under the curve of EPSI between IXO and XI(K) is calculated and subtracted from the SUM.

42   Go to line 57.

43   From the loop in lines 21-23 or lines 27 or 28. NDIM is the effective dimension of the variable being integrated; to use IQSF NDIM must be larger than 2.

44   If the value of NDIM<1 go to line 45, if NDIM=1 go to line 48, and if NDIM>1 go to line 53. To get NDIM=0 both X and IXO must be between XI(I-1) and the midpoint between XI(I-1) and XI(I); initially. I=K. For NDIM=1 either: X and IXO are between XI(I-1) and XI(I) with IXO greater than the average of XI(I-1) and XI(I) - producing a SUM<0 (see lines 38-41); or X is between XI(I-1) and XI(I) and IXO is between XI(I) and the midpoint between XI(I) and XI(I+1) - producing a SUM > 0 (see lines 32-35).

45-46 From 44. Since EPSI=0 at X, linear extrapolation with the EPSI value at XI(I) will give an EPSI at IXO. With this and the values for X and IXO the area, and so the total area, has been found.

47   Go to line 170.

48-50   From line 44. If the first case for NDIM=1 occurs
the total area (i.e. integral) is found in line 48
in the same manner as for NDIM=0. For the second
case the area found for the interval from XI(I) to
IXO (see lines 32-35) is added to the area bounded
by the right triangle with corners X, XI(I), and
EPSI(I), with the right angle at XI(I).

52   In both cases the total integral has been found,
so the program proceeds to line 170.

53   From line 44. IJK=0 means X is between XI(I-1)
and XI(I), and because the integration only starts
at XI(I) the area between X and XI(I) is calculated
and becomes the total integral until further
integration can be carried out.

55   If IXO is not equal to any value of XI go back to
line 29, if it is equal to one, continue.

57   From lines 42 or 55. Under the conditions
specified, both X and IXO occur in the first range
of XI values. If this is the case proceed to line
70.

59   If only X is in the first range of XI values go to
line 81.

61   If X and IXO are in the second range of XI values
go to line 125.

63   If only X is in the second range of XI values go
to line 136.

65   If X (and so IXO) is in the third range of XI values go to line 160.

67-69   All possible combinations of values of X and IXO have been covered. However, if somehow the program does reach these lines an error message to locate the problem is printed and the program is sent to line 174.

70   From line 57. If NDIM> 2 IQSF can be used, and so proceed to line 74.

72   NDIM=2, so the rest of the integration can be performed in this line.

73   Integration is complete so proceed to line 170.

74-77   Because IQSF works from an array, starting at the first space and proceeding as far as is specified, the values of the array EPSI must be shifted so that EPSI(I) becomes EPSI(1), EPSI(I+1) becomes EPSI(2), etc.

78   The subroutine to perform the integration is called. The parameters sent to this subroutine are, respectively, the integration step size, the values of the function being integrated, a storage space for integration results, and the effective dimension of the array to be integrated (i.e. the number of values from I to K inclusive for I and K in the same XI range). The result from IQSF is returned in INTDI(NDIM).

79   The result of this integration is added to results
        found previously, if any.

80   Integration is complete; proceed to line 170.

81   From line 59. The 0.1 incremental steps go from
        (originally) EPSI(1) to EPSI(101). If I=100 IQSF
        cannot be used. If I<100 IQSF can be used so .
        operations proceed to line 86.

83   Since IQSF cannot be used, the integrations to the
        end of the first XI range are completed in this
        line.

84   When EPSI is shifted for further integration the
        original EPSI(101) must become EPSI(1), etc. NDIM
        is set to 101 so the term EPSI(NDIM) can be used
        initially.

85   Go to line 93.

86-90   From line 81. NDIM is set to the proper value and
        EPSI is shifted accordingly.

91-92   IQSF is used and the results are added to the
        previous SUM.

93   From lines 85 or 92. If IXO is in the third XI
        range go to line 107.

95   If K > 102 IQSF can be used so proceed to line 99.

97   If K=102 the integration between the spaces
        initially called EPSI(101) and EPSI(102) must be
        done in this manner. NDIM is used instead of 101
        because the array may have been shifted. This

step completes integration.

98   Go to line 170.

99-106   From line 95. The array EPSI is shifted the proper

           number of spaces, NDIM is reset, the integration

           is performed with the results added to previous

           answers, and with all integration completed

           operations proceed to line 170.

107-113   From line 93. EPSI is shifted, NDIM is reset, IQSF

           is used, and the results added to SUM.

114   If K=191 the integrations are complete and

           operations proceed to line 170; if K=192 integrations

           cannot be completed with IQSF so operations proceed

           to line 115; if K > 192 IQSF can be used and

           operations proceed to line 117.

115-116   From line 114. These complete the integration and

           sends operations to line 170.

117-124   From line 114. In a fashion similar to lines

           99-106 these lines complete the integration and send

           operations to line 170.

125-128   From line 61. If NDIM is of insufficient size for

           use of IQSF the integrations are completed here

           and operations are sent to line 170. If NDIM is

           large enough to use IQSF go to line 129.

129-135   EPSI is shifted, integrations are completed, and

           operations are sent to line 170.

136   From line 63. If I<190 IQSF can be used, and so

proceed to line 141.

138   If I=190 integrations for the second XI range are
completed in this line.

139-140 NDIM is reset appropriately and operations proceed
to line 148.

141-147 From line 136. Integration procedures for the
second XI range are completed.

148   From lines 140 or 147. If K > 192 IQSF can be
used and so operations proceed to line 152.

150-151 With K=192, final integration is performed and
operations proceed to line 170.

152-159 From line 148. Final integration for the third XI
range is performed, the result is added to
previous results and operations proceed to line 170.

160   From line 65. If NDIM>2 IQSF can be used so
operations move to line 164.

162-163 Since NDIM=2, final integration is performed in
this manner and operations go to line 170.

164-169 From line 160. Integration procedures are
performed for the third XI range. This completes
integrations for this range.

170   From lines 52, 73, 80, 98, 106, 116, 124, 128, 135,
151, 159, 163, or 169. Whenever the integration
has been completed the program has been sent here.
The final result of the integration, SUM, is
placed in INTDI(1) where it can be retrieved by

the MAIN program.

171   The error marker IJK is set to 0 to indicate
subroutine SIGI has operated correctly.

172   A further safety check is made, if $\mathrm{SUM} \geqslant 0$ the
program proceeds to line 177.

174-175   From lines 69 and 172. Something drastically wrong
has happened. An error message is printed with
much relevant data.

176   From lines 12, 19, 26, and 175. Because some sort
of error has occurred the error marker is set to
111.

177-178   The program returns to the MAIN section and
subroutine SIGI ends.

Subroutine DIVIN

This subroutine is called from line 72 in the MAIN
program and is used to calculate the integral

$$\int_{0}^{x_o} \xi e^{-\psi} \psi' d\xi$$

The factors transferred to this subroutine are the
series of values $\xi e^{-\psi} \psi'$ (called XIEPS) corresponding to the
ξ values, the integration cutoff x_o (called IXO), and the

series of ξ values (called XI). XI characteristics have been described elsewhere, see for example the introduction to the explanation of the subroutine SIGI in this Appendix.

Lines	Function and/or relation to theory
1	Subroutine declaration statement and transfer of necessary data. INTDI and IJK are as described for SIGI, line 1.
2-3	Declaration of arrays and double precision.
4	The storage space for integration results is set to 0.
5-7	A loop is set up that searches for a value of XI greater than or equal to IXO. If the XI value is less than IXO the search continues; if an XI value equals IXO the program goes to line 25; and if no XI value equals IXO, the first XI value greater than IXO sends operations to line 12.
8-11	If IXO is larger than all values of XI an error message is printed, the error marker is set to 111, and operations are sent to line 65
12-24	From line 6. Since IXO occurs between 2 values of XI (i.e. $XI(I-1)<IXO<XI(I)$), integration cannot be exact and so these lines perform the same sort of computations, and for the same reasons, as lines 29-41 in subroutine SIGI.
25	From lines 19 or 24. If $I > 101$ then the entire

first range of XI values is to be integrated over.
In this case proceed to line 32.

27   Since less than the entire first range is to be integrated over, NDIM is assigned the proper value.

28   If $NDIM > 2$ IQSF can be used; operations go to line 33.

30   Since NDIM=2, the total integration is performed by this line.

31   Since all integrations are completed, go to line 64.

32   From line 25. NDIM is set to the appropriate value.

33   From lines 28 or 32. Integrations for the first XI range are performed using IQSF.

34   Integration results are added to previous results, if any.

35   If $I \leqslant 101$ then integrations are complete and the program is sent to line 64.

37   If $I \geqslant 191$ the entire second range of XI values is to be integrated over. In this case proceed to line 44.

39   Since less than the entire second range is to be integrated, NDIM is set to the appropriate value.

40   If IQSF can be used go to line 45.

42-43   The integration for the second XI range is completed and added to previous results and the operations are sent to line 64.

44   From line 37. NDIM is set to the appropriate value.

45-48	From lines 40 or 44. The array XIEPS is shifted the proper number of places.
49	IQSF is called to integrate over the second XI range.
50	The results of this integration are added to the previous SUM.
51	If $I \leqslant 191$ all necessary computations have been made and operations proceed to line 64.
53	Since IXO occurs in the third XI range, more integration needs to be performed, and so NDIM is set appropriately.
54	.If IQSF can be used go to line 58.
56-57	Final integration and summation are completed and operations proceed to line 64.
58-61	From line 54. XIEPS is shifted the proper number of places.
62	Final use of IQSF, on the set of values corresponding to the third XI range.
63	Final summation of results.
64	From lines 31, 35, 43, 51, 57, or 63. The final result is assigned to INTDI(1) for access by the MAIN program upon leaving this subroutine.
65-66	From lines 11 or 64. Operations return to the MAIN program and subroutine DIVIN ends.

Subroutine IQSF

This subroutine is called from lines 78, 91, 104, 112, 122, 133, 146, 157, and 168 in the subroutine SIGI and from lines 33, 49, and 62 in the subroutine DIVIN.

It is part of IBM's Scientific Subroutine Package where it is called "QSF". This subroutine performs integrations numerically following the method of Simpson's rule. Further details and explanations may be found in the SSP manual on page 87.

*Modifications to YAHOO to get BGIN

The changes in lines 4 to 69 are due to σ_{bg} being used as a constant. Wording changes in the format statements reflect this difference in usage of the value read in for SBG in line 19.

The changes in lines 111 through 132 are performed because of the change in the Newton-Raphson method. With respect to the terms of page 15 (and so pages 10 and 11) the variables in the program are:

$$SUM1 \equiv f$$

$$SUM2 \equiv f_x$$

$$EI \equiv \epsilon_i$$

$$UI \equiv \mu_i$$

$$BI(I) \equiv \beta_i$$

$$SIGISO(I) \equiv \sigma_{iso}(x_i)$$

In the last line of the group replacing lines 123 through 132 the size of SUM2 is tested. If it is too small the value of

$$SUM1/(2.*SUM2)$$

(in the new version of line 142) would probably exceed the size limit of the computer.

The new version of line 142 produces a new iterative value of σ_c in the manner described on page 15.

All changes and deletions in the rest of the program are obvious consequences of the use of σ_{bg} as a constant.

The lines inserted between lines 277 and 278 keep track of the latest values of α, σ_c, and χ^2.

*Modifications to YAHOO to get TAFCHEC

The lines inserted between lines 24 and 25 read in and reprint Taff's values for α, σ_c, and σ_{bg}.

The variable inserted between lines 56 and 57 is a marker used to determine whether or not TAFCHEC has completed all the functions YAHOO performs for a given x_o. If it has, then ITAFF's value is changed from 5 to 500, σ_{iso} is calculated for Taff's α and the current x_o, a model isothermal gas sphere is created from this σ_{iso} and Taff's values of σ_c and σ_{bg}, and the χ^2 is found from comparison of the data to this model.

The lines inserted between lines 102 and 103 and again between lines 107 and 108 check to see if TAFCHEC has completed the YAHOO functions. If so, the values for σ_{iso} using Taff's value of α have been calculated (lines 79 to 107 inclusive) and the program can calculate the χ^2.

Since line 281 completes the YAHOO functions, ITAFF's value is reassigned, α is set to Taff's value, and operations return to line 79 to calculate σ_{iso}.

After this has been done operations go to the line flagged 820. Here χ^2 is set to 0 and a loop calculates the theoretical number of galaxies in each ring from the model produced with Taff's values. χ^2 is then calculated from these theoretical values of Taff and the actual number of galaxies (the data set NOBS).

Once the χ^2 has been found both it and the Taff value of α are printed. The α value is printed as a safety check since its variable name in the program, ALPHA, is changed continuously during the program's execution. Taff's values of σ_c and σ_{bg} are not reprinted because the spaces they are assigned to (TSC and TSBG respectively) remain unchanged once they are read in.

After printing Taff's α and the calculated χ^2, the program increments x_0 and continues with the YAHOO functions for it.

Appendix D

Plate and cluster information

The table on the next page lists information relating to the plates used in this thesis. Also listed are the redshift of each cluster as well as the distance and richness classifications and the 1950 positions; the last three items are from Abell (1958).

All plates used were taken by Dr. G.A. Welch.

Object	plate number	Exposure (minutes)	Emulsion	Filter	z	D	R	Centre R.A.	(1950) Dec.
A2052	PS5736	20	103a-D	Wr.12	.0351	3	0	$15^h14^m.0$	$+07°12'$
	PS6868	120	IIIa-J	Wr.4					
A2593	PS7154	20	103a-D	Wr.12	.044	3	0	23 22.0	+14 22
	PS7173	120	IIIa-J	Wr.4					
A2626	PS6875	20	103a-D	Wr.12	.055	3	0	23 34.0	+20 53
	PS7142	120	IIIa-J	Wr.4					
A154	PS7145	20	103a-D	Wr.12	.056	3	1	01 08.3	+17 24
	PS7156	120	IIIa-J	Wr.4					

Appendix E

Strip counts

Tabulated in this appendix are the results of the strip counts. Since the strips were centred on the major galaxy in three of the clusters, this central galaxy was counted twice for each orientation, once for each strip which contained half of it (strips 6 and 7). For the fourth cluster the centre held a binary galaxy. This cluster had the strips centred between the members' of the binary and each member of the binary was treated like all other galaxies in the cluster; each was only counted once for each orientation.

After the three sets of strip counts for each cluster (one set of counts for each magnitude limit) are two items: a list of four cluster centres, the three from the different magnitude limits and the Abell (1958) centre; and a diagram of the centre area of the cluster, at twice the print scale, locating the four centres. The diagrams are centred on the locations of the centres of strip counting and the boxes for the Abell centres come from the one digit difference in accuracy stated in the tables of cluster centres. The squares of the diagram correspond to the 1.5 cm width of the grid used to make the strip counts.

Mag. limit	Strip No.	Orientation			
		E→W	N→S	NE→SW	SE→NW
b	1	1	2	4	2
	2	3	1	2	0
	3	4	5	4	1
	4	3	2	5	4
	5	8	6	4	9
	6	6	9	7	8
	7	9	8	9	10
	8	4	5	1	5
	9	5	1	3	3
	10	1	3	5	3
	11	0	2	1	1
	12	2	2	0	0
	total	46	46	45	46
D	1	9	12	7	12
	2	14	12	14	9
	3	11	10	14	7
	4	21	21	25	15
	5	27	21	24	26
	6	27	31	18	29
	7	22	20	23	30
	8	18	18	12	22
	9	15	10	12	11
	10	9	22	12	16
	11	9	4	16	6
	12	11	11	11	4
	total	193	192	188	187
f	1	26	28	23	26
	2	40	28	36	25
	3	33	21	42	19
	4	34	41	42	33
	5	43	52	40	54
	6	54	66	30	61
	7	50	46	58	58
	8	42	42	37	49
	9	37	30	32	33
	10	22	35	29	30
	11	28	24	27	18
	12	26	21	27	13
	total	435	434	423	417

A2052

Cluster centre (1950)

Source	R.A.	Dec.
Abell	$15^h14^m.0$	$+07°12'$
b	15 14.33	+07 12.3
D	15 14.36	+07 12.7
f	15 14.31	+07 12.0
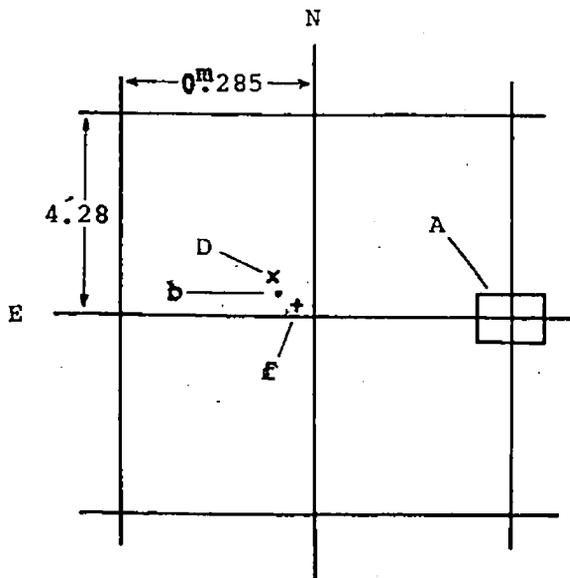
Mag. limit	Strip No.	Orientation			
		E→W	N→S	NE→SW	SE→NW
	1	0	2	2	2
	2	1	3	2	2
	3	4	3	3	0
	4	1	5	1	1
	5	7	3	5	9
	6	7	5	9	9
b	7	6	8	8	7
	8	5	6	3	6
	9	6	4	7	4
	10	5	3	2	1
	11	3	4	0	2
	12	1	2	4	2
	total	46	48	46	45
	1	8	20	12	16
	2	15	19	20	23
	3	22	26	23	25
	4	31	39	21	50
	5	41	30	37	54
	6	64	57	80	49
D	7	71	49	50	39
	8	44	32	31	42
	9	36	32	47	35
	10	29	37	34	32
	11	23	39	28	17
	12	16	18	22	23
	total	400	398	405	405
	1	11	26	22	29
	2	38	33	22	32
	3	31	36	33	33
	4	38	46	35	63
	5	52	41	47	58
	6	73	56	79	54
f	7	71	45	56	50
	8	54	45	39	43
	9	38	41	58	39
	10	32	46	45	38
	11	30	50	35	23
	12	25	31	23	27
	total	493	496	494	489

A2593

Cluster centre (1950)

Source	R.A.	Dec.
Abell	$23^h22^m.0$	$+14°22'$
b	23 21.76	+14 21.7
D	23 21.80	+14 21.1
f	23 21.82	+14 21.4
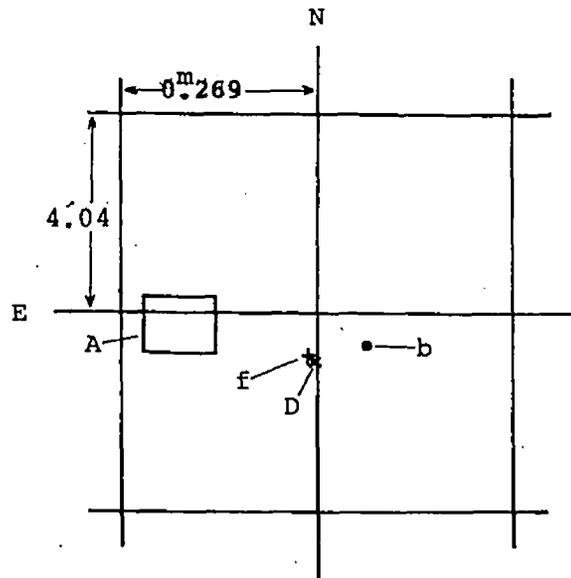
Mag. limit	Strip No.	Orientation			
		E→W	N→S	NE→SW	SE→NW
b	1	0	1	3	3
	2	1	1	1	2
	3	2	3	3	1
	4	4	5	6	5
	5	5	4	1	5
	6	5	9	3	9
	7	13	7	6	11
	8	5	5	7	4
	9	4	2	7	0
	10	6	6	8	4
	11	1	4	5	1
	12	6	4	2	3
	total	52	51	52	48
D	1	18	16	17	23
	2	34	14	19	32
	3	20	18	37	26
	4	26	27	31	31
	5	22	33	30	37
	6	36	38	30	36
	7	58	43	24	46
	8	25	32	33	32
	9	26	28	33	20
	10	30	29	32	23
	11	22	31	23	23
	12	17	27	27	11
	total	334	336	336	340
f	1	60	39	39	33
	2	71	39	43	35
	3	62	39	63	50
	4	48	50	57	47
	5	41	63	57	57
	6	77	69	68	78
	7	97	92	67	87
	8	76	75	67	85
	9	50	71	72	56
	10	69	82	59	61
	11	44	53	52	64
	12	37	59	54	41
	total	732	731	698	694

A2626

Cluster centre (1950)

Source	R.A.	Dec.
Abell	$23^h34^m.0$	+20°53´
b	23 33.86	+20 51.1
D	23 33.99	+20 51.2
f	23 34.01	+20 51.1
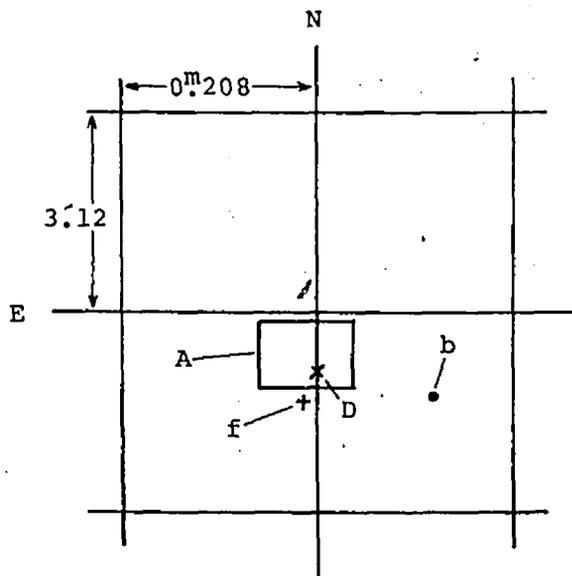
Mag. limit	Strip No.	E→W	N→S	NE→SW	SE→NW
	1	1	5	6	1
	2	1	4	5	4
	3	7	6	5	2
	4	4	3	2	3
	5	3	0	3	5
	6	13	10	6	8
b	7	3	7	14	6
	8	10	4	5	6
	9	3	4	3	5
	10	3	2	1	5
	11	3	4	4	4
	12	1	3	1	3
	total	52	52	55	52
	1	15	13	24	17
	2	15	14	18	23
	3	25	14	18	14
	4	26	24	20	28
	5	18	21	23	17
	6	40	38	40	32
D	7	34	33	46	41
	8	34	24	20	23
	9	14	23	22	19
	10	18	16	10	20
	11	8	20	18	14
	12	12	17	7	17
	total	259	257	266	265
	1	15	20	29	26
	2	28	27	20	21
	3	27	23	23	22
	4	32	30	25	28
	5	38	23	29	26
	6	49	42	47	48
f	7	42	35	59	50
	8	35	31	25	30
	9	18	37	32	26
	10	24	24	8	26
	11	23	32	23	13
	12	17	24	11	17
	total	348	348	331	333

A154

Cluster centre (1950)

Source	R.A.	Dec.
Abell	$01^h08^m.3$	$+17^{\circ}24'$
b	01 08.38	+17 24.0
D	01 08.41	+17 23.8
f	01 08.42	+17 23.5
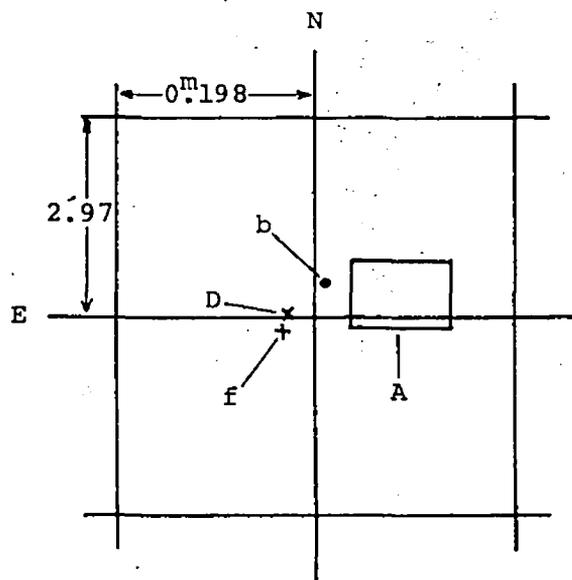
Appendix F

Ring counts

The ring count results are tabulated by quadrant and ring in this appendix for all three magnitude limits of each cluster. The tables contain the ring number, the outer radius in arcminutes of that ring, the number of galaxies in each quadrant, and the number of galaxies for the ring. At the bottom of the tables are the total numbers of galaxies both per quadrant and in the total area counted.

A2052

Ring	ROUT	b					D					f				
		NE	NW	SE	SW	Σ	NE	NW	SE	SW	Σ	NE	NW	SE	SW	Σ
1	2.24	0	2	1	1	4	1	1	0	4	6	4	3	3	3	13
2	4.48	0	2	0	1	3	2	6	0	0	8	0	6	2	8	16
3	6.72	1	1	3	0	5	6	1	8	4	19	7	6	9	7	29
4	8.96	2	1	2	0	5	5	5	3	1	14	14	11	7	10	42
5	11.20	1	1	2	2	6	6	3	4	5	18	9	9	6	9	33
6	13.44	2	1	0	1	4	11	2	2	3	18	16	8	1	11	36
7	15.68	2	0	0	2	4	3	4	0	9	16	17	4	5	9	35
8	17.92	2	1	0	1	4	4	2	3	3	12	9	5	5	10	29
9	20.16	1	0	0	1	2	7	2	4	4	17	20	6	6	9	41
10	22.40	2	1	0	0	3	2	4	2	4	12	16	10	9	6	41
11	24.64	1	0	0	1	2	6	2	4	5	17	11	4	11	16	42
12	26.88	0	1	3	0	4	7	3	6	8	24	11	8	11	11	41
13	29.12	1	0	0	0	1	5	2	3	2	12	14	9	10	8	41
14	31.36	1	1	1	1	4	0	2	4	6	12	9	7	21	17	54
15	33.60	0	1	0	0	1	7	7	1	5	20	10	13	8	5	36
16	35.84	0	0	1	0	1	2	1	6	3	12	9	14	23	5	51
17	38.08	0	1	0	1	2	7	4	1	3	15	13	17	14	12	56
18	40.32	0	0	0	1	1	3	8	5	6	22	18	25	10	22	75
19	42.56	0	0	0	0	0	13	8	4	4	29	18	12	21	13	64
20	44.80	0	1	0	1	2	14	7	13	4	38	14	15	28	16	73
Σ		16	15	13	14	58	111	74	73	83	341	239	192	210	207	848

A2593

Ring	ROUT	b				Σ	D				Σ	f				Σ
		NE	NW	SE	SW		NE	NW	SE	SW		NE	NW	SE	SW	
1	2.12	1	1	0	1	3	3	7	0	3	13	5	2	0	2	9
2	4.24	0	2	1	0	3	9	5	7	2	23	7	9	5	3	24
3	6.36	1	5	1	1	8	15	6	10	2	33	9	11	8	4	32
4	8.48	1	0	1	0	2	3	11	7	8	29	7	6	7	4	24
5	10.60	0	0	1	2	3	4	9	12	3	28	11	9	13	8	41
6	12.72	2	0	0	3	5	3	15	12	17	47	14	3	15	18	50
7	14.84	1	0	0	3	4	13	10	9	13	45	6	17	13	16	52
8	16.96	1	2	0	0	3	9	11	3	9	32	12	14	6	8	40
9	19.08	0	1	1	0	2	7	7	7	15	36	9	10	9	14	42
10	21.20	0	0	2	1	3	6	4	7	9	26	9	6	19	11	45
11	23.32	3	2	1	3	9	8	16	7	5	36	18	9	13	12	52
12	25.44	0	0	0	1	1	6	12	7	10	35	15	11	17	10	53
13	27.56	1	0	0	0	1	6	9	8	11	34	15	11	16	13	55
14	29.68	1	1	0	0	2	11	7	11	13	42	13	14	8	15	50
15	31.80	1	0	0	0	1	2	15	14	9	40	15	6	14	12	47
16	33.92	1	1	0	1	3	5	4	5	13	27	10	12	10	19	51
17	36.04	4	2	1	0	7	8	16	6	8	38	17	12	17	18	64
18	38.16	0	1	0	0	1	11	12	6	7	36	10	13	10	15	48
19	40.28	2	2	6	1	11	10	6	11	8	35	6	17	13	11	47
20	42.40	0	3	1	2	6	11	9	12	11	43	13	12	18	13	56
Σ		20	23	16	19	78	150	191	161	176	678	221	204	231	226	882

A2626

Ring	ROUT	b					D					f				
		NE	NW	SE	SW	Σ	NE	NW	SE	SW	Σ	NE	NW	SE	SW	Σ
1	1.63	1	0	0	1	2	0	3	0	1	4	2	4	2	3	11
2	3.27	1	1	1	1	4	4	5	2	6	17	7	5	10	9	31
3	4.90	1	1	1	2	5	4	3	2	4	13	6	8	7	14	35
4	6.53	1	0	0	2	3	7	5	2	5	19	12	8	7	14	41
5	8.16	0	2	0	1	3	4	4	3	10	21	7	15	8	19	49
6	9.80	2	1	0	2	5	11	4	1	8	24	10	9	6	19	44
7	11.43	3	1	1	2	7	7	7	6	7	27	14	14	13	13	54
8	13.06	2	0	1	2	5	7	10	4	14	35	12	14	16	25	67
9	14.70	2	0	2	0	4	11	7	15	6	39	27	16	30	22	95
10	16.33	0	0	2	0	2	9	4	8	8	29	19	8	27	17	71
11	17.96	1	1	0	2	4	10	11	12	10	43	26	21	21	17	85
12	19.59	0	1	2	3	6	6	6	6	14	32	16	14	22	34	86
13	21.23	1	0	3	0	4	9	12	15	13	49	17	24	19	23	83
14	22.86	1	1	1	2	5	10	3	8	8	29	21	9	21	19	70
15	24.49	2	0	1	0	3	8	3	14	14	39	20	14	36	23	93
16	26.12	1	2	3	1	7	13	8	12	10	43	31	21	38	22	112
17	27.76	2	1	3	0	6	17	16	16	10	59	32	27	40	26	125
18	29.39	5	0	0	1	6	11	17	14	15	57	27	27	33	40	127
19	31.02	0	1	4	0	5	17	7	12	17	53	28	12	31	31	102
20	32.66	0	0	4	1	5	15	12	17	21	65	37	29	38	43	147
Σ		26	13	29	23	91	180	147	169	201	697	371	299	425	433	1528

A154

Ring	ROUT	b					D					f				
		NE	NW	SE	SW	Σ	NE	NW	SE	SW	Σ	NE	NW	SE	SW	Σ
1	1.55	0	1	2	0	3	0	5	4	4	13	2	8	4	4	18
2	3.11	1	0	1	1	33	4	5	2	7	18	5	6	2	6	19
3	4.66	1	1	1	2	5	5	4	4	4	17	8	5	2	6	21
4	6.22	1	3	1	0	5	2	9	4	2	17	2	6	9	4	21
5	7.77	0	0	2	1	3	7	5	7	3	22	6	6	11	5	28
6	9.33	1	2	0	0	3	4	4	6	8	22	7	2	5	12	26
7	10.88	1	0	0	2	3	4	5	2	1	12	6	8	1	3	18
8	12.43	1	1	1	0	3	4	8	4	2	18	8	9	8	1	26
9	13.99	3	1	1	1	6	3	5	10	7	25	9	5	8	8	30
10	15.54	0	1	2	3	6	5	5	8	5	23	4	7	8	6	25
11	17.10	3	1	0	0	4	8	4	8	6	26	14	7	12	10	43
12	18.65	4	1	1	0	6	14	7	10	3	34	13	8	19	7	47
13	20.21	1	1	1	1	4	4	6	6	8	24	7	4	4	12	27
14	21.76	0	2	0	1	3	8	4	3	9	24	9	11	13	9	42
15	23.31	1	0	2	1	4	3	5	17	2	27	7	10	11	6	34
16	24.87	0	3	4	0	7	6	10	13	6	35	11	12	14	5	42
17	26.42	1	0	8	1	10	8	10	16	8	42	16	8	13	6	43
18	27.98	1	0	4	1	6	6	3	13	9	31	6	7	12	16	41
19	29.53	1	3	1	2	7	6	13	10	13	42	4	16	14	7	41
20	31.09	0	1	2	1	4	5	6	11	4	26	11	8	15	8	42
Σ		21	22	34	18	95	106	123	158	111	498	155	153	185	141	634

Appendix G

Background counts

Below are tabulated the background' counts for each cluster's three magnitude limits. These counts were taken in 9 cm by 9 cm squares in each corner of each print (8.75 cm squares for A2052). Besides the number counts (N) and the background densities in galaxies/arcmin^2 (σ), the total counting area in arcmin^2 are presented for each cluster. The densities of the Σ column are those used as the initial estimates for the program YAHOO and also as the fixed values of σ_{bg} in the program BGIN.

The background counts were also used to calculate ΔM in Table 12.

A2052

Total area counted over = 2480.0 arcmin^2

Mag. limit		Corner				Σ
		NE	NW	SE	SW	
b	N	6	2	2	1	11
	σ	9.68(-3)	3.23(-3)	3.23(-3)	1.61(-3)	4.44(-3)
D	N	33	12	50	8	103
	σ	5.32(-2)	1.94(-2)	8.06(-2)	1.29(-2)	4.15(-2)
f	N	87	48	105	38	278
	σ	1.40(-1)	7.74(-2)	1.69(-1)	6.13(-2)	1.12(-1)

A2593

Total area counted over = 2348.0 arcmin^2

Mag. limit		Corner				Σ
		NE	NW	SE	SW	
b	N	4	4	3	4	15
	σ	6.81(-3)	6.81(-3)	5.11(-3)	6.81(-3)	6.39(-3)
D	N	27	60	53	42	182
	σ	4.60(-2)	1.02(-1)	9.03(-2)	7.16(-2)	7.75(-2)
f	N	39	95	71	60	265
	σ	6.64(-2)	1.62(-1)	1.21(-1)	1.02(-1)	1.13(-1)

A2626

Total area counted over = 1401.8 arcmin^2

Mag. limit		Corner				Σ
		NE	NW	SE	SW	
b	N	11	5	7	3	26
	σ	3.14(-2)	1.43(-2)	2.00(-2)	8.56(-3)	1.85(-2)
D	N	92	66	68	74	300
	σ	2.63(-1)	1.88(-1)	1.94(-1)	2.11(-1)	2.14(-1)
f	N	202	146	123	151	622
	σ	5.76(-1)	4.17(-1)	3.51(-1)	4.31(-1)	4.44(-1)

A154

Total area counted over = 1270.2 arcmin^2

Mag. limit		Corner				Σ
		NE	NW	SE	SW	
b	N	5	0	8	8	21
	σ	1.57(-2)	0.00(-0)	2.52(-2)	2.52(-2)	1.65(-2)
D	N	32	26	40	37	135
	σ	1.01(-1)	8.19(-2)	1.26(-1)	1.17(-1)	1.06(-1)
f	N	36	29	62	47	174
	σ	1.13(-1)	9.13(-2)	1.95(-1)	1.48(-1)	1.37(-1)

Appendix H

Complete results

The data for each magnitude limit of a given
cluster were used in the programs YAHOO and BGIN for two
cases each; using data for all 20 rings and just using data
for the inner 10 rings. Each cluster has therefore 12 sets
of results. Table 12 displays parts of these results but
this appendix lists the complete results.

The tables display, in columns from left to right:
ML - the magnitude limit for this set of rows; ΔM - the
difference in magnitude between the b limit and the D and f
magnitude limits; Prog - the program used; NR - the number
of rings used; NG - the number of galaxies used; α - the
best fit scale factor; σ_c - the best fit central density;
σ_{bg} - for YAHOO the best fit background density, for BGIN
the counted background as obtained from the counts in
Appendix G (in BGIN this value is necessarily the same for
both 10 and 20 ring cases for a given cluster and magnitude
limit); x_0 - the best fit integration cutoff to the
isothermal gas-sphere model; χ^2 - the calculated χ^2 obtained
from comparing the best fit model to the data; and Prob -
the probability that any χ^2 would be smaller than the one
actually found. For the last column the number of degrees
of freedom used is (NR-κ), where $\kappa=5$ for YAHOO and $\kappa=4$ for
BGIN.

In these tables the units used are:

ΔM - magnitudes

α - arcmin

σ_c - galaxies/arcmin^2

σ_{bg} - galaxies/arcmin^2

ML	ΔM	Prog	NR	NG	α	σ_c	σ_{bg}	x_o	χ^2	Prob
b	–	YAHOO	10	40	1.311(−1)	3.567(−0)	8.516(−3)	200	2.063	.1596
			20	58	2.179(−1)	2.069(−0)	2.419(−3)	200	8.480	.0969
		BGIN	10	40	1.717(−1)	2.689(−0)	4.44 (−3)	200	2.207	.1003
			20	58	1.824(−1)	2.464(−0)	4.44 (−3)	200	10.22	.1451
D	1.6	YAHOO	10	140	2.899(−0)	2.841(−1)	8.203(−3)	30	4.446	.5129
			20	341	2.876(−0)	2.386(−1)	4.314(−2)	10	24.61	.9446
		BGIN	10	140	2.662(−0)	2.693(−1)	4.15 (−2)	10	4.605	.4046
			20	341	2.952(−0)	2.361(−1)	4.15 (−2)	10	24.88	.9280
f	2.3	YAHOO	10	315	2.404(−0)	4.968(−1)	1.277(−1)	10	8.056	.8468
			20	848	2.714(−0)	4.670(−1)	1.134(−1)	10	17.52	.7113
		BGIN	10	315	2.714(−0)	4.710(−1)	1.12 (−1)	10	8.368	.7876
			20	848	2.773(−0)	4.583(−1)	1.12 (−1)	10	17.59	.6516

A2593

ML	ΔM	Prog	NR	NG	α	σ_c	σ_{bg}	x_o	χ^2	Prob
b	–	YAHOO	10	36	1.468(-0)	1.963(-1)	3.146(-3)	30	5.122	.5988
			20	78	1.431(-0)	1.789(-1)	1.187(-2)	20	30.80	.9907
		BGIN	10	36	1.503(-0)	1.897(-1)	6.39 (-3)	20	5.120	.4715
			20	78	1.192(-0)	2.043(-1)	6.39 (-3)	200	32.30	.9909
D	1.8	YAHOO	10	312	3.095(-0)	6.347(-1)	5.737(-3)	70	12.66	.9732
			20	678	2.615(-0)	6.390(-1)	4.519(-2)	60	18.00	.7373
		BGIN	10	312	3.173(-0)	5.519(-1)	7.75 (-2)	10	12.90	.9553
			20	678	2.271(-0)	6.981(-1)	7.75 (-2)	20	19.71	.7664
f	2.1	YAHOO	10	359	3.747(-0)	4.975(-1)	4.548(-2)	160	8.775	.8816
			20	882	3.911(-0)	4.571(-1)	6.340(-2)	80	15.08	.5543
		BGIN	10	359	3.680(-0)	4.336(-1)	1.13 (-1)	10	8.820	.8160
			20	882	3.995(-0)	4.051(-1)	1.13 (-1)	10	17.27	.6317

A2626

ML	ΔM	Prog	NR	NG	α	σ_c	σ_{bg}	x_o	χ^2	Prob
b	–	YAHOO	10	40	1.407(−0)	2.095(−1)	9.671(−3)	160	4.298	.4926
			20	91	1.294(−0)	2.215(−1)	1.185(−2)	140	6.662	.0336
		BGIN	10	40	1.287(−0)	2.137(−1)	1.85 (−2)	30	4.322	.3668
			20	91	1.271(−0)	2.162(−1)	1.85 (−2)	30	6.787	.0228
D	1.8	YAHOO	10	228	1.035(−0)	5.499(−1)	2.022(−1)	160	5.511	.6433
			20	697	1.574(−0)	4.659(−1)	1.668(−1)	140	19.01	.7867
		BGIN	10	228	9.660(−1)	5.696(−1)	2.14 (−1)	50	5.520	.5210
			20	697	1.152(−0)	5.186(−1)	2.14 (−1)	20	26.14	.9479
f	2.3	YAHOO	10	498	1.080(−0)	1.128(−0)	5.364(−1)	10	8.811	.8832
			20	1528	1.454(−0)	1.204(−0)	3.786(−1)	80	29.80	.9873
		BGIN	10	498	9.637(−1)	1.238(−0)	4.44 (−1)	200	9.861	.8694
			20	1528	1.208(−0)	1.118(−0)	4.44 (−1)	20	34.50	.9954

| ML	ΔM	Prog	NR	NG	α	σ_c	σ_{bg}	x_o	χ^2	Prob
b	–	YAHOO	10	40	5.214(-2)	1.083(+1)	3.769(-2)	200	1.947	.1436
			20	95	6.333(-1)	4.577(-1)	1.790(-2)	180	9.057	.1255
		BGIN	10	40	6.748(-1)	4.455(-1)	1.65 (-2)	200	2.981	.1888
			20	95	6.748(-1)	4.450(-1)	1.65 (-2)	200	9.120	.0916
D	1.3	YAHOO	10	187	6.174(-2)	3.638(+1)	1.704(-1)	200	4.745	.5522
			20	498	5.064(-1)	2.549(-0)	1.049(-1)	180	17.00	.6811
		BGIN	10	187	5.549(-1)	2.313(-0)	1.06 (-1)	120	5.323	.4969
			20	498	5.453(-1)	2.342(-0)	1.06 (-1)	160	17.02	.6157
f	1.5	YAHOO	10	232	1.006(-1)	2.421(+1)	1.709(-1)	190	3.819	.4242
			20	634	1.917(-1)	1.058(+1)	1.505(-1)	200	17.63	.7174
		BGIN	10	232	1.444(-1)	1.569(+1)	1.37 (-1)	200	3.939	.3151
			20	634	2.639(-1)	7.364(-0)	1.37 (-1)	200	18.13	.6779

Appendix I

Density profiles

In Table 16 the accepted core radii are presented, having been calculated from the parameters produced by the program version BGIN when all 20 rings are used. In this appendix the counted density profiles are presented for all three magnitude limits for each cluster. These were produced from the ring count data of Appendix F.

Superimposed on these profiles are the best fitting models created from the BGIN(20) parameters specified on the individual graphs (these parameters are included in Appendix H). The model profiles also indicate the core radii r_c (in arcmin) which are transformed to the radii of Table 16.

The error bars on these graphs are set to be equal to the square root of the number of galaxies occurring in a particular ring. Despite the fact that the outer rings have more galaxies than the inner rings, the densities enclosed by the error bars decrease as r_{av} increases because of the increased area, and so smaller densities, covered by these outer rings.

The units on all profiles are: r_{av} and α - arcmin; σ, σ_c, and σ_{bg} - galaxies/arcmin^2.

A2052(b)

$\sigma(r_{av}=0)=2.47$

$\alpha=.1824$

$\sigma_C=2.464$

$\sigma_{bg}=.00444$

$x_0=200$

r_c

A2052(D)

$\alpha=2.952$

$\sigma_C=.2361$

$\sigma_{bg}=.0415$

$x_0=10$

r_{c_A}

A2052(f)

$\alpha = 2.773$

$\sigma_c = .4583$

$\sigma_{bg} = .112$

$x_o = 10$

A2593(b)

$\alpha = 1.192$

$\sigma_c = .2043$

$\sigma_{bg} = .00639$

$x_o = 200$

A2593(D)

$\alpha = 2.271$

$\sigma_C = .6981$

$\sigma_{bg} = .0775$

$x_O = 20$

r_c

σ

r_{av}

A2593(f)

$\alpha = 3.995$

$\sigma_C = .4051$

$\sigma_{bg} = .113$

$x_O = 10$

r_c

σ

r_{av}

A2626(b)

$\alpha=1.271$

$\sigma_c=.2162$

$\sigma_{bg}=.0185$

$x_0=30$

A2626(D)

$\alpha=1.152$

$\sigma_c=.5186$

$\sigma_{bg}=.214$

$x_0=20$

A2626(f)

$\alpha = 1.208$

$\sigma_C = 1.118$

$\sigma_{bg} = .444$

$x_O = 20$

A154(b)

$\alpha = .6748$

$\sigma_C = .4450$

$\sigma_{bg} = .0165$

$x_O = 200$

A154(D)

$\alpha=.5453$

$\sigma_C=2.342$

$\sigma_{bg}=.106$

$x_0=160$

A154(f)

$\sigma(r_{av}=0)=7.50$

$\alpha=.2639$

$\sigma_C=7.364$

$\sigma_{bg}=.137$

$x_0=200$

Appendix J

Sample output

This appendix contains sample output from the programs YAHOO, BGIN, and the BASIC program used to produce the series of values $e^{-\psi}\psi'$.

Both YAHOO and BGIN begin by reprinting the line used to check terminal speed and width, followed by the initial estimate for σ_c. The next number is either called the estimate for σ_{bg} (if YAHOO is being used) or the actual value of σ_{bg} (if BGIN is being used). The number of rings used is then printed. If this is less than the total available in the data file (i.e. less than 20) the remainder of the ring data will be ignored for all computations.

The table that follows reprints the number of galaxies in each ring and that ring's inner and outer radii as a safety check. (Obviously the outer radius of one ring is the inner radius of the next ring outwards.) Also tabulated are the calculated observed densities of these rings and their average radii. These last two columns are used to draw observed density profiles of the type in Appendix I.

Then the main part of the output begins with the printing of the x_0 value for the subsequent series of α

values. The table of numbers following x_o lists the α values found by incrementing $\log(\alpha)$, and, for YAHOO, the σ_c and σ_{bg} values arising from this particular x_o-α combination and the resultant χ^2. There are three sets of these four values in each row; groups are to be read across, and not down, the page. For BGIN's output σ_{bg} has already been set, so printed across the page are four sets of three values: α, the σ_c found by the Newton-Raphson method, and the resulting χ^2.

After the possible range of α values has been printed, the minimum χ^2 in the table and its causative parameters are printed. Then the χ^2 and associated parameters obtained from the α averaging technique are listed.

The program then proceeds to the next x_o value and continues.

A sample output from the BASIC program is included to show that the ξ values are not as exact for the $e^{-\psi}\psi'$ values, as YAHOO and BGIN make them. Remarks on how this program was used for this thesis are included in Appendix B.

    THIS DATA IS THESIS MATERIAL. DO NOT EDIT OR DELETE.(COLIN MIGHT CRY IF YOU DO.)

       INITIAL CENT DENS=   5.00E-01
       INITIAL BG DENS=   4.15E-02

       THERE ARE 20 RINGS

NOBS	ROUT	SIGOB	RAV
6.0000	0.0000	0.3806	1.5839
8.0000	2.2400	0.1692	3.5418
19.0000	4.4800	0.2411	5.7109
14.0000	6.7200	0.1269	7.9196
18.0000	8.9600	0.1269	10.1420
18.0000	11.2000	0.1038	12.3708
16.0000	13.4400	0.0781	14.6030
12.0000	15.6800	0.0508	16.8373
17.0000	17.9200	0.0634	19.0729
12.0000	20.1600	0.0401	21.3095
17.0000	22.4000	0.0514	23.5467
24.0000	24.6400	0.0662	25.7843
12.0000	26.8800	0.0305	28.0224
12.0000	29.1200	0.0282	30.2607
20.0000	31.3600	0.0438	32.4993
12.0000	33.6000	0.0246	34.7381
15.0000	35.8400	0.0288	36.9770
22.0000	38.0800	0.0399	39.2160
29.0000	40.3200	0.0497	41.4551
38.0000	42.5600	0.0618	43.6944
	44.8000		

XO IS EQUAL TO   10

ALPHA	CENT DENS	BG DENS	CHI-SQ	ALPHA	CENT DENS	BG DENS	CHI-SQ	ALPHA	CENT DENS	BG DENS	CHI-SQ
1.902E-01	1.757E+01	6.269E-02	1.185E+02	2.287E-01	5.755E+00	6.269E-02	1.185E+02	2.750E-01	2.906E+00	6.269E-02	1.185E+02
3.306E-01	1.762E+00	6.269E-02	1.185E+02	3.975E-01	1.591E+00	6.208E-02	1.156E+02	4.779E-01	1.228E+00	6.146E-02	1.118E+02
5.745E-01	8.475E-01	6.128E-02	1.104E+02	6.907E-01	1.325E+00	5.780E-02	9.389E+01	8.304E-01	1.068E+00	5.596E-02	7.982E+01
9.984E-01	8.421E-01	5.427E-02	6.795E+01	1.200E+00	6.899E-01	5.225E-02	5.607E+01	1.443E+00	5.684E-01	5.002E-02	4.419E+01
1.735E+00	4.595E-01	4.786E-02	3.397E+01	2.086E+00	3.607E-01	4.610E-02	2.774E+01	2.508E+00	2.823E-01	4.451E-02	2.513E+01
3.015E+00	2.257E-01	4.264E-02	2.474E+01	3.625E+00	1.805E-01	4.078E-02	2.809E+01	4.358E+00	1.441E-01	3.899E-02	3.596E+01
5.240E+00	1.223E-01	3.573E-02	4.370E+01	6.299E+00	1.092E-01	3.138E-02	5.121E+01	7.574E+00	1.015E-01	2.601E-02	5.876E+01
9.105E+00	9.826E-02	1.926E-02	6.617E+01	1.095E+01	9.950E-02	1.040E-02	7.307E+01	1.316E+01	1.057E-01	-1.531E-03	7.917E+01
1.582E+01	1.181E-01	-1.824E-02	8.429E+01	1.902E+01	1.386E-01	-4.191E-02	8.839E+01	2.287E+01	1.695E-01	-7.516E-02	9.167E+01
2.750E+01	2.166E-01	-1.239E-01	9.403E+01	3.306E+01	2.844E-01	-1.929E-01	9.593E+01	3.975E+01	3.847E-01	-2.940E-01	9.715E+01
4.779E+01	5.270E-01	-4.370E-01	9.824E+01	5.745E+00	7.387E-01	-6.490E-01	9.870E+01	6.907E+01	1.038E+00	-9.485E-01	9.923E+01
		MIN FROM PROGRAM:		3.015E+00	-2.257E-01	4.264E-02	2.474E+01				
		MINIMUM METHOD :		2.876E+00	2.386E-01	4.314E-02	2.461E+01				

XO IS EQUAL TO   20

ALPHA	CENT DENS	BG DENS	CHI-SQ	ALPHA	CENT DENS	BG DENS	CHI-SQ	ALPHA	CENT DENS	BG DENS	CHI-SQ
9.512E-02	5.765E+01	6.269E-02	1.185E+02	1.144E-01	1.863E+01	6.269E-02	1.185E+02	1.375E-01	9.315E+00	6.269E-02	1.185E+02
1.653E-01	5.405E+00	6.269E-02	1.185E+02	1.987E-01	4.517E+00	6.214E-02	1.159E+02	2.389E-01	3.358E+00	6.154E-02	1.124E+02
2.873E-01	2.218E+00	6.137E-02	1.111E+02	3.454E-01	3.070E+00	5.852E-02	9.828E+01	4.152E-01	2.483E+00	5.677E-02	8.654E+01
4.992E-01	1.993E+00	5.492E-02	7.518E+01	6.002E-01	1.584E+00	5.310E-02	6.439E+01	7.216E-01	1.267E+00	5.115E-02	5.365E+01
8.675E-01	1.004E+00	4.914E-02	4.339E+01	1.043E+00	7.734E-01	4.747E-02	3.581E+01	1.254E+00	5.920E-01	4.594E-02	3.051E+01
1.508E+00	4.590E-01	4.430E-02	2.671E+01	1.812E+00	3.568E-01	4.265E-02	2.529E+01	2.179E+00	2.775E-01	4.105E-02	2.690E+01
2.620E+00	2.250E-01	3.869E-02	2.893E+01	3.150E+00	1.877E-01	3.601E-02	3.184E+01	3.787E+00	1.599E-01	3.308E-02	3.592E+01
4.553E+00	1.396E-01	2.979E-02	4.098E+01	5.474E+00	1.246E-01	2.609E-02	4.708E+01	6.581E+00	1.142E-01	2.166E-02	5.392E+01
7.912E+00	1.082E-01	1.611E-02	6.110E+01	9.512E+00	1.064E-01	8.795E-03	6.818E+01	1.144E+01	1.092E-01	-1.037E-03	7.480E+01
1.375E+01	1.178E-01	-1.490E-02	8.054E+01	1.653E+01	1.332E-01	-3.431E-02	8.543E+01	1.987E+01	1.503E-01	-6.228E-02	8.929E+01
2.389E+01	1.957E-01	-1.018E-01	9.242E+01	2.873E+01	2.515E-01	-1.592E-01	9.466E+01	3.454E+01	3.341E-01	-2.428E-01	9.419E+01

THIS DATA IS THESIS MATERIAL. DO NOT EDIT OR DELETE.(COLIN MIGHT CRY IF YOU DO.)

```
INITIAL CENT DENS=    5.00E-01
         BG  DENS=    4.15E-02

THERE ARE 20 RINGS
```

NOBS	ROUT	SIGOB	RAV
6.0000	0.0000	0.3806	1.5839
8.0000	2.2400	0.1692	3.5418
19.0000	4.4800	0.2411	5.7109
14.0000	6.7200	0.1269	7.9196
18.0000	8.9600	0.1269	10.1420
18.0000	11.2000	0.1038	12.3708
16.0000	13.4400	0.0781	14.6030
12.0000	15.6800	0.0508	16.8373
17.0000	17.9200	0.0634	19.0729
12.0000	20.1600	0.0401	21.3095
17.0000	22.4000	0.0514	23.5467
24.0000	24.6400	0.0662	25.7843
12.0000	26.8800	0.0305	28.0224
12.0000	29.1200	0.0282	30.2607
20.0000	31.3600	0.0438	32.4993
12.0000	33.6000	0.0246	34.7381
15.0000	35.8400	0.0288	36.9770
22.0000	38.0800	0.0399	39.2160
29.0000	40.3200	0.0497	41.4551
38.0000	42.5600	0.0618	43.6944
	44.8000		

XO IS EQUAL TO   10

ALPHA	CENT DENS	CHI-SQ	ALPHA	CENT DENS	CHI-SQ	ALPHA	CENT DENS	CHI-SQ	ALPHA	CENT DENS	CHI-SQ
1.902E-01	1.874E+01	1.866E+02	2.287E-01	6.139E+00	1.866E+02	2.750E-01	3.100E+00	1.866E+02	3.306E-01	1.879E+00	1.866E+02
3.975E-01	2.259E+00	1.787E+02	4.779E-01	1.506E+00	1.714E+02	5.745E-01	1.193E+00	1.657E+02	6.907E-01	1.865E+00	1.307E+02
8.304E-01	1.292E+00	1.096E+02	9.984E-01	9.804E-01	9.110E+01	1.200E+00	7.958E-01	7.212E+01	1.443E+00	6.394E-01	5.404E+01
1.735E+00	5.013E-01	3.936E+01	2.086E+00	3.837E-01	3.051E+01	2.508E+00	2.941E-01	2.629E+01	3.015E+00	2.296E-01	2.490E+01
3.625E+00	1.784E-01	2.815E+01	4.358E+00	1.378E-01	3.662E+01	5.240E+00	1.086E-01	4.657E+01	6.299E+00	8.686E-02	5.803E+01
7.574E+00	7.043E-02	7.048E+01	9.105E+00	5.808E-02	8.306E+01	1.095E+01	4.884E-02	9.496E+01	1.316E+01	4.198E-02	1.056E+02
1.582E+01	3.690E-02	1.146E+02	1.902E+01	3.318E-02	1.219E+02	2.287E+01	3.050E-02	1.276E+02	2.750E+01	2.855E-02	1.320E+02
3.306E+01	2.717E-02	1.352E+02	3.975E+01	2.619E-02	1.376E+02	4.779E+01	2.551E-02	1.393E+02	5.745E+01	2.503E-02	1.404E+02
6.907E+01	2.470E-02	1.413E+02	8.304E+01	2.446E-02	1.419E+02	9.984E+01	2.430E-02	1.423E+02	1.200E+02	2.419E-02	1.426E+02
1.443E+02	2.411E-02	1.428E+02	1.735E+02	2.406E-02	1.429E+02	2.086E+02	2.402E-02	1.430E+02	2.508E+02	2.400E-02	1.431E+02
3.015E+02	2.398E-02	1.431E+02	3.625E+02	2.396E-02	1.431E+02	4.358E+02	2.396E-02	1.432E+02	5.240E+02	2.395E-02	1.432E+02
6.299E+02	2.395E-02	1.432E+02	7.574E+02	2.394E-02	1.432E+02	9.105E+02	2.394E-02	1.432E+02	1.095E+03	2.394E-02	1.432E+02
1.316E+03	2.394E-02	1.432E+02	1.582E+03	2.394E-02	1.432E+02						
	MIN FROM PROGRAM!		3.015E+00	2.296E-01	2.490E+01						
	MINIMUM METHOD !		2.952E+00	2.361E-01	2.488E+01						

XO IS EQUAL TO   20

ALPHA	CENT DENS	CHI-SQ	ALPHA	CENT DENS	CHI-SQ	ALPHA	CENT DENS	CHI-SQ	ALPHA	CENT DENS	CHI-SQ
9.512E-02	6.149E+01	1.866E+02	1.144E-01	1.987E+01	1.866E+02	1.375E-01	9.936E+00	1.866E+02	1.653E-01	5.765E+00	1.866E+02
1.987E-01	6.419E+00	1.794E+02	2.389E-01	4.223E+00	1.724E+02	2.873E-01	2.866E+00	1.688E+02	3.454E-01	4.701E+00	1.300E+02
4.152E-01	3.228E+00	1.181E+02	4.992E-01	2.424E+00	1.004E+02	6.002E-01	1.894E+00	8.284E+01	7.216E-01	1.477E+00	6.611E+01
8.675E-01	1.135E+00	5.098E+01	1.043E+00	8.499E-01	4.040E+01	1.254E+00	6.352E-01	3.300E+01	1.508E+00	4.808E-01	2.766E+01
1.812E+00	3.638E-01	2.550E+01	2.179E+00	2.753E-01	2.688E+01	2.620E+00	2.127E-01	2.971E+01	3.150E+00	1.668E-01	3.446E+01
3.787E+00	1.324E-01	4.124E+01	4.553E+00	1.066E-01	4.965E+01	5.474E+00	8.693E-02	5.944E+01	6.581E+00	7.178E-02	7.018E+01
7.912E+00	6.006E-02	8.130E+01	9.512E+00	5.098E-02	9.224E+01	1.144E+01	4.396E-02	1.024E+02	1.375E+01	3.861E-02	1.115E+02
1.653E+01	3.457E-02	1.191E+02	1.987E+01	3.157E-02	1.253E+02	2.389E+01	2.936E-02	1.302E+02	2.873E+01	2.777E-02	1.338E+02
3.454E+01	2.663E-02	1.365E+02	4.152E+01	2.582E-02	1.385E+02	4.992E+01	2.525E-02	1.399E+02	6.002E+01	2.485E-02	1.409E+02

OLD IGSFH

Ready

RUNNH
ENTER STEP SIZE, NUMBER OF STEPS
? .1, 100
 .1   100

XI	EXP(-PSI)	PSI	EXP(-PSI)*PSI
10	.238359E-1	.251061	.598429E-2
20	.449904E-2	.113128	.508965E-3
29.9998	.18658E-2	.694362E-1	.129554E-3
39.9997	.104079E-2	.494812E-1	.514997E-4
49.9995	.674007E-3	.383622E-1	.258564E-4
59.9994	.476767E-3	.031363	.149529E-4
69.9997	.357342E-3	.265801E-1	.949818E-5
80.0003	.278964E-3	.231133E-1	.644777E-5
90.0009	.224449E-3	.204869E-1	.459828E-5
100.002	.184835E-3	.184279E-1	.340612E-5
110.002	.155053E-3	.167689E-1	.260007E-5
120.003	.132042E-3	.154024E-1	.203376E-5
130.003	.113862E-3	.014256	.162321E-5
140.004	.992299E-4	.132793E-1	.13177E-5
150.005	.872661E-4	.124364E-1	.108528E-5
160.005	.773514E-4	.117008E-1	.905072E-6
170.006	.69038E-4	.110528E-1	.763049E-6
180.006	.619958E-4	.104766E-1	.649505E-6
190.007	.559762E-4	.996097E-2	.557577E-6
200.008	.507887E-4	.949637E-2	.482308E-6
210.008	.462864E-4	.907531E-2	.420063E-6
220.009	.423532E-4	.869172E-2	.368122E-6
230.009	.388966E-4	.834063E-2	.324422E-6
240.01	.358427E-4	.801792E-2	.287384E-6
250.011	.33131E-4	.772017E-2	.255777E-6
260.01	.307123E-4	.744448E-2	.228637E-6
270.008	.28546E-4	.71884E-2	.2052E-6
280.005	.265981E-4	.694984E-2	.184853E-6
290.003	.248404E-4	.008727	.167101E-6
300	.232488E-4	.651831E-2	.151543E-6
309.998	.218033E-4	.632244E-2	.13785E-6
319.995	.204866E-4	.61382E-2	.125751E-6
329.993	.192837E-4	.596456E-2	.115019E-6
339.991	.181822E-4	.58006E-2	.105468E-6
349.988	.17171E-4	.564551E-2	.96939E-7
359.986	.162405E-4	.549857E-2	.892996E-7
369.983	.153824E-4	.535914E-2	.824367E-7
379.981	.145895E-4	.522665E-2	.762543E-7
389.978	.138554E-4	.510057E-2	.706704E-7
399.976	.131744E-4	.498044E-2	.656141E-7
409.973	.125415E-4	.486585E-2	.610251E-7
419.971	.119525E-4	.47564E-2	.568508E-7
429.969	.114033E-4	.465177E-2	.530454E-7
439.966	.108905E-4	.455162E-2	.495693E-7
449.964	.104109E-4	.445567E-2	.463876E-7
459.961	.996182E-5	.436366E-2	.4347E-7
469.959	.954071E-5	.427535E-2	.407899E-7
479.956	.914532E-5	.419051E-2	.383236E-7
489.954	.877362E-5	.410895E-2	.360503E-7
499.951	.842375E-5	.403047E-2	.339517E-7
509.949	.809406E-5	.39549E-2	.320112E-7
519.947	.778305E-5	.388208E-2	.302144E-7
529.944	.748934E-5	.381186E-2	.285483E-7
539.942	.721167E-5	.37441E-2	.270012E-7