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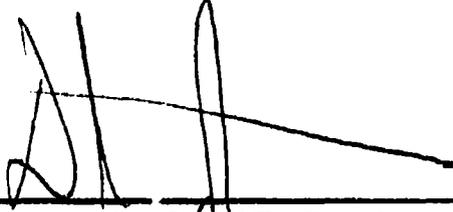
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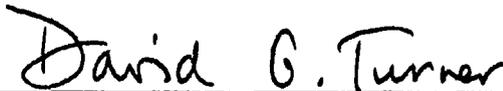
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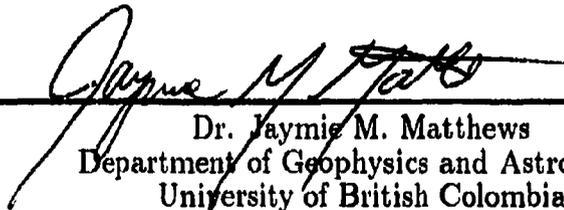
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Abstract

Seismology of Stars in the Hyades and M67 Open Clusters: Theoretical Constraints on the Mixing Length Parameter

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September 1996

The mixing length parameter is a free parameter governing convective energy transport in current stellar models, and is poorly constrained by current photometric observations. Theoretical nonradial pulsation spectra have been calculated for main-sequence stars in the Hyades and M67 open clusters. It is shown that the major eigenfrequency spacing is sensitive to the mixing length parameter, and relatively insensitive to uncertainties in stellar metallicity, age, and helium abundance. Given the predicted uncertainties in observing the major eigenfrequency spacing of stars in the Hyades and M67 clusters, it is shown that proposed seismological studies of these clusters (Gilliland *et al.* 1993; Appourchaux *et al.* 1991) could determine the mixing length parameters for member stars to within ± 0.2 . The determination of the mixing length parameter for a number of different, well-characterized stars would allow for the parameterization of convective properties as a function of other stellar characteristics. Such a parameterization would be beneficial to current efforts to numerically simulate stellar convection from first principles (Demarque 1996; Kim *et al.* 1996).

Contents

| | |
|--|-----------|
| <i>List of Figures</i> | <i>ii</i> |
| <i>Acknowledgements</i> | <i>iv</i> |
| 1 Introduction | 1 |
| 2 Objectives | 6 |
| 2.1 The Advantages of Stellar Clusters | 6 |
| 2.2 Observational Characteristics | 7 |
| 2.3 Selection of Clusters | 12 |
| 2.4 Determination of the Mixing Length Parameter using a Colour-Magnitude Diagram | 12 |
| 3 Results | 17 |
| 3.1 The Stellar Models and Pulsation Spectra | 17 |
| 3.2 Uncertainties in the Predictions | 19 |
| 4 Summary and Conclusions | 23 |
| Appendix A | 26 |
| References | 34 |
| <i>Curriculum Vitae</i> | 37 |

List of Figures

- 1 Diagram showing nodal lines for nonradial oscillations for $m=0$ and (a) $\ell=1$, (b) $\ell=2$, and (c) $\ell=3$. Red regions represent expansion and blue regions represent contraction. White regions represent nodal lines. Note that the azimuthal degree equals the number of nodal lines. 8
- 2 Colour-colour diagram for models run using Hyades cluster parameters. Each group of 11 points represents the variation of the mixing length parameter from 1.5 to 2.5 in increments of 0.1 for masses from $0.7 M_{\odot}$ to $1.5 M_{\odot}$ in increments of $0.1 M_{\odot}$. The arrow shows the direction of increasing mixing length parameter. 14
- 3 Colour-magnitude diagram for models run using Hyades cluster parameters. Each group of 11 points represents the variation of the mixing length parameter from 1.5 to 2.5 in increments of 0.1 for a mass of $0.7 M_{\odot}$ to $1.5 M_{\odot}$ in increments of $0.1 M_{\odot}$. The arrow shows the direction of increasing mixing length parameter. The shaded envelope represents the region within which the uncertainties in metallicity, helium abundance and age prevent accurate determination of the mixing length parameter. 15
- 4 Major spacing-absolute visual magnitude diagram for models run using Hyades cluster parameters. Each group of 11 points represents the variation of the mixing length parameter from 1.5 to 2.5 in increments of 0.1 for masses from $0.7 M_{\odot}$ to $1.5 M_{\odot}$ in increments of $0.1 M_{\odot}$. The arrow shows the direction of increasing mixing length parameter. The shaded envelope represents the region within which the uncertainties in metallicity, helium abundance and age prevent accurate determination of the mixing length parameter. 20

5 Major spacing-absolute visual magnitude diagram for models run using M67 cluster parameters. Each group of 11 points represents the variation of the mixing length parameter from 1.5 to 2.5 in increments of 0.1 for masses from $0.7 M_{\odot}$ to $1.1 M_{\odot}$ in increments of $0.1 M_{\odot}$. The arrow shows the direction of increasing mixing length parameter. The shaded envelope represents the region within which the uncertainties in metallicity, helium abundance and age prevent accurate determination of the mixing length parameter. 21

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1 Introduction

“So it seems to me that this development is a beautiful example of those bright occasions when theoreticians enter a new field before all logically necessary data are at hand, proceed on the basis of some clear working hypothesis, and in this manner gain results essential for the later derivation of a more definitive theory.”

-Martin Schwarzschild, 1961

In 1907, Robert Emden published the first basic text on stellar interiors. In this text Emden proceeded by using the working hypothesis that convection alone was the method by which energy was transported from the centre to the surface of every star. At about the same time, it was shown that radiative transfer can be an efficient energy transport mechanism at the high temperatures found in stars (Schwarzschild 1906). In this work, Karl Schwarzschild developed a criterion which predicts whether a given layer in a star is in convective or radiative equilibrium. Using this criterion, he showed that the atmosphere of the Sun was in radiative equilibrium, contrary to the working hypothesis of Emden (1907). Two decades later, Eddington (1926) wrote the second basic text on stellar interiors. In this text, radiative equilibrium was applied in every star from centre to surface. The question of radiative transfer versus convective transport inspired work by Cowling (1935, 1936) and Biermann (1932, 1935) which fostered the qualitative picture that many stars, our Sun being a prime example, consist of a radiative core surrounded by a convective envelope and a thin radiative atmosphere.

In order to produce more than just a qualitative picture of stellar interiors, it was necessary to model convective energy transport in a quantitative manner. To determine the efficiency of convection as an energy transport mechanism, Biermann (1932) adapted the mixing length theory of convection developed by Ludwig Prandtl (1925). The mixing length theory greatly reduces the complexity of mod-

eling the structure of stars with outer convective regions. It is obviously an oversimplification, but to its credit, its use has produced very accurate predictions of the state of the stellar interior. There now exist many different variations on mixing length theory, but all are based on the principles first developed by Biermann (1932, 1943, 1948), Vitense (1953), and Böhm-Vitense (1958). In this picture, the convective medium is made up of warmer rising and cooler sinking bubbles which travel a distance and then are dissolved into the surrounding medium. The distance that one of these convective elements may travel before dissolution is referred to as the mixing length. This basic picture also assumes that the rising bubbles are identical in size, number, mass, speed and mixing length to the sinking bubbles. Given these assumptions, an expression for convective flux can be formulated in terms of the mixing length (Cox & Guili 1968). Unfortunately, this mixing length cannot be determined by mixing length theory, and is not observable. In the laboratory experiments done by Prandtl (1925), the mixing length scaled as the depth of the convective layer. Taking into account the changes in density that occur in stellar convection zones, changes that did not occur in Prandtl's laboratory fluid, a further assumption that the mixing length is proportional to the local pressure scale height is typically made. This assumption assures that the internal flows of convective elements will have a simple, non-peculiar pattern (Schwarzschild 1961). The constant of proportionality (α) is called the mixing length parameter and is a global parameter. Work by Chen & Sofia (1987) using numerical simulations of turbulent, compressible convection has confirmed the assumption that the mixing length is indeed proportional to the local pressure scale height for deep efficient convection. The mixing length parameter, α , and the helium abundance, Y , of a star are used as free parameters in stellar modeling. Knowledge of stellar radius and luminosity constrain these parameters as described below.

The efficiency of convective energy transport depends on the mixing length pa-

parameter. A large mixing length parameter implies a large temperature gradient in a convective region. A change in the mixing length parameter has little effect on the luminosity of a star. The spatial extent of a convection zone is determined by the efficiency of that convection zone to transport the energy from its bottom to its top. This is determined by the temperature gradient. Basically, a larger temperature gradient will require a smaller distance to transport a given amount of energy. Thus, a larger mixing length parameter will decrease the stellar radius. For a more complete and physically accurate description see Larson (1974). Since the luminosity is relatively independent of choice of the mixing length parameter, a decrease in stellar radius caused by a larger mixing length parameter will result in an increase in stellar effective temperature. This is observable as a change in the colour of the star.

The mean molecular weight of a star is dependent on the helium mass fraction. Since the majority of a typical star is composed of hydrogen, an increase in the helium mass fraction at the expense of the hydrogen mass fraction will increase the mean molecular weight of a star. Since the luminosity of a star is strongly dependent on its mean molecular weight (see Clayton 1983), a change in the helium abundance affects the luminosity of a star. This is observable as a change in the absolute magnitude of the star.

The ability to constrain these two free parameters is contingent upon the accurate determination of observables such as mass, metallicity, luminosity and temperature. These free parameters are best determined for the Sun. For most field stars, it is necessary to make the assumption that the values of the solar helium abundance and mixing length can be scaled by some simple means to be valid for use in constructing stellar models of a particular field star. This assumption leads to an increased uncertainty in the determination of other relevant parameters such as mass, metallicity and age for that field star. One goal of current research (Demarque 1996;

Kim *et al.* 1996) is to develop a numerical method to model the convective envelope of the Sun and other stars from first principles. Results produced by such an effort will be free from the uncertainties of the mixing length formalism. Thus, it would be very useful to determine the mixing length for a number of different, well-characterized stars. This would allow for the parameterization of convective properties as a function of other stellar parameters and help direct the work on simulating convection from first principles.

The discovery that the surface of the Sun oscillates with a period of about five minutes (Leighton *et al.* 1962) does not obviously fit into a discussion of the mixing length parameter. For some time, the five minute oscillations were thought to be local resonances caused by the overshoot of convective elements into the atmosphere. It was demonstrated by Frazier (1968) that this explanation was inconsistent with spectral analysis of the oscillations. Frazier proposed that the oscillations were primarily a superposition of a large number of standing resonant acoustic waves trapped in subphotospheric layers. Prompted by this, Ulrich (1970) examined the acoustic properties of the subphotospheric layers of the Sun and determined that such acoustic oscillations could only exist along discrete lines in a diagram of horizontal wavenumber versus frequency. This pattern was confirmed observationally by Deubner (1975), but discrepancies between the observations and predictions showed that the contemporary models of the Sun were not correct. This was the birth of the field known as helioseismology. In the last two decades it is a field which has proven to be a window into the inner workings of the Sun. It provides a new set of observables which can be used to constrain the free parameters Υ and α .

Knowledge of the solar acoustic spectrum has provided a detailed look at the interior structure of the Sun. Its accurate determination has allowed for the testing of new ideas which push the envelope of modern physics (Demarque *et al.* 1994; Guenther *et al.* 1996). Unfortunately, the intrinsically small amplitudes of these

oscillations make them difficult to observe on stars other than the Sun (Gelly *et al.* 1988). However, the wealth of information that stands to be gained has prompted continual improvement in observational techniques and instrumentation. These efforts have concentrated on two categories of stars: those which are closest to us, and those in clusters. It is to the latter which this thesis is devoted.

2 Objectives

2.1 The Advantages of Stellar Clusters

Given that only recently has strong evidence for the detection of nonradial, solar-type pulsation been made (Kjeldsen *et al.* 1995), it is logical to ask what advantage there would be to observing stars in open clusters. The open clusters are more distant than many solar-type field stars, and hence pulsation would be less easily detected. However, observing clusters of stars does afford certain advantages.

The primary advantage of observing stellar clusters is that the interpretive value of observed oscillation data increases as knowledge of fundamental stellar parameters improve (Gough 1987; Brown 1991; Guenther & Demarque 1995). Many stellar clusters have been comprehensively studied and the fundamental parameters such as distance, age, and metallicity are known far better for stars in these clusters than for an individual field star. As well, with a stellar ensemble belonging to a cluster, we can assume that all stars have the same distance, age, metallicity and initial helium abundance. For a unique description of a typical star, one requires precise knowledge of six parameters: the stellar mass, age, metallicity, initial helium abundance, distance to the star, and a mixing length parameter (Gilliland & Brown 1992). Thus, differences between stars belonging to a cluster can be studied with four of the six parameters removed. With the addition of oscillation data to the list of independent observables, the opportunity exists to examine the assumptions originally made in defining a typical star.

Secondly, stellar clusters provide a field of view which is populated with *related* stars at a fairly high spatial density. By increasing the number of stars that can be observed in a single exposure, the efficiency of the observations is improved. Given that proposed observations (Gilliland 1995) would require a large time commitment from each of the largest telescopes, efficiency is a necessity.

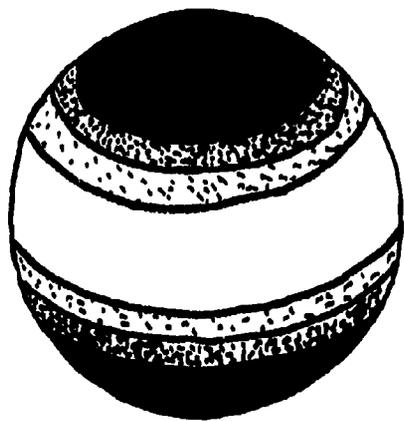
Thus, the increase in difficulty of observing oscillations in stars within clusters, as compared to closer field stars, is compensated for by the increase in the value of the information attained by such observations. Since the observation of stellar pulsation characteristics in cluster stars is such a formidable task, the choice of pulsation characteristic should be limited to that which is realistically obtainable at present.

2.2 Observational Characteristics

The Sun provides insight into what information can be gleaned from the pulsation spectra of other stars and how difficult it will be to gather that information. Study of the solar Five Minute Oscillation has shown that it is a superposition of about ten million individual resonant waves. Each of these resonant waves, or normal modes of oscillation, can be characterized by three wave numbers: n , ℓ and m . The radial order, n , roughly gives the number of nodes in the radial direction. The azimuthal degree, ℓ , is related to the horizontal wavenumber, k_h , at a radius r by

$$k_h = \frac{\sqrt{\ell(\ell + 1)}}{r}. \quad (1)$$

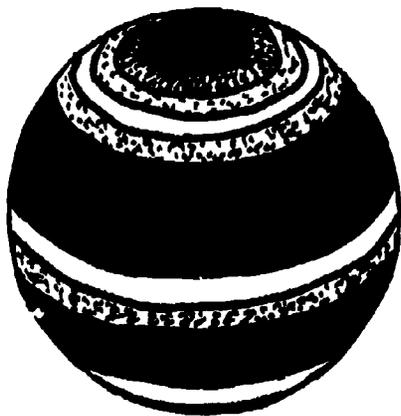
The case $\ell=0$ describes a mode in which the oscillation expands and contracts spherically in the radial direction. The azimuthal order, m , is related to the number of nodes around the equator. In principle, the frequency of a mode of oscillation depends on all three wavenumbers. If rotation and other departures from spherical symmetry are ignored, then the frequency is independent of the azimuthal order. Figure 1 depicts cases $\ell=1, 2$ and 3 for $m=0$ nonradial oscillations. In this thesis, consideration will be limited to stars for which departures from spherical symmetry are typically small. Thus, this work will assume that the frequency is independent of azimuthal order.



(a) $l=1$



(b) $l=2$



(c) $l=3$

Figure 1: Diagram showing nodal lines for nonradial oscillations for $m=0$ and (a) $l=1$, (b) $l=2$, and (c) $l=3$. Red regions represent expansion and blue regions represent contraction. White regions represent nodal lines. Note that the azimuthal degree equals the number of nodal lines.

The Five Minute Oscillation is comprised of resonant waves for which the restoring force is pressure. These are referred to as p-mode oscillations. Each p-mode is a trapped sound wave; thus, each p-mode carries information about the average sound speed over its path of travel. Since higher degree p-modes are increasingly confined to a region close to the surface, by observing $\ell=0$ modes one can determine the global average sound speed. By then observing $\ell=1$ modes one can determine the average sound speed in a region encompassing all but the most central region. With these two averages one can determine the sound speed local to the region where the $\ell=1$ mode is evanescent. One can resolve the run of sound speed with radius by continuing this process to higher and higher degree provided that it is possible to spatially resolve the modes with high azimuthal degree. The Sun is the one star for which it is currently possible to spatially resolve modes with azimuthal degree higher than about three or four. Observations of stars other than the Sun are limited by the lack of this spatial resolution. On an unresolved stellar surface, oscillations which have many nodes simultaneously visible are averaged out, and thus yield no information. Thus, this work will focus on modes with degrees $\ell=0,1,2,3$. This limits the determination of the structure of stars other than the Sun to more global properties.

Vandakurov (1967) showed that modes of low-degree (*i.e.* $\ell \ll n$) should satisfy a dispersion relation of the form

$$\nu_{n,\ell} = \left(n + \frac{1}{2}\left(\ell + \frac{1}{2}\right) + \alpha\right)\nu_0 + \epsilon_{n,\ell}, \quad (2)$$

where $\nu_{n,\ell}$ is the frequency of the mode with radial order n and azimuthal degree ℓ , α is related to the polytropic index of the outer layers of the stellar envelope and is of the order 1, $\epsilon_{n,\ell}$ is a second order correction term (Tassoul 1980), and ν_0 is the characteristic asymptotic frequency spacing, defined here as

$$\nu_0 = \left[2 \int_0^R \frac{dr}{c_s} \right]^{-1} \quad (3)$$

where R is the radius of the star and c_s is the sound speed at distance r from the centre. The characteristic asymptotic frequency spacing is thus a global constant for a given star. One can show that to first order, the dispersion relation presented by Vandakurov (1967) reduces to

$$\nu_{n,\ell} \sim \left[n + \frac{1}{2}\ell \right] \nu_0 + C, \quad (4)$$

where C is independent of both degree and radial order, and is small for $\ell \ll n$. Thus, a set of equally spaced frequencies are found for a given $\ell \ll n$, and the spacing between each frequency,

$$\nu_{n,\ell} - \nu_{n-1,\ell} \sim \nu_0 \quad (5)$$

is ν_0 , the characteristic asymptotic frequency spacing. In this work, the characteristic asymptotic frequency spacing is approximated using

$$\bar{\nu}_0 = \frac{\sum_{n=15}^{30} [\nu_{n,\ell} - \nu_{n-1,\ell}]}{16} \sim \nu_0, \quad (6)$$

where $\bar{\nu}_0$ is called the major spacing.

Using equation 4 for different degrees one can show that

$$\nu_{n,\ell} - \nu_{n,\ell-1} \sim \frac{\nu_0}{2}. \quad (7)$$

Thus, a star will exhibit a power spectrum which resembles a picket fence, and the spacing between pickets will be $\frac{\nu_0}{2}$. Promising evidence for such a distribution of oscillation frequencies has been observed recently (Kjeldsen *et al.* 1995) on the

star η Bootes. The first parameter determined from the frequency spectrum of η Bootes was the major spacing. Essentially, a correlation function was used to distinguish the presence of the regularly spaced peaks in the power spectrum caused by a genuine p-mode signal from the observational noise. A well-defined hump in the correlation function indicates the presence of a regular spacing, and this hump occurs at a spacing equal to half of the major spacing. In practice, when looking at real data, the picket fence described above has noise added to it. The correlation function tries to match an equally-spaced picket fence to the noisy observed power spectrum, and counts the number of peaks that match up for each different spacing of picket fence. The distribution with the spacing that matches the largest number of peaks in the observed power spectrum has a spacing equal to half the major spacing, $\frac{\nu_0}{2}$. Once this spacing is determined, peaks in the power spectrum resulting from actual oscillations are more easily identified. In this way, the major spacing is the most immediately available, and least uncertain, observable parameter to be acquired from a stellar oscillation spectra.

The major spacing is related to stellar mass and radius by

$$\bar{\nu}_0 \propto \left(\frac{M}{R^3}\right)^{\frac{1}{2}}. \quad (8)$$

That is, the major spacing is directly proportional to the mean density of the star. The major spacing of a star is therefore a useful, and realistically observable, parameter which is not currently being exploited for solar-type oscillators. Furthermore, the strong dependence of the major spacing on the stellar radius suggests that it may function as a probe of the mixing length parameter, since the stellar radius is a strong function of the choice of mixing length. It will be shown in this work that the major spacing is much less sensitive to realistic changes in helium abundance, age and metallicity than it is to realistic changes in the mixing length

parameter. Thus, the major spacing is a parameter which can be used to decouple the uncertainty in the mixing length parameter from the uncertainties in other stellar parameters.

2.3 Selection of Clusters

To best constrain the mixing length parameter, open cluster selection should be limited to clusters for which observable parameters are accurately known. Two such clusters are the targets of proposed observational campaigns: M67 (Gilliland *et al.* 1995) and the Hyades (Appourchaux *et al.* 1991). This thesis is focussed on these two clusters. Values used in this thesis for cluster age, metallicity and helium abundance are given below.

For M67, the cluster age was taken to be 4.0 ± 0.5 Gyr (Dinescu *et al.* 1995), and the cluster metallicity was taken to be $Z=0.017 \pm 0.004$ (Chaboyer *et al.*, 1995). The helium abundance and its uncertainty were calculated using the above metallicity value and its uncertainty in the enrichment equation $(Y - Y_{\odot}) = 2.5(Z - Z_{\odot})$ (Chaboyer *et al.* 1995).

For the Hyades, the cluster age was taken to be 0.67 Gyr (Boesgaard 1989) with an uncertainty of ± 0.05 Gyr. The cluster metallicity was taken to be $Z=0.024$ (Taylor 1994) with an uncertainty of ± 0.004 . The helium abundance and its uncertainty were calculated using the above metallicity value and its uncertainty in same manner as for M67.

2.4 Determination of the Mixing Length Parameter using a Colour-Magnitude Diagram

To demonstrate that neither colour-magnitude diagram analysis, nor colour-colour diagram analysis can sufficiently constrain the mixing length parameter given ob-

servational uncertainties, both a theoretical colour-magnitude diagram and *UBV* colour-colour diagram were generated for the Hyades. For the “best” values as given in the previous section, stellar models were constructed for masses from $0.7 M_{\odot}$ to $1.5 M_{\odot}$ in $0.1 M_{\odot}$ increments. For each of the masses, models having mixing length parameters from 1.5 to 2.5 in increments of 0.1 were generated. Evolutionary tracks were constructed using the Yale Rotating Evolution Code (YREC) (Guenther, Jaffe & Demarque 1989) in its nonrotating configuration using the OPAL opacities, Kurucz (1991) opacities for temperatures below $\log T = 4.0$, and the Anders & Grevesse (1989) solar mixture of metals. Colours and absolute magnitudes were calculated using the method described in Green *et al.* (1987). Appendix A shows model characteristics for all stellar models constructed using Hyades cluster parameters.

Figure 2 shows the *UBV* colour-colour diagram of the constructed models. The spread caused by varying the mixing length parameter is parallel to the direction of increasing stellar mass; hence, isochrone fitting will yield no extra information about the mixing length parameter. The *UBV* colour-colour diagram was chosen to demonstrate this, but all colour-colour diagrams in the *UBVRI* system were found to suffer similarly from this problem.

Figure 3 shows the colour-magnitude diagram of the constructed models. The spread caused by varying the mixing length parameter is not completely parallel to the direction of increasing stellar mass. Discrimination of the mixing length parameter to uncertainty levels of ± 0.1 would be possible with *U-B* colour determinations accurate to 0.02 magnitude if no uncertainty existed in other modeling parameters.

To investigate the effects of the known uncertainties in the modeling parameters, additional stellar models were constructed. The masses of these models ranged from $0.7 M_{\odot}$ to $1.5 M_{\odot}$ in $0.1 M_{\odot}$ increments. Each model was evolved with a mixing length parameter of 2.0. For each mass, four extreme models were evolved and their pulsation spectra were calculated. One model was evolved with both the cluster age

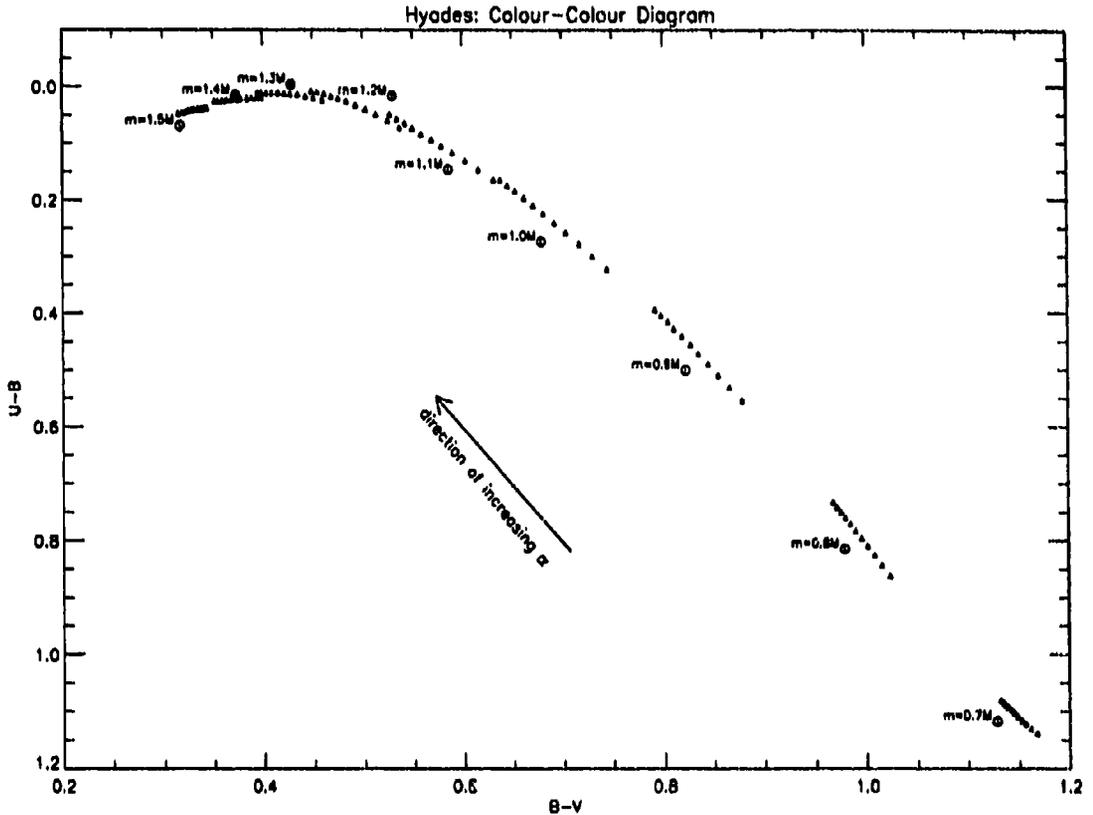


Figure 2: Colour-colour diagram for models run using Hyades cluster parameters. Each group of 11 points represents the variation of the mixing length parameter from 1.5 to 2.5 in increments of 0.1 for masses from $0.7 M_{\odot}$ to $1.5 M_{\odot}$ in increments of $0.1 M_{\odot}$. The arrow shows the direction of increasing mixing length parameter.

and metallicity at the maximum value allowed by observational uncertainties. One model was evolved with both the cluster age and metallicity at the minimum value allowed by observational uncertainties. One model was evolved with the cluster age at a minimum and the metallicity at a maximum. One model was evolved with the cluster age at a maximum and the metallicity at a minimum. It was assumed that low values of metallicity only occur in conjunction with low values of helium abundance, and conversely that high values of metallicity only occur in conjunction with high values of helium abundance. This assumption is consistent with the current understanding of galactic chemical enrichment. The uncertainties in the metallicity, age and helium abundance define an area of uncertainty shown

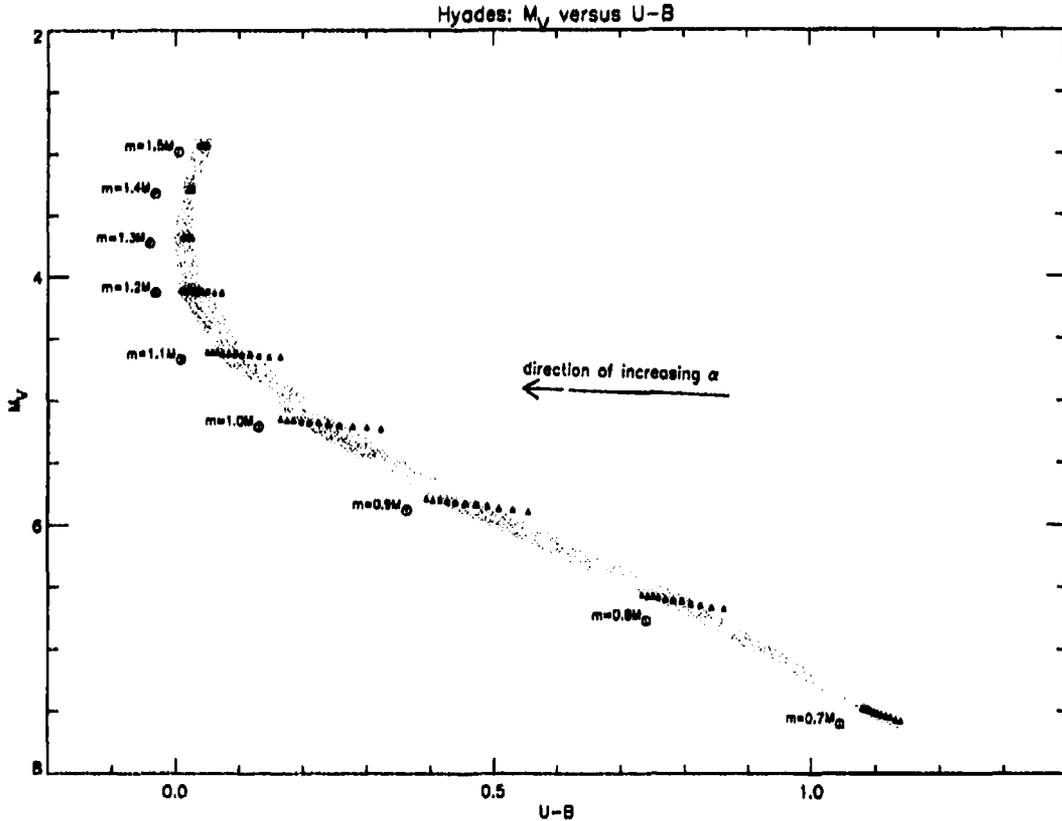


Figure 3: Colour-magnitude diagram for models run using Hyades cluster parameters. Each group of 11 points represents the variation of the mixing length parameter from 1.5 to 2.5 in increments of 0.1 for a mass of $0.7 M_{\odot}$ to $1.5 M_{\odot}$ in increments of $0.1 M_{\odot}$. The arrow shows the direction of increasing mixing length parameter. The shaded envelope represents the region within which the uncertainties in metallicity, helium abundance and age prevent accurate determination of the mixing length parameter.

as a shaded region in Figure 3. Taking this area of uncertainty to represent the outermost error boundary implicitly assumes a linear response to the perturbations in metallicity, helium abundance and age. This assumption is justified because the perturbations are relatively small, so to first order the response of the system will be directly proportional to the relative size of the perturbation. It is apparent from this figure that a colour-magnitude diagram cannot sufficiently decouple the uncertainty in the mixing length parameter from the uncertainties in other cluster parameters. At best, this method could constrain the mixing length parameter to ± 0.3 for stars of masses between $0.9 M_{\odot}$ and $1.0 M_{\odot}$. This precision rapidly degrades for stars of

masses higher than $1.1 M_{\odot}$ and lower than $0.9 M_{\odot}$. Such a level of precision is not sufficient to probe changes in the mixing length parameter with mass, and may not be sufficient to determine whether the mixing length parameter is constant for stars of the same mass.

3 Results

3.1 The Stellar Models and Pulsation Spectra

The purpose of this study is to determine whether the observation of pulsation spectra of main-sequence stars in nearby open clusters would provide a constraint on the mixing length parameter. To do this, evolutionary tracks were constructed using YREC in the configuration described in section 2.4. Stellar models were evolved for both the Hyades and M67 open clusters.

For the Hyades, evolutionary tracks were constructed for stars of masses from $0.7 M_{\odot}$ to $1.5 M_{\odot}$ using an increment of one-tenth of a solar mass. For each of these masses, tracks were evolved with the mixing length parameter being varied from 1.5 to 2.5 in increments of 0.1. Hence, 99 models were produced in mass-mixing length space. Individual models were evolved to a current best estimate for cluster age (Boesgaard 1989) and metallicity (Taylor 1994). The helium abundance for the Hyades was chosen assuming the helium enrichment parameter $R=2.5$ (Chaboyer, Demarque & Pinsonneault 1995), where $R=(\Delta Y/\Delta Z)$.

For M67, evolutionary tracks were constructed for stars of masses from 0.7 to 1.1 times solar using an increment of one-tenth of a solar mass. For each of these masses, tracks were evolved with the mixing length parameter being varied from 1.5 to 2.5 in increments of 0.1. Hence, 55 models were produced in mass-mixing length space. Individual models were evolved to a current best estimate for the cluster age (Dinescu, Demarque, Guenther, & Pinsonneault, 1995) and metallicity (Chaboyer *et al.* 1995). The helium abundance and its uncertainty were calculated using the above metallicity value and its uncertainty in same manner as for M67.

The pulsation spectrum for each evolved model was calculated by using Guenther's nonradial, nonadiabatic stellar pulsation code in its adiabatic configuration (Guenther 1994). Presently, nonadiabatic effects are negligible when compared with

observational error and, thus, have been left out in favour of increased speed of calculation. The pulsation code calculated frequencies of modes with radial orders $n=15$ to 30 and azimuthal orders $\ell=0$ to 3, and these frequencies were used to calculate the major spacing for each evolved model.

Absolute visual magnitudes were calculated using the method described in Green *et al.* (1987). This method is currently used to produce the Yale '95 Isochrones (Demarque 1996).

Figures 4 and 5 show the plot of major spacing versus absolute visual magnitude for stellar models evolved with Hyades cluster parameters and M67 cluster parameters, respectively. These plots demonstrate the dependence of the major spacing on both mass and mixing length.

The major spacing is most sensitive to changes in the mixing length parameter at masses near solar, with sensitivity tapering off at both higher and lower masses. This two-sided taper is produced by the competition of two effects. The first can be explained by looking at the definition of the mixing length parameter. A more massive, more centrally condensed star has a larger value for its pressure scale height than a less massive, less centrally condensed star. Since the mixing length parameter is a measure of the number of pressure scale-heights a convective element moves before it dissolves into its surroundings, a change in the mixing length parameter has a more pronounced effect on the spatial extent of a convection zone in a more massive star. Since a change in the mixing length parameter has little effect on the star below the outer convection zone, the change in radius of the convection zone will change the overall radius of the star. This change in overall radius will affect the major spacing as shown by equation 8. Thus, the spread in major spacing caused by varying the mixing length parameter should increase with mass.

The second effect acts in the opposing direction. It has been well-established by stellar modeling that very-low mass stars are completely convective. Stars with

masses greater than about $0.4 M_{\odot}$ develop a radiative core. With increasing mass, this radiative core becomes a larger fraction of a star and the outer convective zone become smaller in spatial extent. In Figures 4 and 5, the spread in major spacing caused by varying the mixing length parameter begins to decrease with mass for masses greater than about $1.1 \odot$. At this point, though the effect that the mixing length parameter has on the spatial extent of the convective zone is still increasing with mass, the size of the convection zone has become so small as to make the change in overall radius small. Thus, the spread in major spacing caused by varying the mixing length parameter begins to decrease with mass beyond about $1.1 \odot$, and the cigar-like shape is produced.

3.2 Uncertainties in the Predictions

The effect of uncertainties in helium abundance, metallicity and age was explored by evolving and pulsing models with all realistic combinations of these input parameters. For each mass, four extreme models were evolved and pulsed. One model was evolved with both the cluster age, and metallicity (and hence the helium abundance) at the maximum value allowed by observational uncertainties. One model was evolved with both the cluster age, and metallicity at the minimum value allowed by observational uncertainties. One model was evolved with the cluster age at a minimum and the metallicity at a maximum. One model was evolved with the cluster age at a maximum and the metallicity at a minimum. All of these models were evolved using a mixing length parameter of $\alpha = 2.0$. These extreme models were run for both M67 and the Hyades clusters.

By placing the above models of either cluster on a major spacing-absolute visual magnitude diagram and connecting points of equal metallicity and age, but different mass, an envelope is created which defines the region of uncertainty for that cluster.

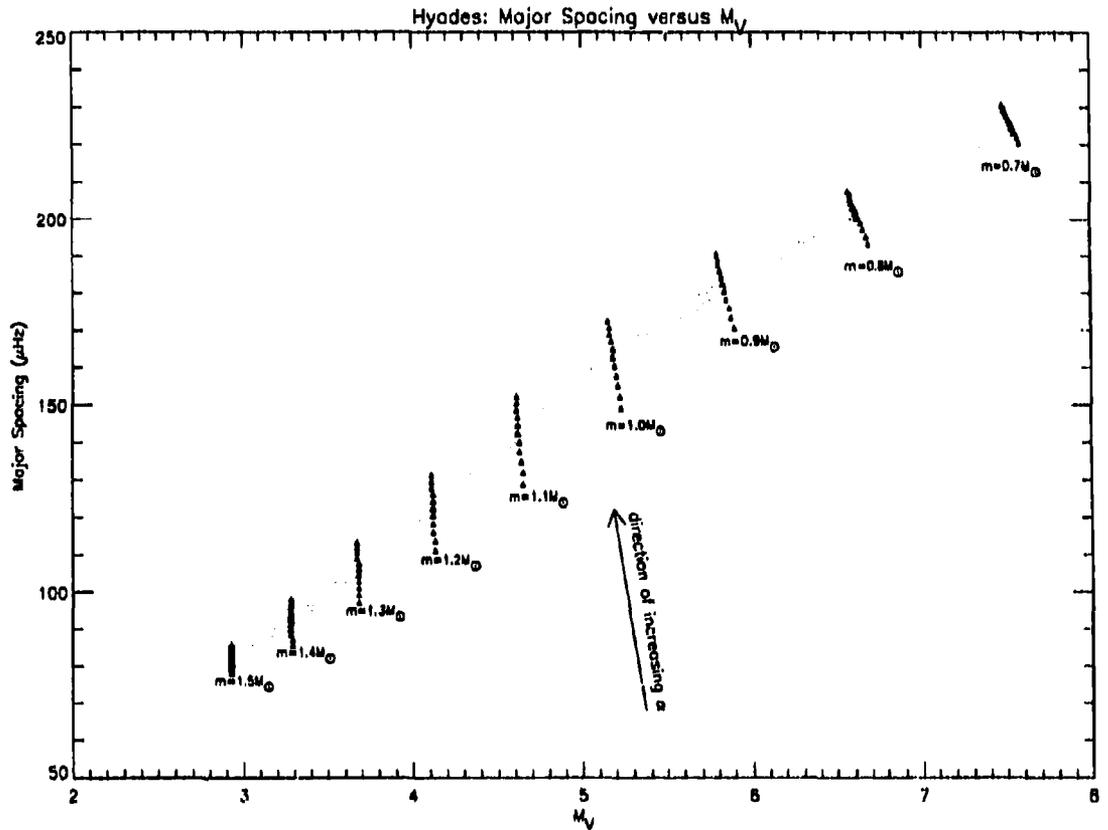


Figure 4: Major spacing-absolute visual magnitude diagram for models run using Hyades cluster parameters. Each group of 11 points represents the variation of the mixing length parameter from 1.5 to 2.5 in increments of 0.1 for masses from $0.7 M_\odot$ to $1.5 M_\odot$ in increments of $0.1 M_\odot$. The arrow shows the direction of increasing mixing length parameter. The shaded envelope represents the region within which the uncertainties in metallicity, helium abundance and age prevent accurate determination of the mixing length parameter.

This region of uncertainty is represented by the shaded regions on Figures 4 and 5. Appendix A tabulates model characteristics for all stellar models constructed.

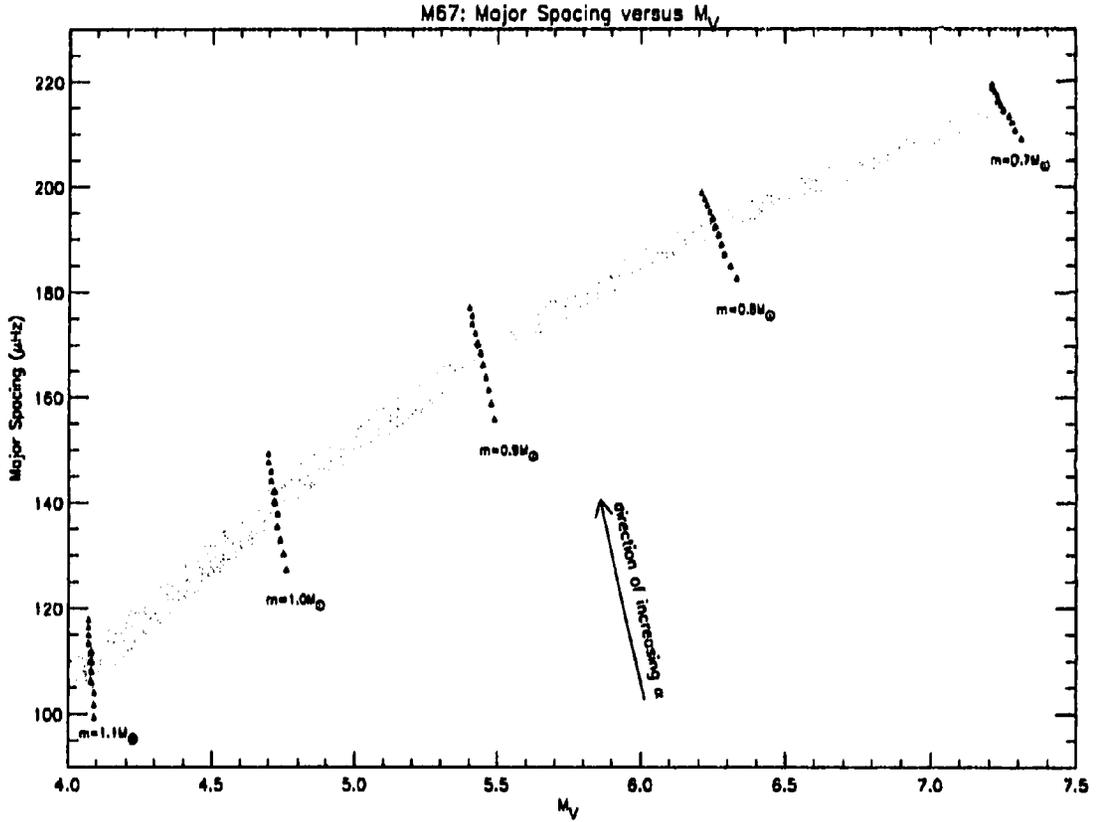


Figure 5: Major spacing-absolute visual magnitude diagram for models run using M67 cluster parameters. Each group of 11 points represents the variation of the mixing length parameter from 1.5 to 2.5 in increments of 0.1 for masses from $0.7 M_\odot$ to $1.1 M_\odot$ in increments of $0.1 M_\odot$. The arrow shows the direction of increasing mixing length parameter. The shaded envelope represents the region within which the uncertainties in metallicity, helium abundance and age prevent accurate determination of the mixing length parameter.

Uncertainties in the determination of absolute magnitudes for M67 and Hyades cluster stars will limit the ability of seismological observations to accurately constrain the mixing length parameter. The ability to distinguish trends in the mixing length parameter within a cluster is dependent upon the internal accuracy of the absolute magnitude determinations. As well, the ability to compare the mixing length parameters measured for stars in one cluster to the Sun or other clusters is dependent on the external accuracy of the absolute magnitude determinations. For

the Hyades cluster, internal uncertainties are small enough that they do not significantly impact the determination of trends in the mixing length parameter. Recent investigations of the Hyades constrain the external error to less than ± 0.07 mag (Peterson *et al.* 1993; Dombrowski *et al.* 1991; Gatewood *et al.* 1992). For M67, internal uncertainties are also small. Most recent investigations of M67 constrain the external error to less than ± 0.10 mag (Nissen *et al.* 1987; Vandenberg & Poll 1989; Demarque *et al.* 1992; Montgomery *et al.* 1993; Tripicco *et al.* 1993). Results from the Hipparcos satellite mission should greatly reduce the uncertainty for the Hyades. Uncertainty in the absolute magnitudes of M67 cluster members will constrain the ability of the technique to determine the mixing length parameter, but will not significantly affect the ability to distinguish trends in the mixing length parameter within the cluster.

4 Summary and Conclusions

In proposing a Next Generation Network, Gilliland *et al.* (1995) suggested that such a campaign necessarily required an effort on the part of theorists to justify such a time commitment of the largest telescopes. One such justification has been shown here. Seismological data from stars in stellar clusters could be used to parameterize convection as a function of other stellar properties. Such a parameterization could test whether numerical simulations of convection currently being developed for the Sun can adequately model the convection occurring on other stars. A detailed understanding of the physics underlying stellar convection will eliminate the need for the mixing length formalism. This will allow for more accurate and reliable prediction of stellar parameters such as age and luminosity.

The previous attempt by Gilliland *et al.* (1995) to observe stellar pulsation in cluster members has produced promising results, but no strong evidence that detailed pulsation spectra can be obtained for such stars using current techniques. For that reason, this work has concentrated on determining whether or not the major spacing alone can provide new, useful information about stellar structure. The major spacing will be the most easily determined information to come out of the observation of stellar pulsation. Also, since the major spacing is an average property of the oscillations, its determination will improve as the oscillation frequencies become better determined.

The work done here has shown that the mixing length parameters for stars in the Hyades and M67 clusters can be determined by using the major spacing as a constraint. If the pulsation spectra of stars in these clusters could be measured with the same accuracy as the solar pulsation spectra, mixing length parameters could be determined to within ± 0.1 . This limit is imposed by the uncertainties in metallicities, helium abundances, and ages of these stars. By using simulated pulsa-

tion spectra, Gilliland *et al.* (1995) estimated the uncertainty in the measurement of the major spacing of stars in M67 at about $\pm 2\mu\text{Hz}$. With such an uncertainty, mixing length parameters could be determined to within ± 0.2 . The recent observations of η Boo by Kjeldsen *et al.* (1995) support the claims that the major spacing can be measured for stars other than our Sun. Observations done by Kjeldsen *et al.* (1995) show that the major spacing for η Boo is determined to an accuracy of about $\pm 1\mu\text{Hz}$. The ability to measure the mixing length parameter of these stars will increase as observations improve. Given the uncertainties currently expected in such observations, determination of mixing length using the major spacing is a realistic possibility. It has also been shown here that such a determination cannot be made realistically using CMD fitting.

The question must be asked whether the mixing length should be expected to vary from star to star. It is possible that the characteristics of convection in the Sun can be transferred from star to star by scaling to the pressure scale height. Chaboyer *et al.* (1995) constructed isochrones for 40 globular clusters and fit them to their corresponding observations. In that work it is shown that a solar mixing length parameter, $\alpha = 1.7$, yields a good fit for all examined clusters. The evidence for this is the good agreement between theoretical and observed post-main-sequence stars, primarily on the giant branch. However, this work does not consider the possibility that the mixing length parameter may vary from star to star. The mixing length parameter that they derive is a best median value for the cluster as a whole. Their work shows directly that many individual stars do exist that could be better modeled by using non-solar mixing length parameters than by using the solar mixing length parameter.

Other evidence for a mixing length which does not scale to solar as expected has been shown by detailed evolutionary models of α Cen A and B (Edmonds *et al.* 1992). It was shown that α Cen A requires a smaller mixing length parameter

than does its companion α Cen B. Given that both stars in this system should have the same age, metallicity and initial helium abundance, such a distinction could be taken as an indication that the mixing length parameter may vary with mass.

There exist many important factors which are not incorporated into stellar modeling. Uncertainty in the treatment of convection may mask the effects of magnetic fields, diffusion, gravitational settling, convective overshoot and rotation. It is unlikely that with such a simplistic formalism for the treatment of convection one could produce stellar models which are accurate enough to probe such effects. The construction of a model for convection based on fundamental physics can help to clear the way for such detailed analyses. A parameterization of the mixing length parameter using cluster members will provide the large data set needed to establish a detailed, universal model of stellar convection.

The results of this work alone are not reason enough to justify the large observational commitment necessary to determine the pulsation spectra of cluster members in M67 or in the Hyades. This work does provide evidence that successful observations will produce unique constraints on stellar structure. Effort is needed on the part of the theoretical community to provide other such justifications. By providing a large database of stars with well-known structure, the observation of solar-type pulsation on stars in clusters will help to define what problems are present in the current theories of stellar structure and evolution.

Appendix A

Table 1: Model characteristics for stars evolved using Hyades parameters.

| Mass [M_{\odot}] | Z | Y | age [Gyr] | α | U-B [mag] | B-V [mag] | V-R [mag] | R-I [mag] | M_v [mag] | i_0 [μHz] |
|-------------------------|-------|----|--------------|----------|--------------|--------------|--------------|--------------|----------------|-----------------------------|
| 0.7 | 0.028 | 31 | 0.62 | 2.0 | 1.139 | 1.162 | 0.700 | 0.590 | 7.54 | 222.73 |
| 0.7 | 0.028 | 31 | 0.72 | 2.0 | 1.139 | 1.162 | 0.700 | 0.589 | 7.53 | 222.46 |
| 0.7 | 0.024 | 29 | 0.67 | 1.5 | 1.139 | 1.167 | 0.708 | 0.600 | 7.58 | 220.46 |
| 0.7 | 0.024 | 29 | 0.67 | 1.6 | 1.131 | 1.161 | 0.703 | 0.595 | 7.57 | 221.96 |
| 0.7 | 0.024 | 29 | 0.67 | 1.7 | 1.123 | 1.156 | 0.699 | 0.591 | 7.55 | 223.28 |
| 0.7 | 0.024 | 29 | 0.67 | 1.8 | 1.116 | 1.152 | 0.695 | 0.587 | 7.54 | 224.53 |
| 0.7 | 0.024 | 29 | 0.67 | 1.9 | 1.109 | 1.148 | 0.692 | 0.584 | 7.53 | 225.65 |
| 0.7 | 0.024 | 29 | 0.67 | 2.0 | 1.103 | 1.145 | 0.689 | 0.581 | 7.52 | 226.61 |
| 0.7 | 0.024 | 29 | 0.67 | 2.1 | 1.098 | 1.142 | 0.686 | 0.578 | 7.51 | 227.56 |
| 0.7 | 0.024 | 29 | 0.67 | 2.2 | 1.093 | 1.139 | 0.683 | 0.576 | 7.50 | 228.38 |
| 0.7 | 0.024 | 29 | 0.67 | 2.3 | 1.089 | 1.136 | 0.681 | 0.574 | 7.49 | 229.16 |
| 0.7 | 0.024 | 29 | 0.67 | 2.4 | 1.085 | 1.134 | 0.679 | 0.572 | 7.49 | 229.86 |
| 0.7 | 0.024 | 29 | 0.67 | 2.5 | 1.081 | 1.131 | 0.677 | 0.570 | 7.48 | 230.50 |
| 0.7 | 0.020 | 27 | 0.62 | 2.0 | 1.106 | 1.152 | 0.699 | 0.594 | 7.60 | 227.14 |
| 0.7 | 0.020 | 27 | 0.72 | 2.0 | 1.106 | 1.152 | 0.699 | 0.594 | 7.60 | 226.85 |
| 0.8 | 0.028 | 31 | 0.62 | 2.0 | 0.809 | 1.000 | 0.567 | 0.480 | 6.60 | 199.03 |
| 0.8 | 0.028 | 31 | 0.72 | 2.0 | 0.807 | 0.999 | 0.567 | 0.479 | 6.60 | 198.77 |
| 0.8 | 0.024 | 29 | 0.67 | 1.5 | 0.862 | 1.023 | 0.589 | 0.499 | 6.68 | 193.17 |
| 0.8 | 0.024 | 29 | 0.67 | 1.6 | 0.843 | 1.015 | 0.582 | 0.493 | 6.67 | 195.20 |
| 0.8 | 0.024 | 29 | 0.67 | 1.7 | 0.826 | 1.008 | 0.576 | 0.489 | 6.65 | 197.13 |
| 0.8 | 0.024 | 29 | 0.67 | 1.8 | 0.810 | 1.001 | 0.571 | 0.485 | 6.64 | 198.88 |
| 0.8 | 0.024 | 29 | 0.67 | 1.9 | 0.796 | 0.995 | 0.566 | 0.481 | 6.62 | 200.47 |
| 0.8 | 0.024 | 29 | 0.67 | 2.0 | 0.783 | 0.989 | 0.562 | 0.478 | 6.61 | 201.83 |
| 0.8 | 0.024 | 29 | 0.67 | 2.1 | 0.771 | 0.984 | 0.558 | 0.475 | 6.60 | 203.15 |
| 0.8 | 0.024 | 29 | 0.67 | 2.2 | 0.760 | 0.979 | 0.554 | 0.472 | 6.59 | 204.39 |
| 0.8 | 0.024 | 29 | 0.67 | 2.3 | 0.751 | 0.975 | 0.551 | 0.470 | 6.58 | 205.54 |
| 0.8 | 0.024 | 29 | 0.67 | 2.4 | 0.742 | 0.971 | 0.548 | 0.467 | 6.58 | 206.58 |
| 0.8 | 0.024 | 29 | 0.67 | 2.5 | 0.733 | 0.967 | 0.545 | 0.465 | 6.57 | 207.48 |
| 0.8 | 0.020 | 27 | 0.62 | 2.0 | 0.777 | 0.987 | 0.564 | 0.482 | 6.66 | 203.80 |
| 0.8 | 0.020 | 27 | 0.72 | 2.0 | 0.775 | 0.987 | 0.563 | 0.482 | 6.66 | 203.52 |

| Mass [M_{\odot}] | Z | Y | age [Gyr] | α | U-B [mag] | B-V [mag] | V-R [mag] | R-I [mag] | M_v [mag] | $\bar{\nu}_0$ [μHz] |
|-------------------------|-------|----|--------------|----------|--------------|--------------|--------------|--------------|----------------|-------------------------------------|
| 0.9 | 0.028 | 31 | 0.62 | 2.0 | 0.478 | 0.833 | 0.457 | 0.401 | 5.81 | 179.46 |
| 0.9 | 0.028 | 31 | 0.72 | 2.0 | 0.476 | 0.832 | 0.456 | 0.401 | 5.80 | 179.10 |
| 0.9 | 0.024 | 29 | 0.67 | 1.5 | 0.555 | 0.877 | 0.485 | 0.424 | 5.90 | 170.60 |
| 0.9 | 0.024 | 29 | 0.67 | 1.6 | 0.531 | 0.865 | 0.478 | 0.419 | 5.88 | 173.39 |
| 0.9 | 0.024 | 29 | 0.67 | 1.7 | 0.509 | 0.854 | 0.471 | 0.414 | 5.87 | 175.93 |
| 0.9 | 0.024 | 29 | 0.67 | 1.8 | 0.490 | 0.844 | 0.464 | 0.409 | 5.85 | 178.27 |
| 0.9 | 0.024 | 29 | 0.67 | 1.9 | 0.472 | 0.834 | 0.459 | 0.405 | 5.84 | 180.42 |
| 0.9 | 0.024 | 29 | 0.67 | 2.0 | 0.456 | 0.826 | 0.454 | 0.402 | 5.83 | 182.41 |
| 0.9 | 0.024 | 29 | 0.67 | 2.1 | 0.441 | 0.818 | 0.449 | 0.398 | 5.82 | 184.25 |
| 0.9 | 0.024 | 29 | 0.67 | 2.2 | 0.428 | 0.810 | 0.444 | 0.395 | 5.81 | 185.95 |
| 0.9 | 0.024 | 29 | 0.67 | 2.3 | 0.416 | 0.804 | 0.441 | 0.392 | 5.80 | 187.52 |
| 0.9 | 0.024 | 29 | 0.67 | 2.4 | 0.404 | 0.797 | 0.437 | 0.390 | 5.80 | 189.00 |
| 0.9 | 0.024 | 29 | 0.67 | 2.5 | 0.394 | 0.791 | 0.434 | 0.387 | 5.79 | 190.38 |
| 0.9 | 0.020 | 27 | 0.62 | 2.0 | 0.435 | 0.819 | 0.452 | 0.403 | 5.87 | 185.61 |
| 0.9 | 0.020 | 27 | 0.72 | 2.0 | 0.433 | 0.818 | 0.451 | 0.403 | 5.86 | 185.27 |
| 1.0 | 0.028 | 31 | 0.62 | 2.0 | 0.247 | 0.688 | 0.379 | 0.346 | 5.16 | 159.33 |
| 1.0 | 0.028 | 31 | 0.72 | 2.0 | 0.245 | 0.687 | 0.378 | 0.345 | 5.15 | 158.81 |
| 1.0 | 0.024 | 29 | 0.67 | 1.5 | 0.323 | 0.744 | 0.407 | 0.370 | 5.23 | 149.05 |
| 1.0 | 0.024 | 29 | 0.67 | 1.6 | 0.300 | 0.729 | 0.400 | 0.364 | 5.22 | 152.20 |
| 1.0 | 0.024 | 29 | 0.67 | 1.7 | 0.278 | 0.716 | 0.393 | 0.359 | 5.21 | 155.11 |
| 1.0 | 0.024 | 29 | 0.67 | 1.8 | 0.258 | 0.703 | 0.387 | 0.354 | 5.20 | 157.81 |
| 1.0 | 0.024 | 29 | 0.67 | 1.9 | 0.241 | 0.692 | 0.381 | 0.350 | 5.19 | 160.33 |
| 1.0 | 0.024 | 29 | 0.67 | 2.0 | 0.225 | 0.681 | 0.376 | 0.345 | 5.18 | 162.66 |
| 1.0 | 0.024 | 29 | 0.67 | 2.1 | 0.210 | 0.671 | 0.371 | 0.342 | 5.18 | 164.84 |
| 1.0 | 0.024 | 29 | 0.67 | 2.2 | 0.197 | 0.662 | 0.367 | 0.338 | 5.17 | 166.88 |
| 1.0 | 0.024 | 29 | 0.67 | 2.3 | 0.185 | 0.653 | 0.363 | 0.335 | 5.16 | 168.79 |
| 1.0 | 0.024 | 29 | 0.67 | 2.4 | 0.175 | 0.645 | 0.359 | 0.332 | 5.16 | 170.58 |
| 1.0 | 0.024 | 29 | 0.67 | 2.5 | 0.165 | 0.638 | 0.356 | 0.329 | 5.15 | 172.26 |
| 1.0 | 0.020 | 27 | 0.62 | 2.0 | 0.201 | 0.674 | 0.373 | 0.346 | 5.22 | 166.58 |
| 1.0 | 0.020 | 27 | 0.72 | 2.0 | 0.199 | 0.673 | 0.372 | 0.346 | 5.21 | 166.10 |

| Mass [M_{\odot}] | Z | Y | age [Gyr] | α | U-B [mag] | B-V [mag] | V-R [mag] | R-I [mag] | M_v [mag] | $\bar{\nu}_0$ [μHz] |
|-------------------------|-------|----|--------------|----------|--------------|--------------|--------------|--------------|----------------|-------------------------------------|
| 1.1 | 0.028 | 31 | 0.62 | 2.0 | 0.114 | 0.575 | 0.324 | 0.304 | 4.59 | 138.94 |
| 1.1 | 0.028 | 31 | 0.72 | 2.0 | 0.113 | 0.574 | 0.324 | 0.303 | 4.58 | 138.22 |
| 1.1 | 0.024 | 29 | 0.67 | 1.5 | 0.165 | 0.631 | 0.351 | 0.328 | 4.65 | 128.80 |
| 1.1 | 0.024 | 29 | 0.67 | 1.6 | 0.147 | 0.616 | 0.344 | 0.322 | 4.65 | 131.85 |
| 1.1 | 0.024 | 29 | 0.67 | 1.7 | 0.131 | 0.603 | 0.338 | 0.317 | 4.64 | 134.71 |
| 1.1 | 0.024 | 29 | 0.67 | 1.8 | 0.117 | 0.590 | 0.332 | 0.312 | 4.63 | 137.39 |
| 1.1 | 0.024 | 29 | 0.67 | 1.9 | 0.105 | 0.579 | 0.326 | 0.307 | 4.63 | 139.91 |
| 1.1 | 0.024 | 29 | 0.67 | 2.0 | 0.094 | 0.569 | 0.322 | 0.303 | 4.62 | 142.27 |
| 1.1 | 0.024 | 29 | 0.67 | 2.1 | 0.084 | 0.559 | 0.317 | 0.299 | 4.62 | 144.49 |
| 1.1 | 0.024 | 29 | 0.67 | 2.2 | 0.074 | 0.550 | 0.313 | 0.296 | 4.62 | 146.58 |
| 1.1 | 0.024 | 29 | 0.67 | 2.3 | 0.065 | 0.542 | 0.309 | 0.292 | 4.61 | 148.55 |
| 1.1 | 0.024 | 29 | 0.67 | 2.4 | 0.057 | 0.534 | 0.305 | 0.289 | 4.61 | 150.40 |
| 1.1 | 0.024 | 29 | 0.67 | 2.5 | 0.049 | 0.527 | 0.302 | 0.286 | 4.61 | 152.14 |
| 1.1 | 0.020 | 27 | 0.62 | 2.0 | 0.072 | 0.562 | 0.319 | 0.303 | 4.67 | 146.46 |
| 1.1 | 0.020 | 27 | 0.72 | 2.0 | 0.071 | 0.561 | 0.318 | 0.303 | 4.66 | 145.78 |
| 1.2 | 0.028 | 31 | 0.62 | 2.0 | 0.042 | 0.491 | 0.284 | 0.271 | 4.09 | 119.71 |
| 1.2 | 0.028 | 31 | 0.72 | 2.0 | 0.042 | 0.490 | 0.283 | 0.271 | 4.08 | 118.78 |
| 1.2 | 0.024 | 29 | 0.67 | 1.5 | 0.072 | 0.537 | 0.305 | 0.292 | 4.13 | 111.06 |
| 1.2 | 0.024 | 29 | 0.67 | 1.6 | 0.060 | 0.525 | 0.300 | 0.287 | 4.13 | 113.56 |
| 1.2 | 0.024 | 29 | 0.67 | 1.7 | 0.049 | 0.513 | 0.294 | 0.282 | 4.12 | 115.93 |
| 1.2 | 0.024 | 29 | 0.67 | 1.8 | 0.040 | 0.503 | 0.289 | 0.278 | 4.12 | 118.18 |
| 1.2 | 0.024 | 29 | 0.67 | 1.9 | 0.033 | 0.493 | 0.285 | 0.274 | 4.12 | 120.32 |
| 1.2 | 0.024 | 29 | 0.67 | 2.0 | 0.026 | 0.484 | 0.281 | 0.270 | 4.12 | 122.35 |
| 1.2 | 0.024 | 29 | 0.67 | 2.1 | 0.021 | 0.476 | 0.277 | 0.267 | 4.12 | 124.27 |
| 1.2 | 0.024 | 29 | 0.67 | 2.2 | 0.017 | 0.469 | 0.273 | 0.263 | 4.12 | 126.10 |
| 1.2 | 0.024 | 29 | 0.67 | 2.3 | 0.013 | 0.462 | 0.270 | 0.260 | 4.11 | 127.85 |
| 1.2 | 0.024 | 29 | 0.67 | 2.4 | 0.010 | 0.455 | 0.266 | 0.257 | 4.11 | 129.50 |
| 1.2 | 0.024 | 29 | 0.67 | 2.5 | 0.008 | 0.449 | 0.263 | 0.254 | 4.11 | 131.09 |
| 1.2 | 0.020 | 27 | 0.62 | 2.0 | 0.008 | 0.480 | 0.279 | 0.270 | 4.17 | 126.39 |
| 1.2 | 0.020 | 27 | 0.72 | 2.0 | 0.008 | 0.478 | 0.278 | 0.270 | 4.16 | 125.50 |

| Mass [M_{\odot}] | Z | Y | age [Gyr] | α | U-B [mag] | B-V [mag] | V-R [mag] | R-I [mag] | M_v [mag] | $\bar{\nu}_0$ [μHz] |
|-------------------------|-------|----|--------------|----------|--------------|--------------|--------------|--------------|----------------|-------------------------------------|
| 1.3 | 0.028 | 31 | 0.62 | 2.0 | 0.025 | 0.428 | 0.251 | 0.243 | 3.65 | 103.66 |
| 1.3 | 0.028 | 31 | 0.72 | 2.0 | 0.025 | 0.428 | 0.251 | 0.243 | 3.64 | 102.54 |
| 1.3 | 0.024 | 29 | 0.67 | 1.5 | 0.024 | 0.460 | 0.268 | 0.261 | 3.68 | 97.16 |
| 1.3 | 0.024 | 29 | 0.67 | 1.6 | 0.020 | 0.451 | 0.264 | 0.256 | 3.68 | 99.07 |
| 1.3 | 0.024 | 29 | 0.67 | 1.7 | 0.017 | 0.443 | 0.259 | 0.253 | 3.68 | 100.92 |
| 1.3 | 0.024 | 29 | 0.67 | 1.8 | 0.014 | 0.435 | 0.255 | 0.249 | 3.68 | 102.68 |
| 1.3 | 0.024 | 29 | 0.67 | 1.9 | 0.013 | 0.428 | 0.252 | 0.245 | 3.68 | 104.36 |
| 1.3 | 0.024 | 29 | 0.67 | 2.0 | 0.012 | 0.422 | 0.248 | 0.242 | 3.68 | 105.97 |
| 1.3 | 0.024 | 29 | 0.67 | 2.1 | 0.011 | 0.416 | 0.245 | 0.239 | 3.68 | 107.50 |
| 1.3 | 0.024 | 29 | 0.67 | 2.2 | 0.011 | 0.410 | 0.242 | 0.236 | 3.67 | 108.97 |
| 1.3 | 0.024 | 29 | 0.67 | 2.3 | 0.011 | 0.404 | 0.238 | 0.233 | 3.67 | 110.39 |
| 1.3 | 0.024 | 29 | 0.67 | 2.4 | 0.011 | 0.399 | 0.236 | 0.230 | 3.67 | 111.74 |
| 1.3 | 0.024 | 29 | 0.67 | 2.5 | 0.011 | 0.395 | 0.233 | 0.227 | 3.67 | 113.05 |
| 1.3 | 0.020 | 27 | 0.62 | 2.0 | -0.004 | 0.417 | 0.246 | 0.242 | 3.72 | 109.23 |
| 1.3 | 0.020 | 27 | 0.72 | 2.0 | -0.004 | 0.416 | 0.246 | 0.241 | 3.70 | 108.17 |
| 1.4 | 0.028 | 31 | 0.62 | 2.0 | 0.033 | 0.380 | 0.224 | 0.219 | 3.26 | 90.35 |
| 1.4 | 0.028 | 31 | 0.72 | 2.0 | 0.034 | 0.382 | 0.225 | 0.220 | 3.25 | 89.01 |
| 1.4 | 0.024 | 29 | 0.67 | 1.5 | 0.020 | 0.399 | 0.235 | 0.232 | 3.29 | 85.61 |
| 1.4 | 0.024 | 29 | 0.67 | 1.6 | 0.020 | 0.394 | 0.232 | 0.229 | 3.29 | 87.03 |
| 1.4 | 0.024 | 29 | 0.67 | 1.7 | 0.021 | 0.389 | 0.229 | 0.226 | 3.28 | 88.41 |
| 1.4 | 0.024 | 29 | 0.67 | 1.8 | 0.021 | 0.384 | 0.226 | 0.223 | 3.28 | 89.75 |
| 1.4 | 0.024 | 29 | 0.67 | 1.9 | 0.022 | 0.378 | 0.223 | 0.220 | 3.28 | 91.04 |
| 1.4 | 0.024 | 29 | 0.67 | 2.0 | 0.023 | 0.374 | 0.221 | 0.217 | 3.28 | 92.29 |
| 1.4 | 0.024 | 29 | 0.67 | 2.1 | 0.023 | 0.369 | 0.218 | 0.214 | 3.28 | 93.49 |
| 1.4 | 0.024 | 29 | 0.67 | 2.2 | 0.024 | 0.364 | 0.215 | 0.212 | 3.28 | 94.65 |
| 1.4 | 0.024 | 29 | 0.67 | 2.3 | 0.025 | 0.360 | 0.213 | 0.209 | 3.28 | 95.78 |
| 1.4 | 0.024 | 29 | 0.67 | 2.4 | 0.026 | 0.356 | 0.210 | 0.207 | 3.28 | 96.88 |
| 1.4 | 0.024 | 29 | 0.67 | 2.5 | 0.026 | 0.352 | 0.208 | 0.204 | 3.28 | 97.94 |
| 1.4 | 0.020 | 27 | 0.62 | 2.0 | 0.009 | 0.366 | 0.217 | 0.215 | 3.32 | 95.31 |
| 1.4 | 0.020 | 27 | 0.72 | 2.0 | 0.010 | 0.367 | 0.218 | 0.216 | 3.30 | 94.05 |

| Mass [M_{\odot}] | Z | Y | age [Gyr] | α | U-B [mag] | B-V [mag] | V-R [mag] | R-I [mag] | M_v [mag] | ν_0 [μHz] |
|-------------------------|-------|----|--------------|----------|--------------|--------------|--------------|--------------|----------------|-------------------------------|
| 1.5 | 0.028 | 31 | 0.62 | 2.0 | 0.050 | 0.340 | 0.200 | 0.197 | 2.91 | 79.25 |
| 1.5 | 0.028 | 31 | 0.72 | 2.0 | 0.050 | 0.344 | 0.202 | 0.200 | 2.90 | 77.59 |
| 1.5 | 0.024 | 29 | 0.67 | 1.5 | 0.038 | 0.344 | 0.203 | 0.201 | 2.93 | 78.03 |
| 1.5 | 0.024 | 29 | 0.67 | 1.6 | 0.038 | 0.342 | 0.202 | 0.200 | 2.93 | 78.44 |
| 1.5 | 0.024 | 29 | 0.67 | 1.7 | 0.039 | 0.340 | 0.201 | 0.199 | 2.93 | 78.90 |
| 1.5 | 0.024 | 29 | 0.67 | 1.8 | 0.040 | 0.338 | 0.199 | 0.198 | 2.93 | 79.53 |
| 1.5 | 0.024 | 29 | 0.67 | 1.9 | 0.040 | 0.335 | 0.197 | 0.196 | 2.93 | 80.28 |
| 1.5 | 0.024 | 29 | 0.67 | 2.0 | 0.041 | 0.331 | 0.196 | 0.194 | 2.93 | 81.10 |
| 1.5 | 0.024 | 29 | 0.67 | 2.1 | 0.042 | 0.328 | 0.194 | 0.192 | 2.93 | 81.99 |
| 1.5 | 0.024 | 29 | 0.67 | 2.2 | 0.043 | 0.325 | 0.192 | 0.190 | 2.93 | 82.89 |
| 1.5 | 0.024 | 29 | 0.67 | 2.3 | 0.045 | 0.322 | 0.190 | 0.188 | 2.93 | 83.80 |
| 1.5 | 0.024 | 29 | 0.67 | 2.4 | 0.046 | 0.319 | 0.188 | 0.186 | 2.93 | 84.67 |
| 1.5 | 0.024 | 29 | 0.67 | 2.5 | 0.048 | 0.315 | 0.186 | 0.184 | 2.93 | 85.51 |
| 1.5 | 0.020 | 27 | 0.62 | 2.0 | 0.031 | 0.321 | 0.190 | 0.190 | 2.97 | 84.23 |
| 1.5 | 0.020 | 27 | 0.72 | 2.0 | 0.030 | 0.324 | 0.192 | 0.192 | 2.95 | 82.64 |

Table 2: Model characteristics for stars evolved using M67 parameters.

| Mass [M_{\odot}] | Z | Y | α | age [Gyr] | U-B [mag] | B-V [mag] | V-R [mag] | R-I [mag] | M_{ν} [mag] | $\bar{\nu}_0$ [μHz] |
|-------------------------|-------|------|----------|--------------|--------------|--------------|--------------|--------------|--------------------|-------------------------------------|
| 0.7 | 0.013 | 0.26 | 2.0 | 3.5 | 0.931 | 1.062 | 0.632 | 0.542 | 7.24 | 217.00 |
| 0.7 | 0.013 | 0.26 | 2.0 | 4.5 | 0.921 | 1.056 | 0.628 | 0.538 | 7.21 | 215.00 |
| 0.7 | 0.017 | 0.28 | 1.5 | 4.0 | 1.017 | 1.104 | 0.661 | 0.563 | 7.31 | 209.10 |
| 0.7 | 0.017 | 0.28 | 1.6 | 4.0 | 1.007 | 1.097 | 0.656 | 0.558 | 7.29 | 210.70 |
| 0.7 | 0.017 | 0.28 | 1.7 | 4.0 | 0.997 | 1.092 | 0.652 | 0.554 | 7.28 | 212.10 |
| 0.7 | 0.017 | 0.28 | 1.8 | 4.0 | 0.989 | 1.087 | 0.648 | 0.551 | 7.27 | 213.40 |
| 0.7 | 0.017 | 0.28 | 1.9 | 4.0 | 0.981 | 1.083 | 0.644 | 0.547 | 7.25 | 214.50 |
| 0.7 | 0.017 | 0.28 | 2.0 | 4.0 | 0.974 | 1.079 | 0.641 | 0.545 | 7.24 | 215.50 |
| 0.7 | 0.017 | 0.28 | 2.1 | 4.0 | 0.967 | 1.075 | 0.638 | 0.542 | 7.23 | 216.40 |
| 0.7 | 0.017 | 0.28 | 2.2 | 4.0 | 0.961 | 1.072 | 0.635 | 0.540 | 7.23 | 217.30 |
| 0.7 | 0.017 | 0.28 | 2.3 | 4.0 | 0.955 | 1.069 | 0.632 | 0.538 | 7.22 | 218.10 |
| 0.7 | 0.017 | 0.28 | 2.4 | 4.0 | 0.950 | 1.066 | 0.630 | 0.536 | 7.21 | 218.80 |
| 0.7 | 0.017 | 0.28 | 2.5 | 4.0 | 0.945 | 1.064 | 0.628 | 0.534 | 7.21 | 219.50 |
| 0.7 | 0.021 | 0.30 | 2.0 | 3.5 | 1.013 | 1.095 | 0.650 | 0.549 | 7.26 | 214.80 |
| 0.7 | 0.021 | 0.30 | 2.0 | 4.5 | 1.003 | 1.090 | 0.646 | 0.546 | 7.23 | 213.20 |
| 0.8 | 0.013 | 0.26 | 2.0 | 3.5 | 0.515 | 0.870 | 0.488 | 0.434 | 6.26 | 195.00 |
| 0.8 | 0.013 | 0.26 | 2.0 | 4.5 | 0.496 | 0.860 | 0.482 | 0.430 | 6.21 | 192.00 |
| 0.8 | 0.017 | 0.28 | 1.5 | 4.0 | 0.644 | 0.928 | 0.524 | 0.457 | 6.33 | 182.60 |
| 0.8 | 0.017 | 0.28 | 1.6 | 4.0 | 0.625 | 0.919 | 0.518 | 0.452 | 6.31 | 184.90 |
| 0.8 | 0.017 | 0.28 | 1.7 | 4.0 | 0.607 | 0.910 | 0.511 | 0.447 | 6.29 | 187.10 |
| 0.8 | 0.017 | 0.28 | 1.8 | 4.0 | 0.591 | 0.902 | 0.506 | 0.443 | 6.28 | 189.00 |
| 0.8 | 0.017 | 0.28 | 1.9 | 4.0 | 0.576 | 0.894 | 0.501 | 0.439 | 6.27 | 190.80 |
| 0.8 | 0.017 | 0.28 | 2.0 | 4.0 | 0.562 | 0.888 | 0.496 | 0.436 | 6.26 | 192.40 |
| 0.8 | 0.017 | 0.28 | 2.1 | 4.0 | 0.549 | 0.881 | 0.492 | 0.433 | 6.25 | 193.90 |
| 0.8 | 0.017 | 0.28 | 2.2 | 4.0 | 0.538 | 0.876 | 0.488 | 0.431 | 6.24 | 195.20 |
| 0.8 | 0.017 | 0.28 | 2.3 | 4.0 | 0.527 | 0.870 | 0.485 | 0.428 | 6.23 | 196.50 |
| 0.8 | 0.017 | 0.28 | 2.4 | 4.0 | 0.517 | 0.865 | 0.482 | 0.426 | 6.22 | 197.60 |
| 0.8 | 0.017 | 0.28 | 2.5 | 4.0 | 0.507 | 0.861 | 0.479 | 0.424 | 6.21 | 198.80 |
| 0.8 | 0.021 | 0.30 | 2.0 | 3.5 | 0.613 | 0.910 | 0.508 | 0.441 | 6.28 | 191.30 |
| 0.8 | 0.021 | 0.30 | 2.0 | 4.5 | 0.594 | 0.900 | 0.502 | 0.437 | 6.23 | 189.00 |

| Mass [M_{\odot}] | Z | Y | α | age [Gyr] | U-B [mag] | B-V [mag] | V-R [mag] | R-I [mag] | M_v [mag] | ν_0 [μHz] |
|-------------------------|-------|------|----------|--------------|--------------|--------------|--------------|--------------|----------------|-------------------------------|
| 0.9 | 0.013 | 0.26 | 2.0 | 3.5 | 0.201 | 0.695 | 0.385 | 0.361 | 5.46 | 172.00 |
| 0.9 | 0.013 | 0.26 | 2.0 | 4.5 | 0.183 | 0.682 | 0.378 | 0.357 | 5.39 | 168.00 |
| 0.9 | 0.017 | 0.28 | 1.5 | 4.0 | 0.338 | 0.768 | 0.423 | 0.387 | 5.49 | 155.80 |
| 0.9 | 0.017 | 0.28 | 1.6 | 4.0 | 0.315 | 0.754 | 0.415 | 0.381 | 5.48 | 158.80 |
| 0.9 | 0.017 | 0.28 | 1.7 | 4.0 | 0.295 | 0.742 | 0.408 | 0.376 | 5.47 | 161.50 |
| 0.9 | 0.017 | 0.28 | 1.8 | 4.0 | 0.277 | 0.731 | 0.402 | 0.371 | 5.46 | 163.90 |
| 0.9 | 0.017 | 0.28 | 1.9 | 4.0 | 0.260 | 0.720 | 0.397 | 0.367 | 5.45 | 166.20 |
| 0.9 | 0.017 | 0.28 | 2.0 | 4.0 | 0.245 | 0.711 | 0.392 | 0.363 | 5.44 | 168.40 |
| 0.9 | 0.017 | 0.28 | 2.1 | 4.0 | 0.231 | 0.702 | 0.387 | 0.359 | 5.43 | 170.30 |
| 0.9 | 0.017 | 0.28 | 2.2 | 4.0 | 0.217 | 0.693 | 0.383 | 0.356 | 5.42 | 172.20 |
| 0.9 | 0.017 | 0.28 | 2.3 | 4.0 | 0.205 | 0.686 | 0.379 | 0.353 | 5.41 | 173.90 |
| 0.9 | 0.017 | 0.28 | 2.4 | 4.0 | 0.194 | 0.679 | 0.376 | 0.350 | 5.41 | 175.50 |
| 0.9 | 0.017 | 0.28 | 2.5 | 4.0 | 0.184 | 0.672 | 0.372 | 0.348 | 5.40 | 177.00 |
| 0.9 | 0.021 | 0.30 | 2.0 | 3.5 | 0.294 | 0.733 | 0.402 | 0.368 | 5.47 | 167.50 |
| 0.9 | 0.021 | 0.30 | 2.0 | 4.5 | 0.275 | 0.721 | 0.396 | 0.363 | 5.39 | 163.30 |
| 1.0 | 0.013 | 0.26 | 2.0 | 3.5 | 0.036 | 0.558 | 0.318 | 0.309 | 4.77 | 146.00 |
| 1.0 | 0.013 | 0.26 | 2.0 | 4.5 | 0.028 | 0.548 | 0.312 | 0.305 | 4.67 | 139.00 |
| 1.0 | 0.017 | 0.28 | 1.5 | 4.0 | 0.146 | 0.636 | 0.353 | 0.336 | 4.76 | 127.20 |
| 1.0 | 0.017 | 0.28 | 1.6 | 4.0 | 0.126 | 0.620 | 0.346 | 0.330 | 4.75 | 130.20 |
| 1.0 | 0.017 | 0.28 | 1.7 | 4.0 | 0.109 | 0.607 | 0.340 | 0.324 | 4.74 | 132.90 |
| 1.0 | 0.017 | 0.28 | 1.8 | 4.0 | 0.095 | 0.594 | 0.334 | 0.319 | 4.73 | 135.40 |
| 1.0 | 0.017 | 0.28 | 1.9 | 4.0 | 0.083 | 0.583 | 0.329 | 0.315 | 4.73 | 137.80 |
| 1.0 | 0.017 | 0.28 | 2.0 | 4.0 | 0.072 | 0.573 | 0.324 | 0.310 | 4.72 | 140.10 |
| 1.0 | 0.017 | 0.28 | 2.1 | 4.0 | 0.062 | 0.563 | 0.319 | 0.306 | 4.72 | 142.10 |
| 1.0 | 0.017 | 0.28 | 2.2 | 4.0 | 0.053 | 0.554 | 0.315 | 0.303 | 4.71 | 144.10 |
| 1.0 | 0.017 | 0.28 | 2.3 | 4.0 | 0.045 | 0.546 | 0.311 | 0.299 | 4.71 | 145.90 |
| 1.0 | 0.017 | 0.28 | 2.4 | 4.0 | 0.037 | 0.539 | 0.308 | 0.296 | 4.70 | 147.60 |
| 1.0 | 0.017 | 0.28 | 2.5 | 4.0 | 0.029 | 0.531 | 0.304 | 0.293 | 4.70 | 149.20 |
| 1.0 | 0.021 | 0.30 | 2.0 | 3.5 | 0.110 | 0.594 | 0.334 | 0.315 | 4.76 | 140.20 |
| 1.0 | 0.021 | 0.30 | 2.0 | 4.5 | 0.101 | 0.584 | 0.328 | 0.312 | 4.66 | 133.10 |

| Mass [M_{\odot}] | Z | Y | α | age [Gyr] | U-B [mag] | B-V [mag] | V-R [mag] | R-I [mag] | M_v [mag] | $\bar{\nu}_0$ [μHz] |
|-------------------------|-------|------|----------|--------------|--------------|--------------|--------------|--------------|----------------|-------------------------------------|
| 1.1 | 0.013 | 0.26 | 2.0 | 3.5 | -0.025 | 0.469 | 0.275 | 0.272 | 4.14 | 116.00 |
| 1.1 | 0.013 | 0.26 | 2.0 | 4.5 | -0.020 | 0.469 | 0.274 | 0.273 | 4.01 | 106.00 |
| 1.1 | 0.017 | 0.28 | 1.5 | 4.0 | 0.061 | 0.546 | 0.309 | 0.302 | 4.09 | 99.40 |
| 1.1 | 0.017 | 0.28 | 1.6 | 4.0 | 0.049 | 0.533 | 0.303 | 0.296 | 4.09 | 101.70 |
| 1.1 | 0.017 | 0.28 | 1.7 | 4.0 | 0.037 | 0.521 | 0.298 | 0.291 | 4.09 | 104.00 |
| 1.1 | 0.017 | 0.28 | 1.8 | 4.0 | 0.027 | 0.510 | 0.293 | 0.286 | 4.08 | 106.10 |
| 1.1 | 0.017 | 0.28 | 1.9 | 4.0 | 0.018 | 0.500 | 0.288 | 0.282 | 4.08 | 108.10 |
| 1.1 | 0.017 | 0.28 | 2.0 | 4.0 | 0.011 | 0.490 | 0.284 | 0.278 | 4.08 | 109.90 |
| 1.1 | 0.017 | 0.28 | 2.1 | 4.0 | 0.004 | 0.481 | 0.280 | 0.274 | 4.08 | 111.70 |
| 1.1 | 0.017 | 0.28 | 2.2 | 4.0 | -0.001 | 0.473 | 0.276 | 0.271 | 4.07 | 113.40 |
| 1.1 | 0.017 | 0.28 | 2.3 | 4.0 | -0.006 | 0.466 | 0.272 | 0.267 | 4.07 | 114.90 |
| 1.1 | 0.017 | 0.28 | 2.4 | 4.0 | -0.009 | 0.459 | 0.269 | 0.264 | 4.07 | 116.40 |
| 1.1 | 0.017 | 0.28 | 2.5 | 4.0 | -0.012 | 0.452 | 0.266 | 0.261 | 4.07 | 117.80 |
| 1.1 | 0.021 | 0.30 | 2.0 | 3.5 | 0.038 | 0.508 | 0.292 | 0.282 | 4.14 | 112.10 |
| 1.1 | 0.021 | 0.30 | 2.0 | 4.5 | 0.054 | 0.522 | 0.298 | 0.288 | 4.04 | 102.10 |

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