STRUCTURE FORMATION WITH SCALAR-TENSOR GRAVITATIONAL THEORIES

by

S. O. Mendes

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Astronomy

Department of Astronomy and Physics

Saint Mary's University

Halifax, Nova Scotia

B3H 3C3

submitted March 2001

0-612-74842-1

Certificate of Examination

Saint Mary's University
Department of Astronomy and Physics

Dr. Malcolm N. Butler
Associate Professor
Department of Astronomy and Physics
Saint Mary's University
(Supervisor)

Dr. Francine R. Marleau
Assistant Professor
Department of Astronomy and Physics
Saint Mary's University

Dr. George F. Mitchell
Professor
Department of Astronomy and Physics
Saint Mary's University

Dr. David B. Guenther
Associate Professor
Department of Astronomy and Physics
Saint Mary's University
(Chair)

Dr. Alan A. Coley
Professor
Department of Mathematics and Statistics
Dalhousie University
(External Examiner)

Acknowledgments

Many people helped me during the preparation of this work, and it is perhaps impossible to thank all of them in these few lines. My apologies to those who contributed in a way or another and whose contribution will remain anonymous for the lack of space. I would like to express my gratitude to:

My thesis supervisor Dr. Malcolm Butler for his support, dedication and for the always enlightening discussions on relevant topics for this thesis.

Dr. Michael West, who is the co-supervisor of this work and whose ideas and suggestions were always useful.

The Natural Sciences and Engineering Research Council of Canada for partial financial support to my graduate studies as well as to two conferences (one in Canada and the other in Brazil) I had the chance to attend.

The Senate and the Department of Astronomy and Physics at Saint Mary's University for partial financial support.

Dr. Christopher Willmer and Dr. Marcio Maia for encouragement and support of my decision to study in Canada.

Juan Ramón, Ian and Andreea for their friendship and the (rare) occasions we could spent together outside the office, as well as to Reneé, who accompained me during my first days in Canada.

Elfrie Waters for having the patience and willingness to sort out all my questions and requests directed to her.

My lovely wife Vanessa, to whom I am greatly indebted for her love, friendship, support and for being always with me during the good and the difficult times during this work.

Dr. Rachid Ouyed for his friendship and support.

Dr. Gary Welch, Dr. David Clarke, Dr. David Turner and all faculty members who helped me in a way or another during the completion of my Master's degree at Saint Mary's University.

Dr. Jean-Pierre de Villiers for useful discussions and support.

Dr. Nick Tothill for his friendship and the good times we had away from work.

Dave Lane, who always guaranteed the functioning of the computational facilities in the department.

Mario, Marcelo, Peter and Andrew for their friendship and all the fun we had.

Dr. Alan Coley and Dr. Eric O'Neill for useful discussions on scalar-tensor theories.

Dr. Alexandre de Oliveira for his friendship and patience to answer to my frequent questions about Maple and general relativity.

The Department of Astronomy at the University of São Paulo for allowing me to finish my Master's thesis using their facilities and at the same time accepting me as a PhD student.

Abstract

We study the evolution of linear density perturbations in a cosmological background described by scalar-tensor theories (STT) of gravity. The evolution of the density constrast δ is obtained using the Jeans formalism for an expanding Universe. Three classes of such theories are investigated: Brans-Dicke (BD) theory, which is the simplest possible STT: dynamical Λ theory, in which the function $\Lambda(\varphi)$ plays the role of a decaying cosmological constant: and a varying-ω theory, where the Brans-Dicke coupling function is no longer a constant. In general, theories with growing fluctuations admit faster growth that in conventional general relativity which may, in turn, allow for structure formation at earlier times. However, there are solutions where growth is exponential, leading to conflicting ages for the Universe. There are also classes of theories with decaying and/or oscillatory modes which are incompatible with the paradigm of structure formation. The evolution of δ depends on ω, and this is shown to constrain the allowed values of the coupling function. We also find a possible connection between the ansatz $a\phi^n = constant$ (where a is the cosmic scale factor and ϕ is the scalar field of STT) and the weak field limit of BD theory.

Contents

List of Figures

List of Tables

1. Introduction

1.1. The Very Early Universe

Cosmology is one of the most challenging and yet obscure branches of science. The quest to understand the origin, evolution, and eventual fate of the Universe is probably an occupation as old as mankind itself. We are privileged to live in an era when technological advances so neatly reveal to us the deep mysteries of the observable Universe, whereas our understanding of the physical laws governing nature seem to account for most of what is observed with our telescopes.

The earliest stages of the Universe can be studied in the framework of inflationary Big Bang models. Although these models are not able to explain where the Universe came from, or why it exists at all, they provide information about epochs as early as 10^{-35} s after its creation. There is no satisfactory theory for the creation itself, but it is possible that the whole Universe is the result of a quantum fluctuation (*e.g.* Tryon 1973 and Vilenkin 1982), in which case the issue belongs to the realm of quantum cosmology.

The theory of inflation (Guth 1981) came to the aid of the standard Big Bang picture in order to fix some cosmological problems with the latter. The original motivation for inflation had nothing to do with cosmology and its main target was the study of magnetic monopoles in the early Universe (see, *e.g.*, Guth 1997). However, attention was soon directed to cosmology once it was realized that the theory could solve two of the most outstanding cosmological problems at that time: the horizon and flatness problems. The former can be understood on the basis of the observed isotropy of the cosmic microwave background (CMB) radiation. This radiation results from the fact that the Universe became transparent at the time of recombination and the CMB photons were no longer trapped into the hot primordial plasma. The horizon size at that time as seen by someone on Earth represents only a small fraction of the sky. That is, two regions in opposite directions in the sky, for example, could not be in causal contact 300,000 years after the Big Bang and yet they have the same temperature within one part in 10^5. The flatness problem, on the other hand, does not involve any contradiction but rather a lack of predictive power in the Big Bang theory, or a strong coincidence in nature. Calculations within the standard Big Bang show that if the total density parameter Ω was slightly larger than the critical value in the early Universe then the expansion would be halted in a time scale of the order of the Planck time ($\sim 10^{-43}$ s). Correspondingly, a slightly open Universe would cool down to the currently observed temperature ($T \simeq 3$ K) at $t \simeq 10^{-11}$ s, and would be much larger and colder by now. The only way our Universe would evolve to its present state is if the density parameter was extremely close to unity at early times, which is by no means predicted by the Big Bang.

The main idea behind inflation is that the Universe has undergone an epoch of exponentially accelerated expansion at a time $t \simeq 10^{-37}$ s that lasted for approximately 10^{-30} s. This extraordinarily fast expansion is believed to have been driven by a scalar field usually called an inflaton. As a result, the Universe is predicted to have had submicroscopic dimensions before the inflationary phase, so that all regions were causally connected. This is the inflationary solution to the horizon problem. Also, the rapid expansion would erase any curvature that might have existed before inflation, driving Ω to unity and solving the flatness problem.

The original inflationary theory was not perfect, and its proposers soon realized that the Universe would be filled with false vacuum bubbles during the inflationary phase, and collisions between these bubbles would break the homogeneity of the

Universe. This has become known as the graceful exit problem, and it took Linde (1982) and Albrecht & Steinhardt (1982) to develop independently the so-called new inflation. The modified inflationary scenario was based on a special shape for the inflaton potential which caused the whole observable Universe to be contained inside a single false vacuum bubble, freeing the theory of the graceful exit problem.

The success of inflation is corroborated observationally by results from CMB experiments. The latest data (*e.g.* Lange *et al.* 2000) favor a flat Universe with a scale-invariant spectrum of primordial fluctuations, in accordance with inflationary predictions. There is indeed strong agreement between CMB results and other extragalactic estimators such as type Ia supernovae and clusters of galaxies (Hu *et al.* 2000), constraining cosmological parameters to a fairly small region of the parameter space. The next generation of CMB satellites (*e.g.* MAP[1] and Planck[2]) along with incoming redshift surveys (*e.g.* SDSS[3]) will be able to further probe inflation and to constrain other cosmological parameters (such as the age of the Universe, the Hubble constant and the density parameter) with unprecedented precision.

The Universe started to evolve as described by the standard Big Bang theory from the end of inflation onward. The inflationary mechanism not only solved the flatness and horizon problems but also reheated the particle soup which is postulated to exist in the early Universe. This background of particles and radiation was merely the result of false vacuum decay. At this stage the Universe was in nearly thermal equilibrium, but expansion made it cool down gradually. It was not before a time

---

[1]See http://map.gsfc.nasa.gov/

[2]See http://astro.estec.esa.nl/Planck/

[3]See http://www.sdss.org/

$t \sim 10^{-2}$ to 10^2 s that the temperature dropped enough so that protons and neutrons could combine to form the first light elements. This period is referred to as the Big Bang nucleosynthesis era. At an estimated time of $\sim 10^{11}$ s the energy density in matter and radiation became equal (equivalence time) and after that the Universe became matter-dominated. As the temperature dropped further, electrons and nuclei were able to combine and form atoms at $t \sim 10^{13}$ s. At that point matter and radiation decoupled, i.e., they started to evolve independently because the expansion rate became greater than the interaction rate between them. This timescale is known as the recombination epoch, and as was mentioned before, the CMB photons started then their nearly straight line trajectories until the present time. It is not clear whether there really existed a time interval between the equivalence and recombination times. Nevertheless, structure formation processes are believed to have been triggered sometime after the equivalence time.

1.2. Structure Formation

The cosmological principle states that the Universe is isotropic and homogeneous on large scales, and that what we observe is typical of structure everywhere. Big Bang models are based on the cosmological principle, and the observational confirmation of this hypothesis is key for the theory. The galaxy distribution is certainly isotropic on the largest scales, and it is conventional wisdom that the distribution tends to homogeneity. However, the major piece of evidence for the cosmological principle (and hence for the Big Bang theory) was the discovery of the CMB radiation by Penzias & Wilson (1965). The (almost) perfectly homogeneous and isotropic radiation of temperature $T \sim 2.73$ K indeed confirmed that the Universe used to be very hot in the past.

We are fortunate that the CMB radiation is not totally homogeneous and isotropic because it would be otherwise hard to explain how the observed large scale structure formed. Shortly after its discovery, astronomers pursued the task of finding anisotropies in the CMB radiation, which should be an imprint of the small density fluctuations in the early Universe that gave rise to the formation of structures. Almost 30 years elapsed between the discovery by Penzias & Wilson and the detection of the desired anisotropies by the COBE satellite (Smoot *et al.* 1992). The COBE mission yielded an all-sky map of the CMB on angular scales of approximately 10 degrees. Balloon-borne experiments have complemented the COBE measurements on smaller scales (*e.g.* de Bernardis *et al.* 2000 and Hanany *et al.* 2000), and the combined results constrain several important cosmological parameters such as the baryonic and dark matter density, the Hubble constant, the cosmological constant and the age of the Universe.

The physical origin of the primordial fluctuations can be explained with the help of quantum mechanics (*e.g.* Bardeen, Steinhardt & Turner 1983). The idea is that inflation ends at different places in different times due to quantum effects, and since particles are created at the end of inflation, it could generate inhomogeneities in the early Universe. The fluctuations are very small, but they are amplified by gravity. The small density perturbations will create slightly overdense regions, which will in turn attract more and more matter towards their centres of mass.

It can be shown classically that the density contrast can be decomposed into plane waves (Jeans 1902 and Bonnor 1957), and that it is unstable to growth in amplitude for wavelengths greater than a critical value. This amplification mechanism is known as gravitational instability and will be discussed in some more detail in chapter three. Similar results can be obtained using general relativity (henceforth GR), with the difference that now the problem is treated as perturbations to space-time itself and to the energy-momentum tensor (*e.g.* Peebles 1993).

1.3. Gravitational Theories

Gravity is the weakest interaction amongst the three known fundamental forces in nature (the other two being the strong and electroweak forces). However. The long-range of the gravitational interaction and the fact that the gravitational charge is always positive[4] gives gravity a key role in the understanding of the large scale structure in the Universe. Any attempt to study structure formation in the early Universe has to make use of some gravitational theory. In the original Jeans formalism (Jeans 1902), gravity enters through the Poisson equation. This is a purely Newtonian approach as far as a gravitational theory is concerned. However, the original Jeans equation was designed for a static Universe. The fact that our Universe is expanding affects the way structure is formed: expansion will slow down the collapse of objects. An extension of the Jeans formalism for an expanding Universe was first developed by Lifshitz (1946), where GR was used. Bonnor (1957) used Newtonian cosmology to show that the same basic results could be obtained with no need for a relativistic gravitational theory.

The basic equation for the growth of density perturbations in an expanding Universe is

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\rho\delta = 0. \tag{1.1}$$

where ρ is the density of the Universe. $\delta = \delta\rho/\rho$ is the density contrast. G is Newton's gravitational constant. a is the scale factor and dots represent time

[4]classically, one could say that the gravitational force is always attractive.

derivatives. Note that this equation can be obtained either within a fully relativistic context or using Newtonian gravity. One of the reasons is that for epochs after the equivalence time (when structure formation starts taking place) the Universe is matter-dominated and pressure effects become negligible. In other words, the main contribution to the energy-momentum tensor comes from its time-time component, which is solely the density ρ. In order to solve eq. (1.1), however, we must know how both a and ρ vary with time. Newtonian gravity does not predict the time evolution of these quantities, although we can postulate the scale factor evolution by deriving the Friedmann equation from classical considerations. We thus need to have a background cosmological model to solve eq. (1.1), which usually comes from a specific relativistic gravitational theory.

The standard background cosmological models used to solve the Jeans equation for an expanding Universe are usually based on solutions to the Friedmann equation, which is in turn derived from GR. Although Einstein's theory is regarded as the best gravitational theory to date, being repeatedly tested and confirmed through the years, it is not the only one available. There are several proposed modifications to the Einstein field equations, one of the oldest being the introduction of a matter creation field (or C-field) into the original equation in the context of the steady state theory (Hoyle 1948). It is common knowledge today that the steady state theory is not to be taken too seriously because of its difficulty in explaining the 2.73 K background radiation, which finds a rather natural explanation in the context of Big Bang (i.e. general relativistic) models. More recent attempts to rescue the steady state theory are embedded in the so-called quasi-steady state theory (Hoyle, Burbidge & Narlikar 1993). There are in fact a large number of alternative gravitational theories, and we cannot review all of them here. For a comprehensive review of gravitational physics involving different theories see, *e.g.*, Will (1981).

One of the alternatives to GR that interests us is the one developed by Brans & Dicke (1961). The Brans-Dicke (henceforth BD) theory is a relativistic gravitational theory which, as opposed to GR, incorporates Mach's principle, which states that the local dynamics of a system is determined by the global distribution of matter in the Universe. The field equations in BD theory are characterized by a time-varying scalar field ϕ coupled to the metric tensor and a dimensionless constant coupling parameter ω (giving the general label of scalar-tensor to this type of theory). The weak field approximation of the theory shows that Newton's gravitational constant is actually a function of ϕ and ω:

$$G = \frac{1}{\phi}\frac{4 + 2\omega}{3 + 2\omega}. \tag{1.2}$$

and Solar System experiments constrain ω to be greater than 500 (Reasenberg *et al.* 1979). One attractive aspect of BD theory is that it explains satisfactorily the precession of Mercury's perihelion. However, the coupling parameter is shown to be constrained systematically to small values ($\omega < 2$, including negative values) when the theory is applied to the study of some models of extended inflation (Susperregi & Mazumdar 1998) and cosmic strings (Dahia & Romero 1999), or to explain the present accelerated expansion of the Universe (Batista, Fabris & Ribeiro 2000; Sen & Seshadri 2000; Sen, Sen & Sethi 2000). This might be indicating that a more general scalar-tensor theory should be considered in order to maintain consistency between theory and observations.

1.4. Scalar-Tensor Theories

The most general scalar-tensor theory has two arbitrary functions of the scalar field: $\omega(\phi)$ and $\lambda(\phi)$. The former is a generalization of the coupling constant from BD theory, and the second one can be shown to play the role of a cosmological constant. In fact, current data on gravitational lensing statistics (Helbig 1999) and type Ia supernovae (Efstathiou *et al.* 1999) favor a non-zero cosmological constant, and any alternative gravitational theory should take that into account. This might be another sign that BD theory should be extended, since it is a special case in which ω is constant and $\lambda = 0$. The equivalent to the gravitational constant still takes the form of eq. (1.2), with ω being substituted by $\omega(\phi)$. We see that G in this case can be a complicated function of ϕ depending on the form of $\omega(\phi)$, which opens the possibility to put theoretical predictions in agreement with observational constraints for the present value of ω.

Applications of STT to the study of structure formation have been carried out in a number of papers. Starobinsky & Yokoyama (1994) studied the spectrum of fluctuations generated during inflation within BD theory, and Chiba, Sugiyama & Yokoyama (1998) extended their work for a more general STT. Also in the context of a general STT, Seshadri (1992) investigated the generation of density perturbations during an inflationary epoch due to quantum fluctuations of the scalar field ϕ. This theory is restricted, however, due to the number of assumptions regarding the fluctuations $\delta\phi$. Sen & Seshadri (2000) and Bertolami & Martins (2000), motivated by the observed acceleration of the Universe, studied which type of self-interacting BD potential would be in agreement with observations. Increasing modes for the density contrast were found in both works. Chin & Kamionkowski (1999) studied CMB anisotropies in a BD background, and found that both the width and height of the power spectrum acoustic peaks are changed as compared to GR. They also pointed out that there is a degeneracy in the determination of cosmological parameters for high angular scales (or equivalently low multipoles), and that higher resolution observations with Planck and MAP and polarization measurements by Planck will be able to break this degeneracy and make this cosmological test to BD theory as good as Solar System tests. Two important results for the present work are the papers by Gaztañaga & Lobo (2001) and Esposito-Farèse & Polarski (2000). The former have obtained a relativistic equation for the growth of density perturbations in a BD background which is identical to the classical Jeans equation for an expanding Universe in a BD cosmology, while the latter generalized this result for the case when ω is not constant. Thus we are justified in using the Jeans equation with a STT background to study structure formation.

In the present work we are interested primarily on how different scalar-tensor theories affect the process of structure formation in the early Universe. We review in the following chapters three classes of scalar-tensor theories: (1) simple BD models, (2) models by Diáz-Rivera & Pimentel (1999), in which $\lambda(\phi)$ is explored as a decaying cosmological constant, and (3) models with varying ω from Barrow & Mimoso (1994). We apply the Jeans formalism and determine how structure grows in the early Universe for each of these models. The expressions we obtain for the density contrast depend on ω, which provides us with a new confrontation between theoretical predictions and observational limits of the coupling function. Also, we study the evolution of the Jeans mass within the physically motivated models. We then summarize our results and present a discussion on the power-law relation between a and ϕ for BD theory based on a perturbative treatment of the field equations.

2. The Models

The action for the most general STT is (Will 1981):

$$I = (16\pi)^{-1}\int[\phi R - \phi^{-1}\omega(\phi)g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2\phi\lambda(\phi)](-g)^{1/2}d^4x + I_{NG}, \tag{2.1}$$

where Greek indices run from 0 to 3 and represent spacetime coordinates. Commas represent usual partial derivatives, while semicolons indicate covariant derivatives (see *e.g.* Weinberg 1972). g is the determinant of the metric tensor $g_{\mu\nu}$ ($g^{\mu\nu} = g_{\mu\nu}{}^{-1}$), R is the Ricci scalar, ω and λ are arbitrary functions of the scalar field ϕ, and I_{NG} is the action for non-gravitational interactions. The function $\omega(\phi)$ is associated with the gravitational strength (see comment below), while $\lambda(\phi)$ plays the role of a cosmological constant.

Variations on I with respect to $g_{\mu\nu}$ and ϕ yield the field equations

$$G_{\mu\nu} = \frac{8\pi T_{\mu\nu}}{\phi} + \lambda(\phi)\Box g_{\mu\nu} + \omega(\phi)\phi^{-2}(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda}) + \phi^{-1}(\phi_{;\mu\nu} - g_{\mu\nu}\Box\phi) \tag{2.2}$$

$$\Box\phi + \frac{2\phi^2 d\lambda/d\phi - 2\phi\lambda(\phi)}{3+2\omega(\phi)} = \frac{8\pi T - \phi_{,\mu}\phi^{,\mu}d\omega/d\phi}{3+2\omega(\phi)}. \tag{2.3}$$

The left-hand side of eq. (2.2) is the usual Einstein tensor, and T is the trace of the energy-momentum tensor $T_{\mu\nu}$. Note that ϕ^{-1} is dimensionally equal to the gravitational constant, and in fact G is a time-varying scalar in this type of theory with its strength depending on the value of ϕ and the dimensionless coupling function $\omega(\phi)$ at a given time (see *e.g.* eq. (1.2), chapter one). If we assume the isotropic and homogeneous Robertson-Walker metric

$$ds^2 = -dt^2 + a^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\phi d\phi^2)\right] \tag{2.4}$$

as a solution to the above equations, we obtain the modified Friedmann equations:

$$H^2 + H\frac{\dot\phi}{\phi} - \frac{\omega(\phi)}{6}\left(\frac{\dot\phi}{\phi}\right)^2 + \frac{k}{a^2} - \frac{\lambda(\phi)}{3} - \frac{8\pi\rho}{3\phi} = 0 \tag{2.5}$$

$$\ddot{\phi}+\left[3H+\frac{\dot{\omega}(\phi)}{2\omega(\phi)+3}\right]\dot{\phi}-2[\lambda(\phi)-\phi d\lambda(\phi)/d\phi]=\frac{8\pi\rho(4-3\gamma)}{2\omega(\phi)+3} \tag{2.6}$$

$$\dot{H}+H^2+\frac{\omega(\phi)}{3}\left(\frac{\dot{\phi}}{\phi}\right)^2-H\frac{\dot{\phi}}{\phi}+\frac{\lambda(\phi)}{3}=-\frac{8\pi\rho}{3\phi}\frac{(3\gamma-2)\omega(\phi)+3}{2\omega(\phi)+3}+\frac{1}{2}\frac{\dot{\omega}(\phi)}{2\omega(\phi)+3}\frac{\dot{\phi}}{\phi} \tag{2.7}$$

$$\dot{\rho}+3\gamma H\rho=0. \tag{2.8}$$

where

$$p=(\beta-1)\rho \tag{2.9}$$

$$H\equiv\frac{\dot{a}}{a} \tag{2.10}$$

and β is a constant defining the barotropic equation of state.

Different types of solutions to eqs. (2.5)-(2.8) can be obtained depending on the underlying assumptions about $\lambda(\phi)$, $\omega(\varphi)$ and the relation between a and ϕ. In what follows, we briefly describe the specific solutions that are going to be used for the study of the evolution of density perturbations.

2.1. Brans-Dicke Theory

The gravitational theory developed by Brans & Dicke (1961) is the simplest possible STT. The motivation for BD theory was the incorporation of Mach's principle into a theory of gravity, as was mentioned in chapter one. The BD theory can be represented by eqs. (2.5)-(2.8) in the special case when ω is constant and λ is zero. Brans & Dicke (1961) showed that these cosmological equations admit power-law solutions for a and ϕ provided that $\dot{\phi}a^3=0$ as $a\to 0$, and a power-law relation between these two quantities is indeed commonly assumed in order to obtain exact solutions to scalar-tensor theories (*e.g.* Dehnen & Obregón 1971, 1972, Díaz-Rivera & Pimentel 1999 and appendix A). We are going to describe models by Dehnen & Obregón (1971, hereafter DO71) in this section, from which we will extract the necessary information for the study of structure formation in chapter three.

Solutions with and without an initial singularity were studied in DO71 and DO72, respectively. In both studies, a power-law relation between the cosmic expansion factor and the effective gravitational constant (ultimately the scalar field ϕ, see eq. 1.2) was assumed:

$$8\pi G a^n = C, \tag{2.11}$$

where n and C are constants. They also used matter conservation in order to eliminate the density ρ from eq. (2.2) (in other words, they assumed a dust-fluid-like equation of state for the Universe):

$$\rho a^3 = \rho_0 {a_0}^3 = B, \tag{2.12}$$

with ρ_0 and a_0 being the present values of the density of the Universe and its curvature radius, respectively. ρ_0 was in fact used to constrain ω in DO71.

We are going to focus on models with an initial singularity in the present work. According to DO71 and eqs. (2.11) and (2.12), the gravitational constant, the scale factor and the density of the Universe in the flat BD case can be written respectively as

$$G = \frac{C}{8\pi}\left(\frac{CB}{4}\frac{4+3\omega}{2+\omega}t^2\right)^{-\frac{1}{1+3\omega}} \tag{2.13}$$

$$a = \left(\frac{CB}{4}\frac{4+3\omega}{2+\omega}t^2\right)^{\frac{1+\omega}{4+3\omega}} \tag{2.14}$$

$$\rho = B\left[64\left(\frac{1}{CB}\frac{2+\omega}{4+3\omega}\right)^3 t^{-6}\right]^{\frac{1+\omega}{4+3\omega}}. \tag{2.15}$$

The exponent in eq. (2.11) for this particular case is simply $n = \frac{1}{1+\omega}$ (Brans & Dicke 1961 and DO71). For closed models the set of equations becomes

$$G = \frac{C}{8\pi}\left(-\frac{2}{2+\omega}\right)^{\frac{1}{2}} t \tag{2.16}$$

$$a = \left(-\frac{2}{2+\omega}\right)^{\frac{1}{2}} t \tag{2.17}$$

$$\rho = B\left(-\frac{2+\omega}{2}\right)^{\frac{3}{2}} t^{-3}. \tag{2.18}$$

where $n = -1$ was used in eq. (2.11). Only this value for n yields exact solutions for models with positive curvature (DO71). One obvious consequence for closed models is the constraint $\omega < -2$ from the above equations. An open Universe is excluded according to DO71.

2.2. Dynamical Λ Theory

120 orders of magnitude represents the discrepancy between the predicted value of the cosmological constant Λ from particle physics and limits imposed by observations. This is one of the greatest puzzles in contemporary physics (see Sahni & Starobinsky 2000 for a recent review). One of the proposals to solve the so-called cosmological constant problem is the hypothesis that Λ is a dynamical scalar field which decreases with time (*e.g.* Ratra & Peebles 1988). Models with time-decreasing Λ were exhaustively studied by Overduin & Cooperstock (1998) in the context of general relativity. Díaz-Rivera & Pimentel (1999, hereafter DRP99) conjectured that scalar-tensor theories would be more suitable for the study of a dynamical Λ because of the available function $\lambda(\phi)$ in eq. (2.1). In this case, one does not have to postulate the desired behavior of Λ, as it appears naturally.

DRP99 assumed a relationship between a and G similar to the one used in DO71:

$$a\phi^n = C = constant. \tag{2.19}$$

The only difference is that now the explicit relation is given in terms of ϕ instead of

G. The scalar-tensor theory they used is more general than the BD theory in the sense that now $\lambda(\phi) \neq 0$. At the same time it is not the most general theory because the coupling parameter ω is still assumed to be a constant. It was further assumed that $\lambda(\phi)$ is some power of ϕ.

It is worth mentioning that the family of cosmological solutions found in that case is very restricted. Although vacuum solutions were found for all values of the exponent n in eq. (2.19) (which are of no interest in the present work for obvious reasons), exact solutions considering matter with a barotropic equation of state are found only for $n = 1/2$. That is a severe restriction on the coupling between ϕ and a, implying that

$$a \propto \phi^{-\frac{1}{2}}. \tag{2.20}$$

There is no reason to believe that nature behaves just like that, except for mathematical convenience.

We are going to use here two classes of exact solutions with barotropic equation of state from DRP99, namely the dust-fluid and stiff-matter-fluid cases. As was the case before, we are only interested in the evolution of the scale factor, the scalar field (i.e. the effective gravitational constant) and the density of the Universe in order to solve the Jeans equation in chapter three. Solutions for the dust-fluid case in a flat Universe are given by

$$G = c_1 \frac{4 + 2\omega}{3 + 2\omega} t^4 \tag{2.21}$$

$$a = c_2 t^2 \tag{2.22}$$

$$\rho = c_3(1 - 2\omega)t^{-6}, \tag{2.23}$$

where c_1, c_2 and c_3 are constants[5] and the effective gravitational constant takes the

---

[5] c_1 is an integration constant while c_2 and c_3 are combinations of c_1 and the

form of eq. (1.2).

Although DRP99 found two types of solutions for the dust-fluid case with curvature (depending on $f = k/(3+2\omega)$ being positive or negative), they represent the same thing for our purposes. The two models evolve exactly in the same way, with the difference that a positive f implies an initial singularity state in the Universe, whereas there is no singularity for $f < 0$. The solution in this case can be written as

$$G = \frac{1}{8}\frac{g^2}{c_1}\frac{2+\omega}{3+2\omega} \tag{2.24}$$

$$a = \frac{1}{4}\frac{g}{c_2} \tag{2.25}$$

$$\rho = \frac{16}{\pi}\frac{kc_3}{g^3}. \tag{2.26}$$

where

$$g = 1 - \frac{c_1 k t^2}{4C^2(3+2\omega)} \tag{2.27}$$

and the constants c_1, c_2 and c_3 are defined as in the previous solution. G again assumes the asympotic form from BD theory, and that will always be the case throughout this work.

The last type of solution with dynamical Λ from DRP99 we wish to study is the one with a barotropic equation of state given by $p = \rho$ (stiff-matter-fluid). At a first glance, it does not seem realistic to consider such models since we expect pressure effects to be negligible after the equivalence time between matter and radiation. However, we do not know exactly how the scalar field will contribute to pressure, and if the current acceleration of the Universe is due to the presence of such field.

---

constant C from eq. (2.19)

then it is certainly dominating over matter at the present time. This picture would therefore admit the need to include pressure effects even after the equivalence time.

We are going to explore only non-flat cosmologies here, because of the unphysical solution for $k = 0$ found by DRP99. The relevant expressions in this case are

$$G = \frac{g}{c_1} \frac{4 + 2\omega}{3 + 2\omega} \tag{2.28}$$

$$a = c_2 g^{1/2} \tag{2.29}$$

$$\rho = \frac{1}{\pi} k c_3 g^{-3}, \tag{2.30}$$

where

$$g = 1 + \frac{c_1 k t^2}{C^2(3 + 2\omega)} \tag{2.31}$$

and c_1, c_2 and c_3 are defined as in the two previous solutions. A similar discussion as in the dust-fluid case regarding the singularity problem was also presented by DRP99 in this case, but again the issue is not of interest here.

2.3. Non-Trivial ω Theory

Another way of generalizing STT's is by using the full form of the coupling function $\omega(\phi)$ rather than keeping it constant as in BD theory. It has been shown that we must have $\omega \to \infty$ and $d\omega/d\phi \to 0$ today in order to have this type of STT approaching GR and agreeing with observational tests (Nordvedt 1970). Exact cosmological solutions are difficult to address in this case due to the relative arbitrariness of $\omega(\phi)$. Here we are going to study a particular one-parameter family of flat solutions found by Barrow & Mimoso (1994, henceforth BM94) which, as in the previous sections, are going to be used for the study of structure formation in the Universe. Although BM94 presented a broad range of solutions depending on the choice of $\omega(\phi)$, we are going to use here only one of the two dust-fluid solutions worked out on that paper. We will restrict ourselves to the presentation of the solutions and refer the reader to BM94 for further development. The relation

$$3 + 2\omega(\phi) \sim \frac{4b}{3}\left(\frac{\phi}{c_1}\right)^b \tag{2.32}$$

is assumed for this model, and the important equations in this case become

$$G = \frac{1}{4c_4 b}\frac{3(\phi/c_1)^{-b} + 4c_4 b}{\phi} \tag{2.33}$$

$$a = c_2 t^{2/3}(\ln t)^{\frac{b-1}{3b}} \tag{2.34}$$

$$\rho = c_3[(\ln t)^{\frac{b-1}{b}} t^2]^{-1}. \tag{2.35}$$

where c_1, c_2, c_3 and c_4 are integration constants. $b > 0$ is the free parameter (constant) of this theory and

$$\phi = c_1(\ln t)^{\frac{1}{b}} \tag{2.36}$$

describes the evolution of the scalar field.

More general STT's would be a mixture of the two previous models (dynamical Λ and nontrivial ω). However, exact cosmological solutions involving both $\omega(\phi)$ and $\lambda(\phi)$ are not available in the literature and it is not the purpose of the present work to extend STT's, but rather to study their implications for structure formation. A summary of the models we are going to investigate is presented in table 1.

Model	Theory	Curvature	Equation of State
I	BD	$k = 0$	Dust-Fluid
Ia	BD	$k = 1$	Dust-Fluid
II	Dynamical Λ	$k = 0$	Dust-Fluid
IIa	Dynamical Λ	$k = \pm 1$	Dust-Fluid
IIb	Dynamical Λ	$k = \pm 1$	Stiff-Matter-Fluid
III	Non-Trivial ω	$k = 0$	Dust-Fluid

Table 1: Summary of Background Cosmological Models

3. Structure Formation a la Jeans

3.1. The Jeans Formalism

The idea that the structure we observe in the Universe today was formed through gravitational instability was first introduced by Jeans (1902). The physics needed to understand the Jeans formalism is very simple. The Universe is treated as a self-gravitating ideal fluid in equilibrium, so that it can be described by the equations of fluid mechanics:

$$\dot{\rho} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \tag{3.1}$$

$$\dot{\vec{v}} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla} p - \vec{\nabla} V \tag{3.2}$$

$$\nabla^2 V = 4\pi G \rho. \tag{3.3}$$

Eqs. (3.1)-(3.3) are the continuity equation, Euler's equation and Poisson's equation, respectively. ρ, $\vec{v}$ and p characterize the fluid's density, velocity and pressure, and V is the gravitational potential generated by the fluid. If we make use of linear perturbations in the above set of equations of the type

$$\rho \to \rho + \delta\rho \tag{3.4}$$

$$\vec{v} \to \vec{v} + \delta\vec{v} \tag{3.5}$$

$$p \to p + \delta p \tag{3.6}$$

$$V \to V + \delta V, \tag{3.7}$$

where

$$\delta\rho/\rho \approx \delta\vec{v}/v \approx \delta p/p \approx \delta V/V \ll 1. \tag{3.8}$$

we get the following equation, after some algebraic manipulation[6]:

$$\ddot{\delta\rho} - \nabla^2 \delta p - 4\pi G \rho \delta\rho = 0. \tag{3.9}$$

If we now use the equation of state for a monatomic ideal gas

$$p = \frac{KT}{m} \rho \tag{3.10}$$

and consider an adiabatic expansion

$$p \propto \rho^{\gamma} \; . \tag{3.11}$$

where K is Boltzmann's constant, T is the gas temperature, m is the atomic mass of the gas particles and γ is the adiabatic index, then eq. (3.9) becomes

$$\ddot{\delta\rho} - \frac{v_s^2}{\gamma} \nabla^2 \delta\rho - 4\pi G \rho \delta\rho = 0. \tag{3.12}$$

The sound speed v_s in the above equation is defined in terms of the derivative of pressure with respect to density at constant entropy:

$$v_s^2 \equiv \frac{\partial p}{\partial \rho} = \gamma \frac{p}{\rho}. \tag{3.13}$$

Eq. (3.12) resembles the equation of a classical wave and it is no coincidence that its solution can be decomposed into plane waves. That is, small density fluctuations will oscillate with time throughout the fluid. The general solution to eq. (3.12) can thus be written as

$$\delta\rho = \delta\rho_0 e^{i(kx - \nu t)}, \tag{3.14}$$

where $\delta\rho_0$ is the initial amplitude of the fluctuations, k is the wavenumber and ν is the angular frequency. Substitution of eq. (3.14) into eq. (3.12) shows that the dispersion relation

$$\nu^2 = v_s^2 k^2 - 4\pi G \rho \tag{3.15}$$

[6] For details on this derivation see *e.g.* Coles & Lucchin (1995)

must be satisfied. When ν is complex ($\nu^2 < 0$), the density contrast will grow exponentially with time, establishing a critical wavelength

$$\lambda_J \equiv \frac{2\pi}{k_J} = v_s\sqrt{\frac{\pi}{G\rho}}. \tag{3.16}$$

and causing fluctuations of wavelengths $\lambda > \lambda_J$ to be unstable to gravitational collapse.

The simple gravitational instability mechanism for the growth of structures from small density perturbations provided by the Jeans theory has to be modified in order to be applicable to cosmology. One of the reasons is that Jeans had to assume that the unperturbed gravitational field V does not contribute to the fluid's self-gravity in order to keep physical and mathematical consistency, with only the perturbed field δV making a significant contribution. Another obvious drawback in the original formulation is that the Universe is expanding, and the Jeans equation was envisaged for a static Universe. In order to take expansion effects into consideration, we must work out the equations in comoving (Lagrangean) coordinates. In this way, ρ, $\vec{v}$ and V should be parametrized as (*e.g.* Weinberg 1972)

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^3 \tag{3.17}$$

$$\vec{v} = \vec{r}\left(\frac{\dot{a}}{a}\right) \tag{3.18}$$

$$V = \vec{\nabla} \cdot \vec{r}\left(\frac{4\pi G\rho}{3}\right). \tag{3.19}$$

where the vector $\vec{r}$ is comoving with the fluid's particles. We then obtain eq. (1.1) if we perform the Jeans analysis using the above equations (*e.g.* Lifshitz 1946, Bonnor 1957, Weinberg 1972 and Coles & Lucchin 1995).

A well known application of the Jeans equation for an expanding Universe is the Einstein-de Sitter solution. We shall briefly introduce this solution here since we will refer to it many times along this work. The scale factor and the density of the Universe evolve in this model according to

$$\frac{a}{a_0} = \left(\frac{3H_0 t}{2}\right)^{2/3} \tag{3.20}$$

$$\rho = (6\pi G t^2)^{-1}. \tag{3.21}$$

Substituting the above equations into eq. (1.1) we obtain

$$\ddot{\delta} + \frac{4t^{-1}}{3}\dot{\delta} - \frac{2t^{-2}}{3}\delta = 0. \tag{3.22}$$

whose solution is given by

$$\delta = C_1 t^{-1} + C_2 t^{2/3}. \tag{3.23}$$

with C_1 and C_2 constants.

We use background cosmological models from scalar-tensor theories (see chapter two) in the Jeans equation for an expanding Universe in order to investigate the evolution of the density contrast in such models. As was mentioned in chapter one. eq. (1.1) can be shown to be valid for both BD and varying-ω theories. However. we lack a formal proof that the same equation can be applied to dynamical Λ theories. Nonetheless. we assume it to be valid here for the purpose of this study. We present analytical solutions for the perturbations whenever is possible. and solve the equations numerically otherwise. The numerical solutions are based on a Runge-Kutta-type algorithm of fourth order and the model constants used on each integration (as well as in the plots of analytical solutions) were assumed to be of order unity. We have also studied the implications of using different sets of model constants for each solution and found that this does not affect the general properties of the solutions. We have adopted the initial conditions

$$\delta(t_{rec}) = 10^{-5} \tag{3.24}$$

$$\dot{\delta}(t_{rec}) = 0, \tag{3.25}$$

where t_{rec} is the age of the Universe at the recombination time. Eq. (3.24) is motivated by results from CMB experiments (Lange *et al.* 2000), whereas eq. (3.25) is an arbitrary choice. We have allowed for different initial values of $\dot{\delta}(t_{rec})$ and the interpretation of the results remained unchanged. It is also assumed that the perturbations have a scale-invariant power spectrum, so that the same amplitude for the perturbations apply to all scales. As was mentioned in the introduction, inflationary theories predict fluctuations with such scale-invariant form (*e.g.* Guth 1981) and recent CMB observations give further support to this type of power spectrum (*e.g.* Lange *et al.* 2000).

3.2. Evolution of The Density Contrast

3.2.1. Model I

Using eqs. (2.13)-(2.15) in eq. (1.1) we obtain

$$\ddot{\delta} + \frac{4+4\omega}{4+3\omega}t^{-1}\dot{\delta} - \frac{4+2\omega}{4+3\omega}t^{-2}\delta = 0, \tag{3.26}$$

whose solution is

$$\delta = C_1 t^{-1} + C_2 t^{\frac{4+2\omega}{4+3\omega}}. \tag{3.27}$$

The two modes become $\delta_1 \propto t^{-1}$ and $\delta_2 \propto t^{2/3}$ in the limit $\omega \to \infty$ (which agrees to the general relativistic prediction for an Einstein-de Sitter Universe). Eq. (3.26) is identical to the equation found by Gaztañaga & Lobo (2001, henceforth GL2001) to describe the evolution of δ in flat Brans-Dicke cosmologies. These authors resorted to the BD field equations and the Raychaudhuri equation (Wald 1984). GL2001 argue that the second term in eq. (3.27) corresponds to the growing mode only if

$|\omega| \gg 1$. We do not see any reason why this would be so. and fig. (3.1) illustrates our point. A model with $\omega = 0$, for instance. would produce rapid growth of the type $\delta \propto t$. We find rather that δ_2 is a growing mode whenever $\omega < -2$ or $\omega > -4/3$. If $|\omega| \gg 1$ then the solution is indistinguishable from the Einstein-de Sitter case as was mentioned above. We would also like to point out a mistake in the interpretation of the results from GL2001 concerning the solution to eq. (3.26) (or equivalently. eq. 62 in their paper). They expressed their solution in terms of a rather than t:

$$\delta = C_1 a^{\frac{2+\omega}{1+\omega}} + C_2 a^{\frac{-4-3\omega}{2+2\omega}}. \tag{3.28}$$

and argued that the second term on the right-hand side of the above equation can be either an increasing or a decreasing mode depending on ω. We point out that if we simply substitute the expression for a as a function of time (eq. 54 in GL2000 or eq. 2.14 in the present paper) we see that this mode is always a decreasing mode proportional to t^{-1} as in eq. (3.27). with no dependence on ω whatsoever.

3.2.2. Model Ia

In this case. we have to solve the equation:

$$\ddot{\delta} + 2t^{-1}\dot{\delta} + \frac{CB}{4}(2+\omega)t^{-2}\delta = 0. \tag{3.29}$$

But

$$CB = 4 \tag{3.30}$$

according to Dehnen & Obregon (1971). so that we can rewrite eq. (3.29) as

$$\ddot{\delta} + 2t^{-1}\dot{\delta} + (2+\omega)t^{-2}\delta = 0. \tag{3.31}$$

which leads to the solution

$$\delta = C_1 t^{-\frac{1}{2}+m} + C_2 t^{-\frac{1}{2}-m}. \tag{3.32}$$

with

$$m = \frac{1}{2}\sqrt{-7 - 4\omega}. \tag{3.33}$$

Note that the constraint

$$\omega < -2 \tag{3.34}$$

discussed in section 2.1 assures that eq. (3.32) represents a real function. which makes the first term in this equation always an increasing mode. The second term corresponds always to a decreasing mode.

3.2.3. Model II

The Jeans equation corresponding to model II is

$$\ddot{\delta} + 4t^{-1}\dot{\delta} - 4\frac{(4 + 2\omega)(1 - 2\omega)}{(3 + 2\omega)}t^{-2}\delta = 0. \tag{3.35}$$

with power-law solutions given by

$$\delta = C_1 t^{-\frac{3}{2}+m} + C_2 t^{-\frac{3}{2}-m}. \tag{3.36}$$

where

$$m = \frac{1}{2}\frac{\sqrt{-(3 + 2\omega)(64\omega^2 + 78\omega - 91)}}{3 + 2\omega}. \tag{3.37}$$

The requirement that δ in the above equation is a real quantity provides interesting constraints on ω in this case:

$$-1.5 < \omega \lesssim 0.73 \tag{3.38}$$

or

$$\omega \lesssim -1.9 \tag{3.39}$$

DRP99 constrained ω to be approximately 1/5 from observational considerations of the mass density parameter Ω_M and the possible contribution of a cosmological constant Ω_Λ. If we use this value for the coupling constant in eq. (3.36), we find that the growing mode will evolve approximately with the 4/5 power of time. That means that structure will form faster in this case when compared to the $t^{2/3}$ rate of an Einstein-de Sitter Universe. Conversely, if ω is ~ 0.26 or ~ -2.07 we reproduce the growth rate of δ for an Einstein-de Sitter Universe.

3.2.4. Model IIa

The non-flat dust-fluid dynamical Λ model requires that we solve the equation

$$\ddot{\delta} - \frac{c_1 k t}{g C^2 (3 + 2\omega)} \dot{\delta} - \frac{8 k c_3}{g c_2} \frac{2 + \omega}{3 + 2\omega} \delta = 0, \tag{3.40}$$

where g is defined as in section 2.2. Fig. (3.2) shows the numerical solution to this equation for different values of the coupling parameter. Solutions with negative and positive curvature are identical, and the following criteria were imposed in order to integrate eq. (3.40):

$$\omega > -3/2 \tag{3.41}$$

and

$$t > \sqrt{3 + 2\omega}. \tag{3.42}$$

The two above equations combined with the unitary values assumed for the model constants eliminates singularities along the integration of eq. (3.40) due to $g = 0$. Structure does not grow in this model. Instead, the density contrast undergoes a damped oscillation until the fluctuations are eventually smoothed out.

Following the sequence from table 1. the next model to be studied is the one defined by the equation

$$\ddot{\delta} + \frac{2c_1kt}{gC^2(3+2\omega)}\dot{\delta} - \frac{4kc_3}{g^2c_1}\frac{(4+2\omega)}{(3+2\omega)} = 0. \tag{3.43}$$

with solution given by:

$$\delta = C_1 \sin x + C_2 \cos x. \tag{3.44}$$

where

$$x = 2C\sqrt{-\frac{2c_3}{c_1^2}(2+\omega)} \arctan \frac{c_1kt}{C\sqrt{(3+2\omega)c_1k}}. \tag{3.45}$$

We will limit ourselves to the regimes where

$$(3+2\omega)c_1k > 0. \tag{3.46}$$

Thus if we further assume the integration constant c_1 to be positive we must have $\omega > -3/2$ whenever $k = 1$. Conversely we must have $\omega < -3/2$ for $k = -1$. Also note that the constant c_3 is related to c_1. C and k (section 2.2 and DRP99):

$$c_3 = \frac{kc_1^2}{8\pi C^2}. \tag{3.47}$$

which means that c_3 and k carry the same sign.

Let us first examine the example with positive curvature. In this case. we learn that the general behavior of the solution is dominated by an exponential growth during the early stages followed by a slower growth rate which will eventually drive the density contrast asymptotically to a constant value. The duration of the two stages (rapid and slow growth rates) will depend on the coupling parameter ω (and. of course. on the unknown constants of the model). We can see in fig. (3.3) that lower values for ω lead to a shorter exponential period for the growth of δ. whereas the exponential phase dominates for greater values of the coupling parameter. Although the results shown on the plots are for specific values of the model constants, we find that the same trends are preserved for different sets of constants.

Examples with $k = -1$ are displayed in figs. (3.4) and (3.5). Fig. (3.4) shows one example of the $-2 < \omega < -3/2$ regime, whereas fig. (3.5) illustrates the case $\omega < -2$. Perturbations do not grow in the former case, while they evolve in the latter case in a way similar to the positive curvature counterpart. We should recall, however, that pressure effects are being taken into consideration in this model according to its very definition. Therefore a relativistic derivation of the density perturbations would be desirable to confirm the applicability of the Jeans equation to this model.

3.2.6. Model III

We will now focus our attention on the last model from table 1, namely the one with the coupling parameter ω varying with time. This model has one free parameter (b), which defines the way ω evolves. It should be stressed that several other models for scalar-tensor theories with nontrivial coupling parameter exist (*e.g.* Seshadri 1992, Barrow & Mimoso 1994, Chiba, Sugiyama & Yokoyama 1998, Navarro, Serna & Alimi 1999 and Esposito-Farèse & Polarski 2000). We do not make any attempt here to present a complete analysis of models with nontrivial ω, but we have rather chosen a particular theory to check its consistency with structure formation via gravitational instability.

For this matter, we have to solve the Jeans equation for model III:

$$\ddot{\delta} + \frac{2}{3t}\left(2 + \frac{1}{\ln t} - \frac{1}{b\ln t}\right)\dot{\delta} - \frac{\pi c_3}{c_1 t^2 \ln t}\left(\frac{3}{bc_4 \ln t} + 4\right)\delta = 0. \qquad (3.48)$$

The above equation was solved numerically and the results are shown in figs. (3.6) and (3.7). The density contrast evolves very fast when $b < 1$, and it grows otherwise in a slower rate. The regimes $b \gg 1$ (say, $b > 100$) do not change the appearance of the plots much, because the terms involving the free parameter in eq. (3.48) vanish, which means that the theory never approaches GR as far as δ is concerned.

3.3. Evolution of the Jeans Mass

We have defined the Jeans length in section 3.1 (eq. (3.16)) as the critical wavelength above which density fluctuations are unstable to gravitational collapse. We can also think in terms of a critical mass, that is, the mass contained inside a sphere whose diameter is the Jeans length:

$$M_J = \frac{\pi}{6}\rho\lambda_J^3. \tag{3.49}$$

The sound speed during the post-recombination epoch, which is necessary to evaluate the Jeans length according to eq. (3.16), is given by

$$v_s = \left(\frac{5}{3}\frac{KT}{m_p}\right)^{1/2}, \tag{3.50}$$

where $K = 1.38 \times 10^{-16}$ erg K^{-1} is Boltzmann's constant, T is the radiation temperature and $m_p = 1.67 \times 10^{-24}$ g is the proton mass. If we now use

$$T = T_0\left(\frac{a_0}{a}\right)^2 \tag{3.51}$$

as the expression for temperature in terms of the present temperature ($T_0 \approx 2.73K$) and the relative size of the Universe compared to the recombination era (a_0/a), we are able to calculate the Jeans Mass by introducing the expressions for a, G and ρ from the models we are studying.

3.3.1. Model I

It follows from eqs. (2.13)-(2.15) that the Jeans mass evolves in BD theory according to

$$M_J = M_0 \left(\frac{BC}{4} \frac{4+3\omega}{2+\omega} t^2 \right)^{-\frac{1}{2}\frac{3\omega}{4+3\omega}}. \tag{3.52}$$

where

$$M_0 = 5 \times 10^{12} \pi^4 a_0^3 C^{-1} \sqrt{\frac{30}{BC}}. \tag{3.53}$$

The evolution rate of GR ($\propto t^{-1}$) is recovered when $\omega \to \infty$, for the Jeans mass then become:

$$M_J = 3.2 \times 10^{13} \frac{a_0^3}{\pi^4} C^2 B t^{-1}. \tag{3.54}$$

Another interesting result arises when $\omega = 0$, which leads to a constant value for M_J. Intermediate values of ω are more difficult to address.

3.3.2. Model Ia

In this case we take eqs. (2.16)-(2.18) to derive

$$M_J = M_0 \left[-\frac{1}{CB} 8.8 \times 10^8 (2+\omega) \right]^{3/2} t^{-3}. \tag{3.55}$$

with

$$M_0 = \frac{5\sqrt{15}}{54} B \pi^4 a_0^3. \tag{3.56}$$

3.3.3. Model II

Following the sequence, the evolution of the Jeans mass for the flat dynamical Λ model is given by:

$$M_J = M_0 \frac{a_0^3 \pi^3}{C^3 c_1^{5/2}} \frac{3+2\omega}{2+\omega} \sqrt{\frac{3+2\omega}{(2+\omega)(1-2\omega)}} t^{-9}. \tag{3.57}$$

with $M_0 \approx 4.3 \times 10^{11}$. We can see immediately that we must have $\omega < 1/2$ so that eq. (3.57) yields a real number. Another critical value is the well known $\omega = -2$, which would lead to an infinite Jeans mass (this is also true for model I). That indeed makes sense if we recall that this value for the coupling parameter implies a null gravitational constant, meaning that structure would never collapse under the influence of gravity (in other words, an infinite mass would be required to produce collapse). Finally, the range

$$-2 < \omega < -1.5 \tag{3.58}$$

is forbidden for that would also cause M_J to be an imaginary number. Eq. (3.57) also shows that the Jeans mass has a strong time dependence in this case, as opposed to the Einstein-de Sitter case for example, and that it goes to zero in the limit $\omega \to -\infty$.

3.3.4. Model IIa

The Jeans mass was not calculated in this case due to the uninteresting behavior of the density contrast for structure formation, as demonstrated by the numerical integration of eq. (3.40).

3.3.5. Model IIb

If we now use eqs. (3.49)-(3.51) and the solutions for a, G and ρ from model IIb, we obtain the evolution of the Jeans mass for the non-flat stiff-matter-fluid case:

$$M_J = M_0 \left(\frac{a_0}{c_2}\right)^3 \left(\frac{c_1}{g}\right)^{3/2} \frac{3+2\omega}{2+\omega} \sqrt{\frac{1}{c_3 k} \frac{3+2\omega}{2+\omega}}. \tag{3.59}$$

with M_0 the same as in eq. (3.57). Note that $g \propto t^2$, which implies $M_J \propto t^{-3}$ for this example. The influence of ω in the above equation is not as simple to access as was with the previous case. The reason is that the coupling parameter acts on eq. (3.59) together with the model constants C and c_1 (through the term $g^{3/2}$). For $k = 1$ ($\omega > -3/2$) the general effect of increasing ω is to slow the decay of M_J (fig. (3.8)). The open case of this model ($k = -1$: $\omega < -3/2$) requires that $\omega > -2$ so that eq. (3.59) yields a real number, which poses a rather restrictive range for the allowed values of the coupling parameter.

3.3.6. *Model III*

Finally, using eqs. (2.33)-(2.35) we obtain that the Jeans mass for the model with a varying ω reads as

$$M_J = M_0 \frac{t^{-1} (\ln t)^{\frac{b-2}{2}}}{(3 + 4b \ln t)^{3/2}}, \tag{3.60}$$

where

$$M_0 = 9.7 \times 10^{12} \pi^{5/2} c_3^{-1/2} (c_1 b)^{3/2} \left(\frac{a_0}{c_2}\right)^3. \tag{3.61}$$

Figs. (3.9)-(3.11) show the influence of b in the Jeans mass for this model, where we consider the same values for the free parameter as used in figs. (3.6) and (3.7). We see that the cases $b = 0.1$ and $b = 1$ are very similar, contrary to what happens when we consider the evolution of the density contrast alone. As we increase the value of the free parameter ($b = 10$, fig. 3.10), the logarithmic function in the numerator of eq. (3.60) starts being significant and the Jeans mass becomes peaked rather than monotonically decreasing. This logarithmic term dominates the behavior of the Jeans mass completely for larger values of b ($b = 100$, fig. 3.11), causing the Jeans mass to increase very rapidly.

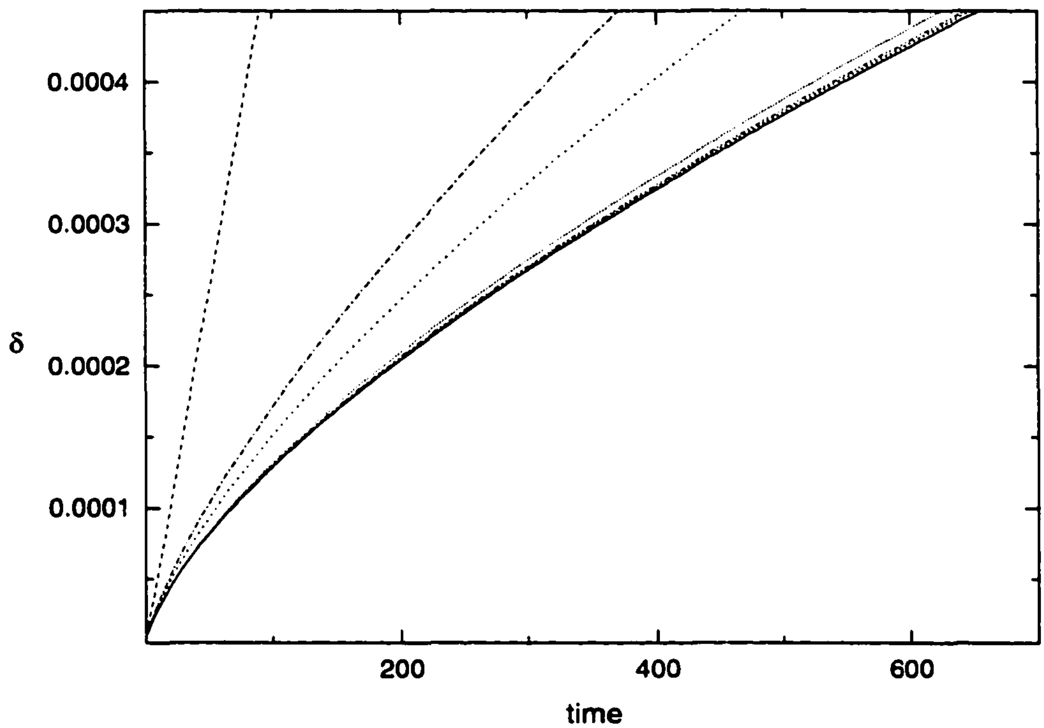

Fig. 3.1.— δ as a function of t/t_{rec} in BD theory for different values of the coupling parameter: $\omega = 0$ (dashed); $\omega = 5$ (dot-dashed); $\omega = 10$ (dotted); $\omega = 100$ (short-dotted); and the cases $\omega = \pm 500$ (dash-dot-dot for plus and short-dashed for minus) and Einstein-de Sitter (solid), which are barely distinguishable on the plot.

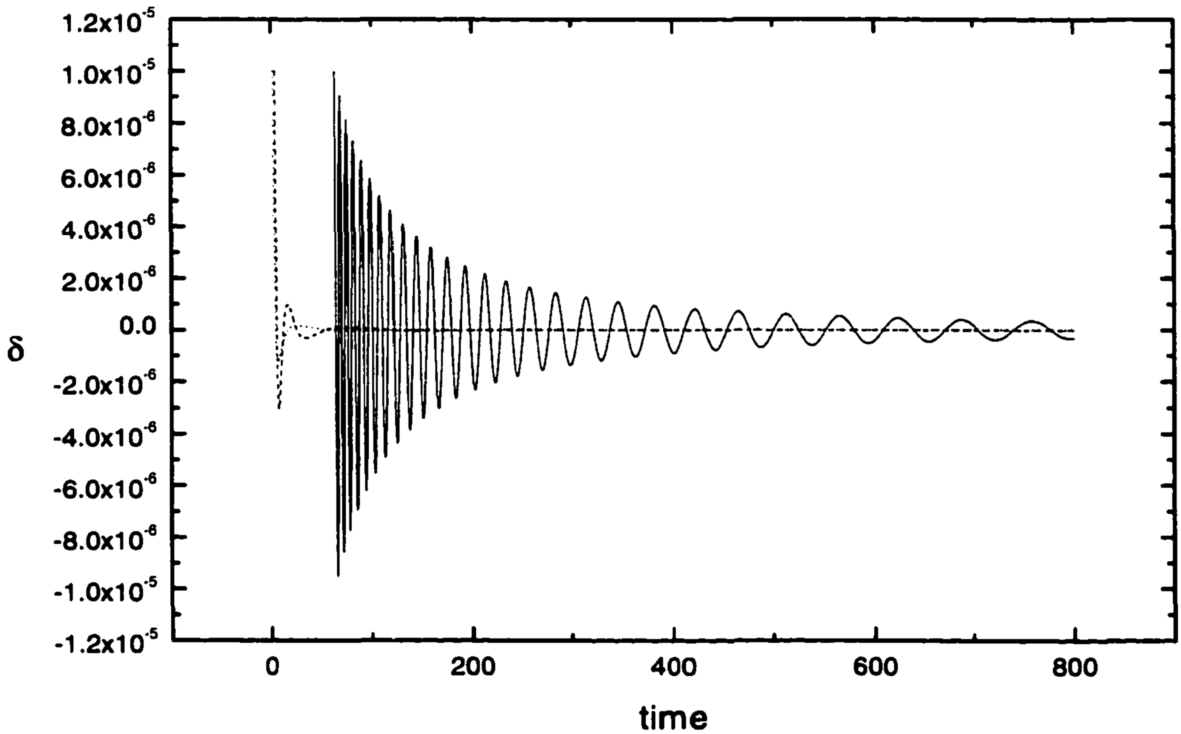

Fig. 3.2.— δ as a function of t/t_{rec} for the closed dynamical Λ model for $\omega = 500$ (solid), $\omega = 0.5$ (dashed) and $\omega = -1$ (dotted)

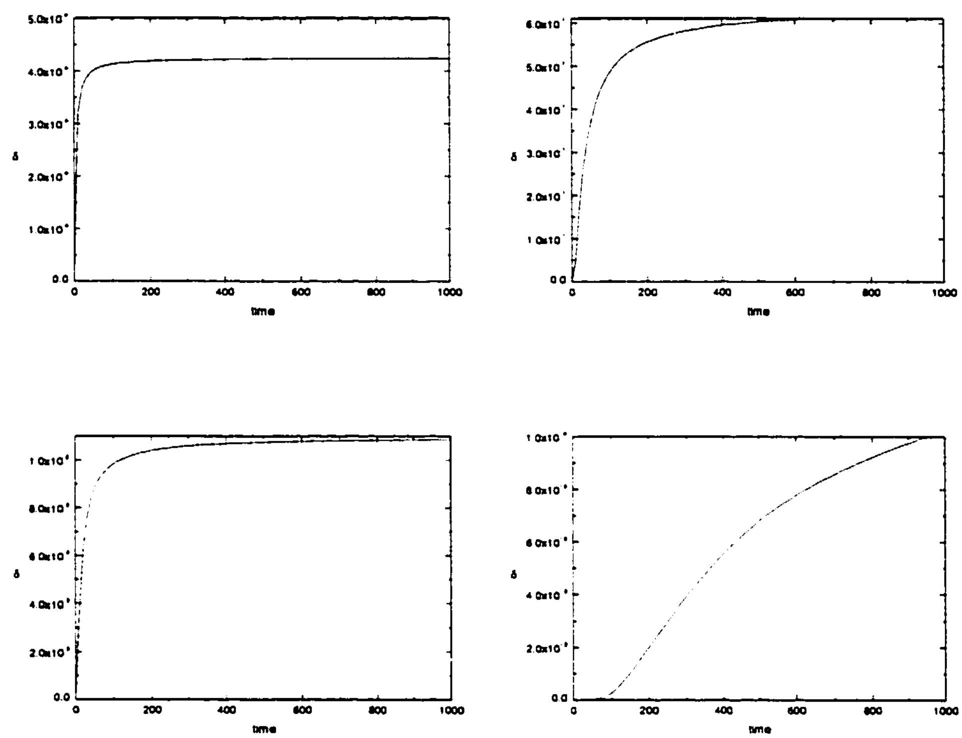

Fig. 3.3.— δ as a function of t/t_{rec} for the stiff-matter-fluid case with $k = 1$ and $\omega = -1$ (upper left), $\omega = 1$ (lower left), $\omega = 10$ (upper right) and $\omega = 100$ (lower right).

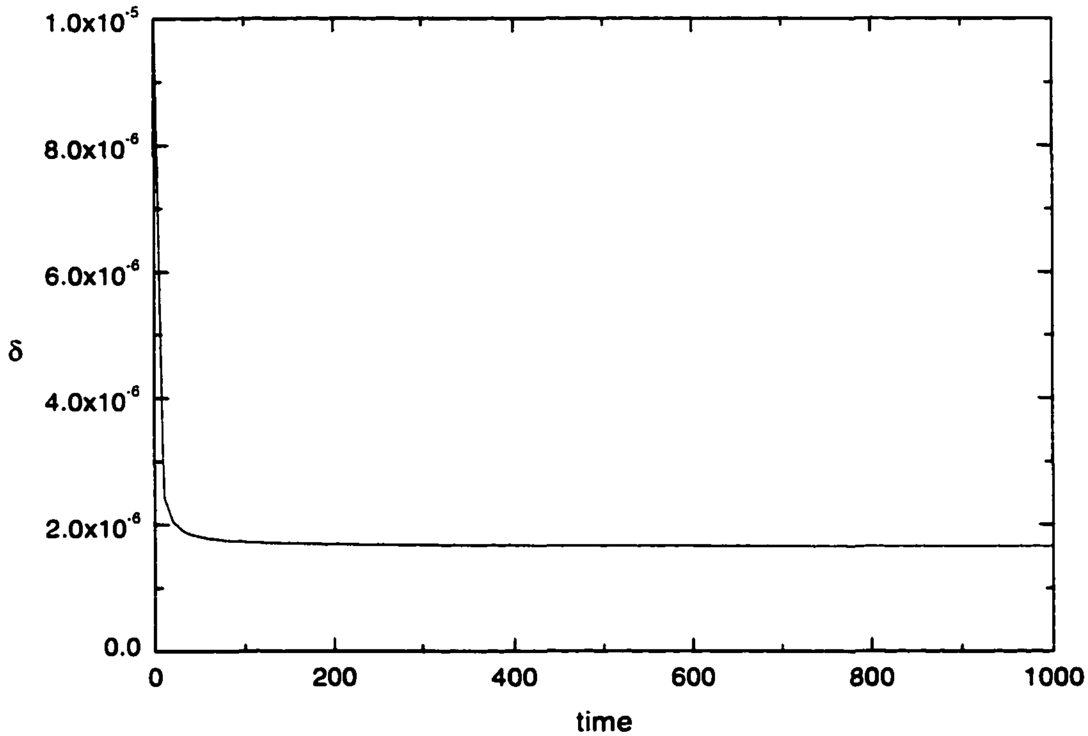

Fig. 3.4.— δ as a function of t/t_{rec} with $k = -1$ in the stiff-matter-fluid case with $\omega = -1.9$.

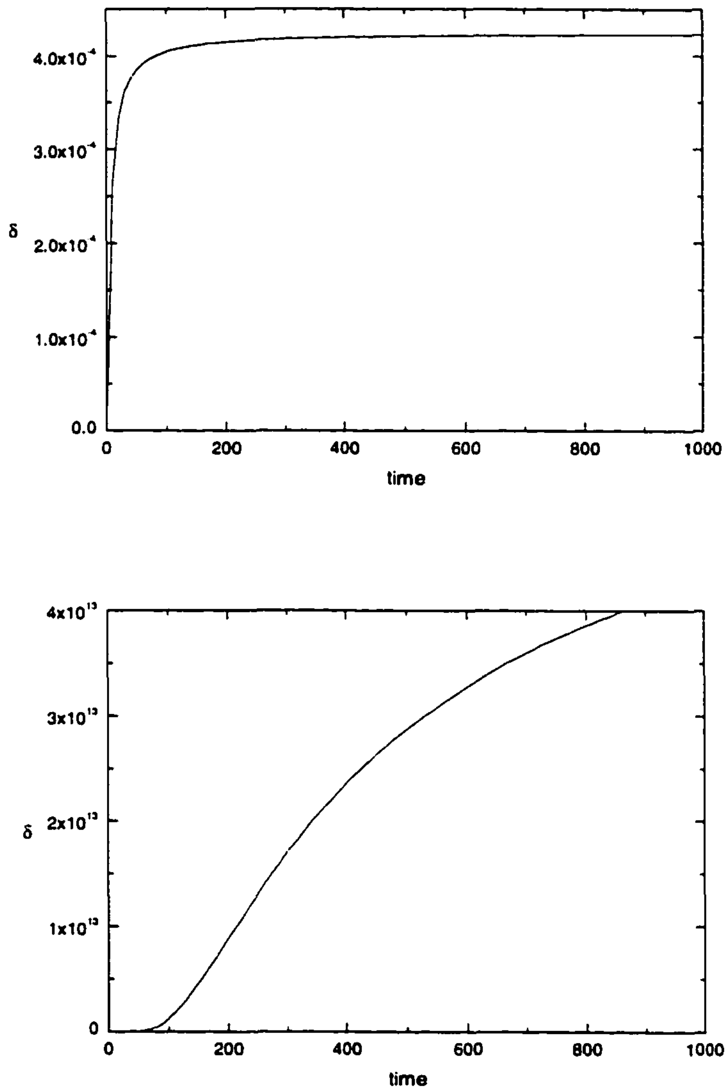

Fig. 3.5.— Same as fig. (3.4) with $\omega = -3$ (upper) and $\omega = -100$ (lower).

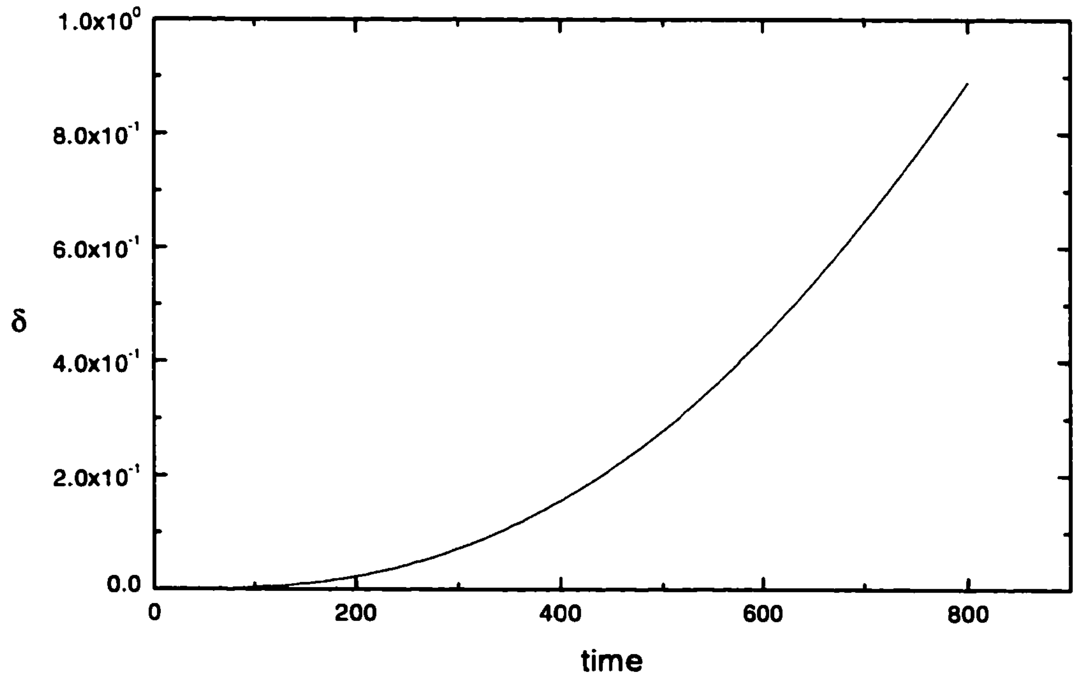

Fig. 3.6.— δ as a function of t/t_{rec} for a flat non-trivial ω model with $b = 0.1$.

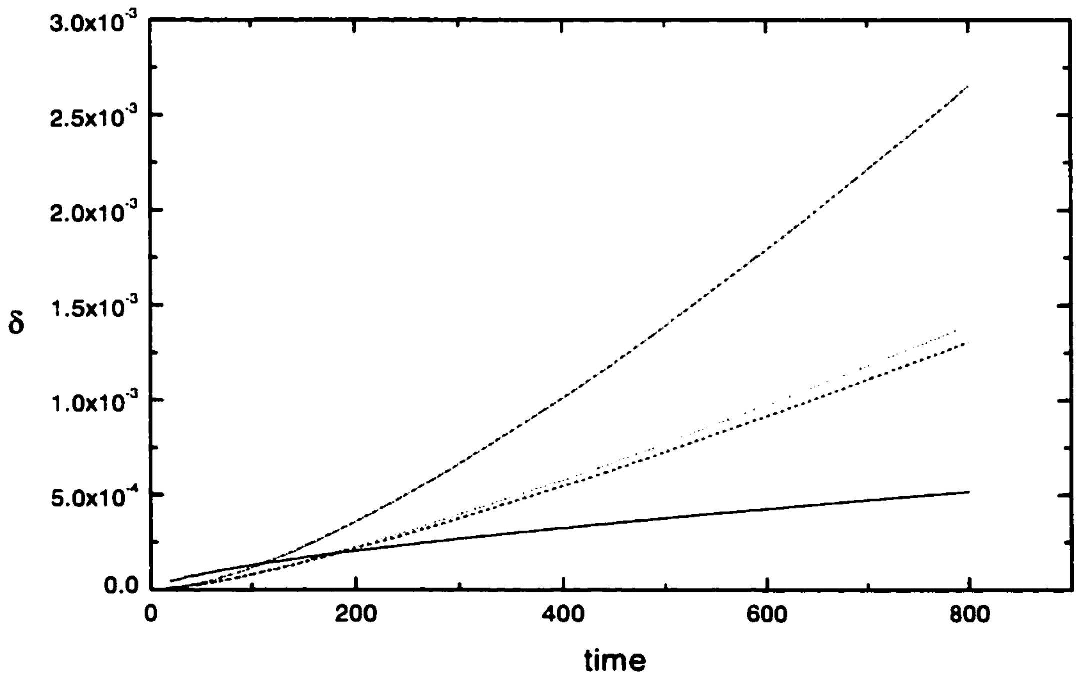

Fig. 3.7.— Same as fig. (3.6) for b = 1 (dot-dashed). b = 10 (dotted). b = 100 (dashed) and an Einstein-de Sitter Universe (solid).

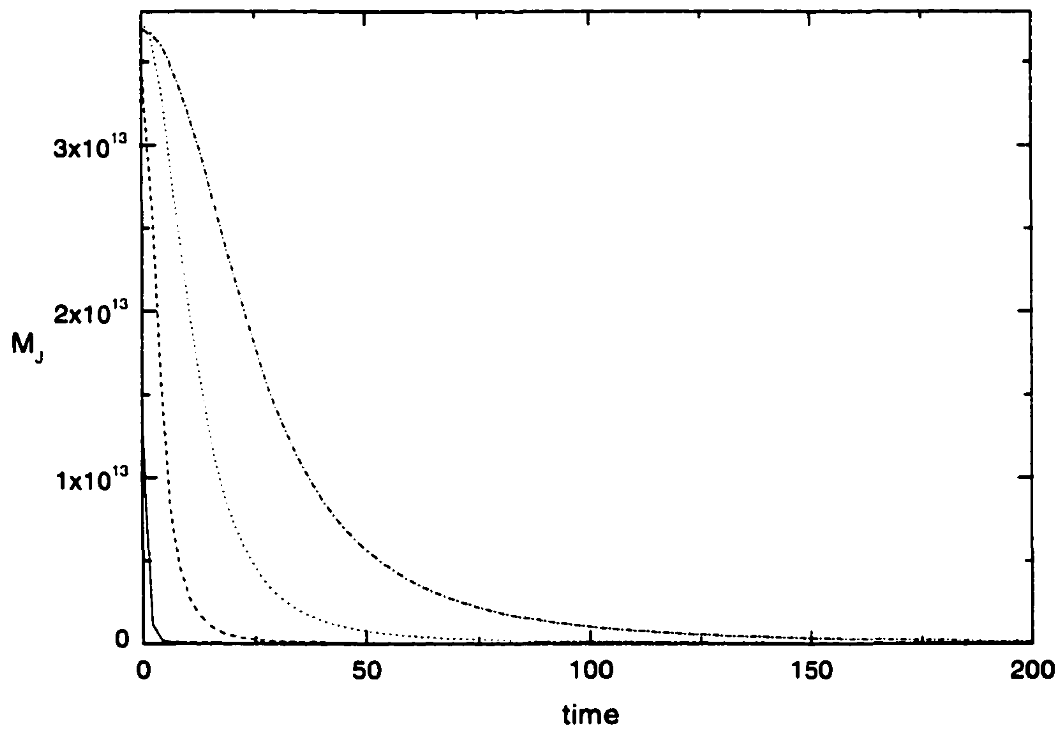

Fig. 3.8.— Evolution of the Jeans mass with t/t_{rec} for the closed stiff-matter-fluid model with $\omega = -1$ (solid). 10 (dashed). 100 (dotted) and 500 (dot-dashed).

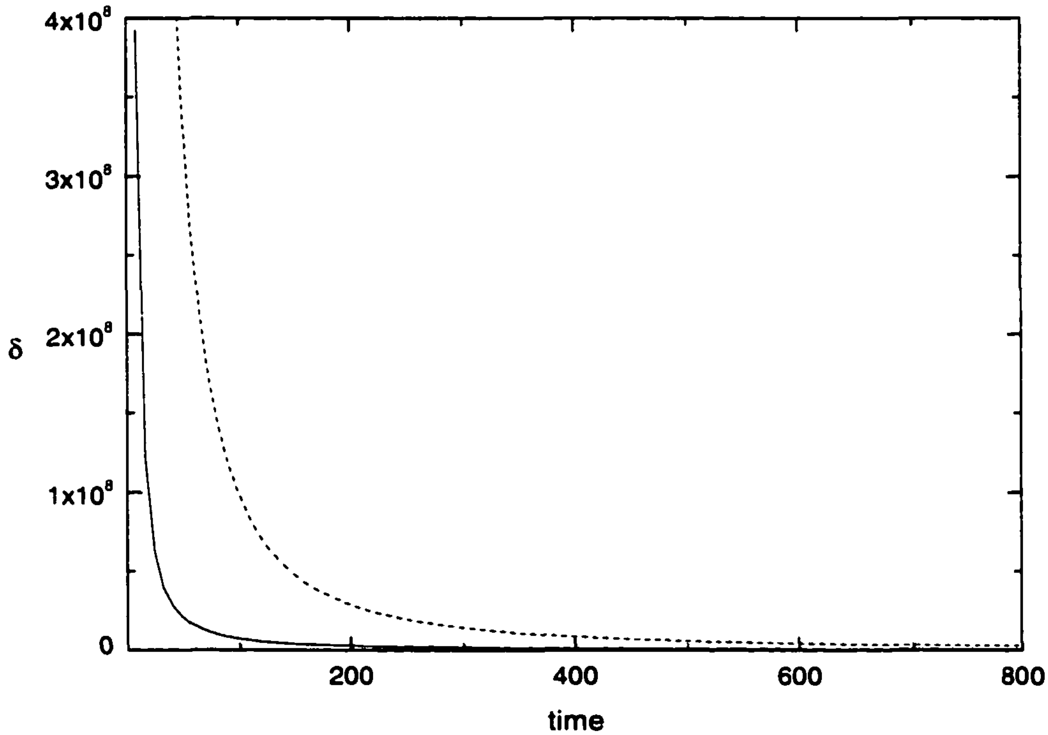

Fig. 3.9.— Evolution of the Jeans mass with t/t_{rec} for the flat non-trivial ω model with $b = 0.1$ (solid) and $b = 1$ (dashed).

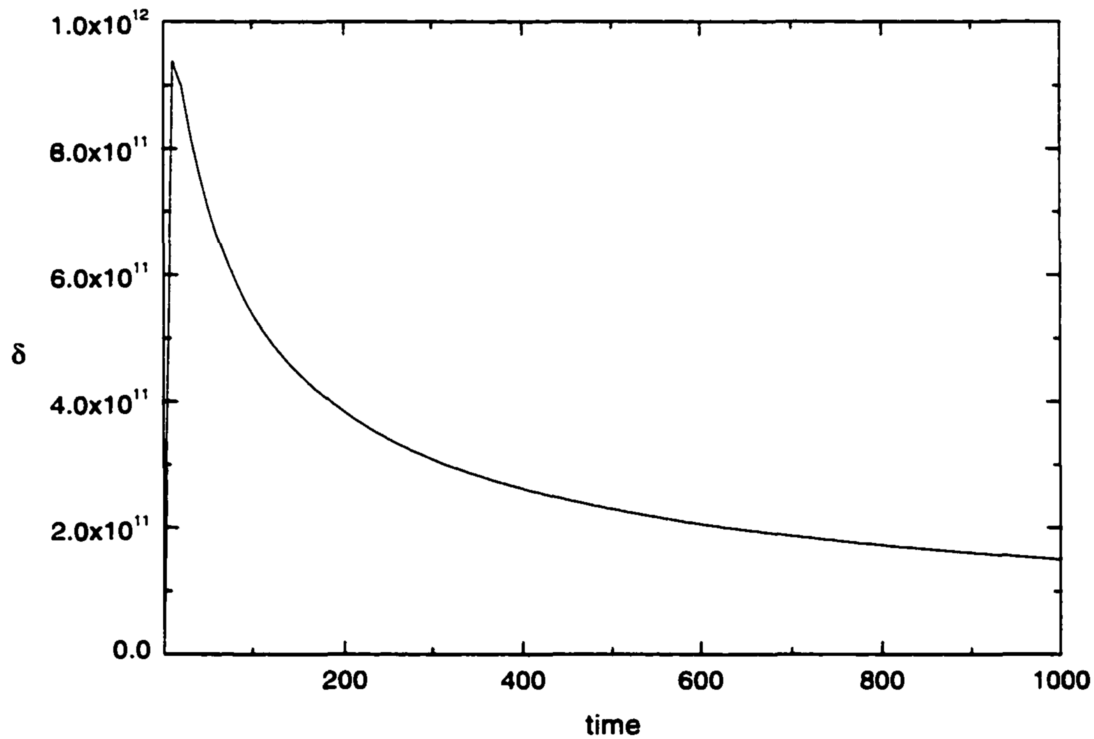

Fig. 3.10.— Evolution of the Jeans mass with t/t_{rec} for the flat non-trivial ω model with $b = 10$.

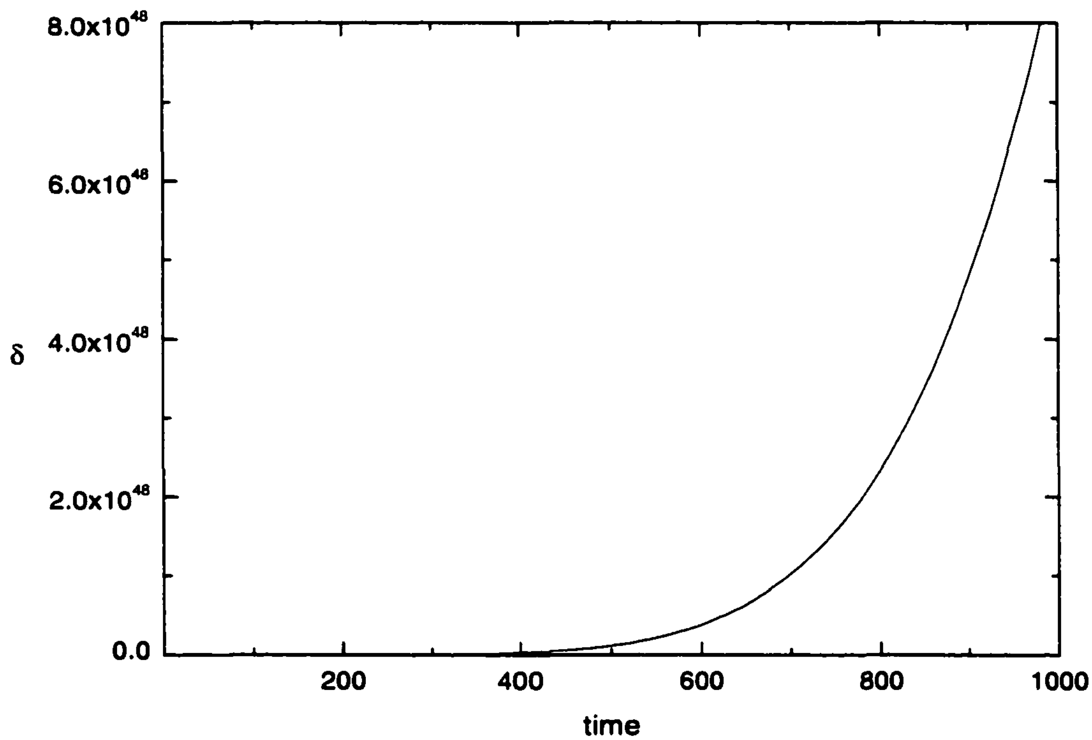

Fig. 3.11.— Evolution of the Jeans mass with t/t_{rec} for the flat non-trivial ω model with $b = 100$.

4. Summary and Conclusions

We have investigated three classes of scalar-tensor cosmological models (Brans-Dicke, dynamical Λ and varying ω) with flat, spherical and hyperbolic geometries. The main goal of this work is the study of the linear growth of density perturbations in such theories, which was undertaken within the Jeans formalism for an expanding Universe. It was found that some models lead to power-law growth rates for the density contrast, with the coupling function ω dictating its evolution. In some other models, structure formation is either supressed or takes place with an exponential growth rate. The results may be summarized as follows:

Brans-Dicke theory: as one of the simplest extensions of general relativity, BD theory is known to approach GR as $\omega \to \infty$. However, this result is verified only for flat curvature. Our analysis for flat BD theory corroborates the work of Gaztañaga & Lobo (2001) and the expected asymptotic behavior of BD theory towards GR. We point out a misinterpretation of the results in Gaztañaga & Lobo (2001) though (see discussion in section 3.2): these authors chose to study the solution for δ as a function of the cosmic scale factor a, and they would have concluded that the mode α_2 in their paper is always equal to -1 if they had considered the explicit time-dependence of a. They also pointed that the mode α_1 in their paper (corresponding to the second term from eq. 3.27 in the present work) is a growing mode only for large values of $|\omega|$, which we do not agree with and give examples to illustrate our point (fig. 3.1). The closed BD model also presents unambiguous increasing and decreasing modes. Fig. (4.1) shows the dependence of the growing modes (denoted by α in the plot) of each model (flat and closed) on ω for comparison[7]. One can see that α goes rapidly to $2/3$ as $|\omega|$ increases for the flat model, whereas it grows indefinitely for the closed counterpart throughout the allowed range of the coupling parameter ($\omega < -2$). Finally, the constants C_1 and C_2 in eqs. (3.27) and (3.32) are ω-dependent, as can be seen if we solve the Jeans equation for models I and Ib using our assumed initial conditions. We thus have

$$C_1 = 5 \times 10^{-4} \frac{2+\omega}{8+5\omega} \tag{4.1}$$

$$C_2 = 1 \times 10^{-5} \frac{4+3\omega}{8+5\omega} \tag{4.2}$$

for model I and

$$C_1 = 2 \times 10^{-5} \left(\frac{1}{\sqrt{-7-4\omega}} + 1 \right) \tag{4.3}$$

$$C_2 = 2 \times 10^{-5} \left(-\frac{1}{\sqrt{-7-4\omega}} + 1 \right) \tag{4.4}$$

for model Ib. There is no further constraint for model Ib from the above equations. However, we must have $\omega \neq -8/5$ for model Ia according to eqs. (4.1) and (4.2).

Dynamical Λ theory: Here we have studied flat, open and closed models with a dust-fluid-like equation of state (models II and IIa) and non-flat models with a stiff-matter-like equation of state (model IIb). The age of the Universe and the Hubble constant are related to each other in model II according to (DRP99):

$$H_0 = \frac{2}{t_0} \tag{4.5}$$

This scenario requires $H_0 \approx 30$ km s^{-1}Mpc^{-1} in order to be consistent with the latest estimates of the age of the Universe ($\sim$ 13.2 billion years, *e.g.* Lineweaver 1999).

---

[7]Note that growing modes exist for flat BD cosmologies only if ω is outside the range $-2 < \omega < -4/3$.

Recent measurements of the Hubble constant point to a value of approximately 68 km $s^{-1}Mpc^{-1}$ (Mould *et al.* 2000). Therefore, in spite of having power-law growing modes for δ, model II can not agree with the inferred values of the age of the Universe and of H_0 simultaneously. The model reproduces the Einstein-de Sitter solution when $\omega = \frac{-65\pm\sqrt{7033}}{72}$. The Jeans mass for this model is proportional to t^{-9}, meaning that it goes rapidly to zero. We have discovered two important aspects of this model: (1) structure formation processes will take place during a short period of time: and (2) the closest to -2 is the coupling parameter the greatest is the Jeans mass in a given time. It is striking that one of the values for ω (≈ -2.07) which reproduces the Einstein-de Sitter equivalent for δ lies just outside the forbidden range imposed by the Jeans mass (eq. 3.58). This assures that if the density contrast is to grow as in the Einstein-de Sitter Universe then the initial Jeans mass had to be high because of the proximity to the resonant value $\omega = -2$. The density contrast never grows in the context of model IIa, but rather undergoes a damped oscillation similar to the oscillations present in the radiation epoch, which are damped by the interaction with the primordial plasma according to standard cosmological models (*e.g.* Coles & Lucchin 1995). Our main finding about model IIb with positive curvature is that there are two distinct stages in the evolution of the density contrast, namely, early exponential evolution followed by a slower growth rate. The duration of the two stages is controlled by ω, but in any case structure grows rather steeply during the earliest stages of evolution and after some time the density contrast ceases to grow. Unlike in standard Big Bang models, δ is not predicted to grow monotonically during the linear regime. Model IIb with negative curvature has the same behavior of the $k = 1$ counterpart for $\omega < -2$, while for $-2 < \omega < -3/2$ there is no growing mode. Values of the coupling parameter greater than -1.5 are excluded for $k = -1$ due to our choice of model constants.

Varying-ω theory: we do find growing modes for this particular varying-ω flat cosmology. The growth rates found are always higher than the Einstein-de Sitter counterpart, specially when the free parameter b of the theory is less than unity. Greater values of b slow the growth rate, but even when $b \to \infty$ the density contrast grows faster than in GR. Perhaps other theories with varying coupling parameter would yield different results, and we leave this for future work.

In summary, we have shown that a treatment as simple as the Jeans analysis for an expanding Universe is useful to test for realistic scalar-tensor cosmologies. Comparisons with observations are not straightforward for every cosmology studied here because of the many model constants involved. However, the very absence of growing modes (or the presence of exponentially increasing modes) can be enough to decide whether a model is viable, or not, for structure formation. Also, the fact that we can have faster growth rates of δ in some of the theories studied when compared to GR means that structure forms at earlier times in these models.

A fully relativistic treatment to the evolution of perturbations within the theories we have analyzed would be a natural extension of the present work, and we leave this for future work. Other theories with varying-ω should also be investigated. Exact solutions involving both $\Lambda(\phi)$ and $\omega(\phi)$ have not been found yet, and they can be promptly tested within the framework we have introduced as soon as they become available.

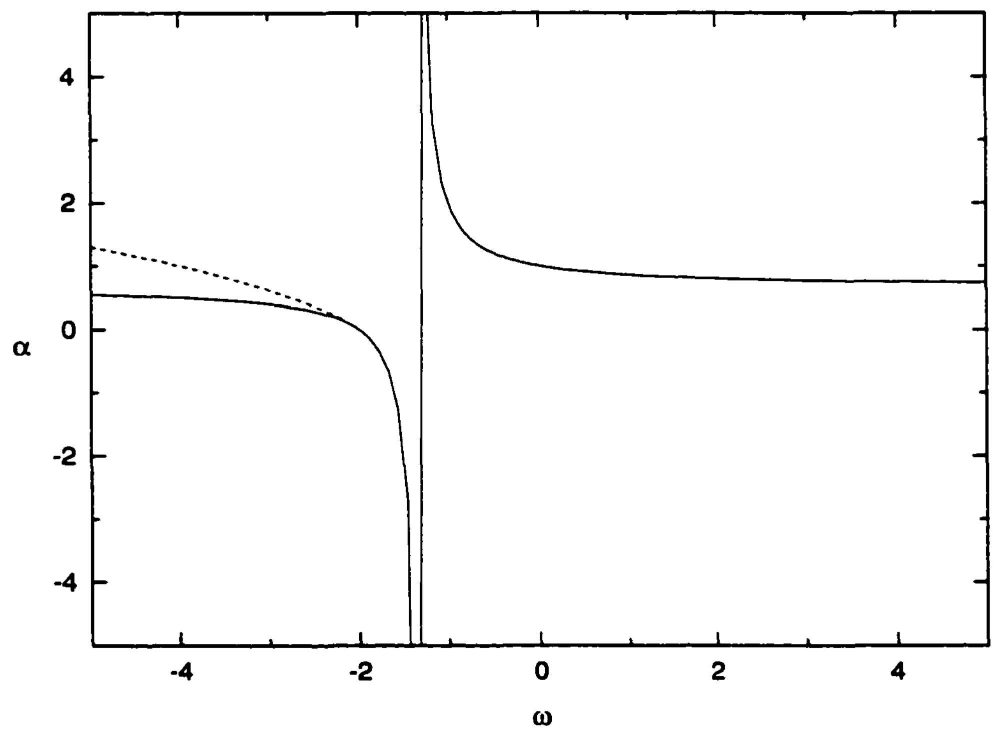

Fig. 4.1.— Growth rates for flat (solid) and closed (dashed) BD cosmologies as a function of the coupling constant.

A. On The Power-Law Relation between a and ϕ in flat dust-fluid Brans-Dicke Cosmology

In order to obtain exact cosmological solutions in scalar-tensor theories, it is usually assumed that eq. (2.19) is valid (*e.g.* Dehnen & Obregon 1971, 1972 and Díaz-Rivera & Pimentel 1999). It has been argued that this relation is a condition for the deceleration parameter to be constant for flat models in BD theory (Johri & Desikan 1994). We show through a linear perturbation treatment that this ansatz is also necessary in order for flat dust-fluid Brans-Dicke models to be consistent with Newtonian physics in the weak field approximation.

Let us allow small perturbations in the metric, in the scalar field and in the energy-momentum tensor:

$$g_{\mu\nu} \Rightarrow g_{\mu\nu} + h_{\mu\nu} \tag{A1}$$

$$\phi \Rightarrow \phi + \delta\phi \tag{A2}$$

$$T_{\mu\nu} \Rightarrow T_{\mu\nu} + \delta T_{\mu\nu}. \tag{A3}$$

and assume all the perturbations to be static and of the same order, so that any terms involving squares or products of perturbations are ignored in the analysis, and their time derivatives vanish. Let us also assume that the perturbations in the Robertson-Walker metric are of the same form as in the weak field approximation of this scalar-tensor theory (i.e. it takes the form of the so-called post-Newtonian parameters of the theory, *e.g.* Will 1981). This is the main ansatz used here and it is by no means the usual approach for treating perturbations in the field equations. In the canonical treatment the perturbed terms are found by plugging them into the field equations and then solving for them. Besides, it is usually assumed that spatial derivatives of the perturbations are negligible compared to their time derivatives.

This is how one can study the evolution of primordial density perturbations in the context of a relativistic gravitational theory (*e.g.* Peebles 1993 and Gaztañaga & Lobo 2001). Therefore the perturbed metric used here has components

$$g_{00} = 1 \tag{A4}$$

$$g_{ij} = -a^2 + \frac{2V(1+\omega)a^2}{2+\omega}; i = j \tag{A5}$$

$$g_{ij} = 0; i \neq j. \tag{A6}$$

We are considering flat spacetime. Note that the above expressions are written in a synchronous gauge, i.e. in time-orthogonal coordinates (*e.g.* Zel'dovich & Novikov 1983 and Peebles 1993). In this way the time-time component of the metric remains unperturbed. V in eq. (A5) is the Newtonian gravitational potential.

For the perturbations in the energy-momentum tensor it is assumed that only its time-time part, namely ρ, makes a significant contribution (i.e. we are neglecting pressure contributions in $T_{\mu\nu}$), so that

$$\delta T_{\mu\nu} = \delta\rho. \tag{A7}$$

We have made no assumption about the perturbation in the scalar field $\delta\phi$.

Substituting eqs. (A4)-(A7) in eqs. (2.2) and (2.3), we get the corresponding perturbed and unperturbed parts of each equation. The unperturbed time-time component of eq. (2.2) is simply eq. (2.5), while the corresponding perturbed equation is given by

$$\nabla^2 V = \left(\frac{a^2}{\phi}\right)\left(\frac{2+\omega}{1+\omega}\right)\left\{4\pi\delta\rho + \frac{1}{2}\left[3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} - \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{8\pi\rho}{\phi}\right]\delta\phi + \frac{1}{a^2}\nabla^2\delta\phi\right\}. \tag{A8}$$

Similarly, the unperturbed part of eq. (2.3) is eq. (2.6), with its perturbed component being

$$\nabla^2\delta\phi = -\frac{2}{3+2\omega}4\pi a^2\delta\rho. \tag{A9}$$

Eq. (A8) and (A9) can be combined and rewritten as

$$\nabla^2 V = \frac{4\pi a^2 \delta\rho}{\phi}\left(\frac{4+2\omega}{3+2\omega}\right) - \frac{1}{2}\left[3\left(\frac{\dot{a}}{a}\right)^2 + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2\right]\left(\frac{2+\omega}{1+\omega}\right)\frac{a^2}{\phi}\delta\phi, \tag{A10}$$

where we have used the identity

$$3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} - \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{8\pi\rho}{\phi} = -\left[3\left(\frac{\dot{a}}{a}\right)^2 + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2\right] \tag{A11}$$

from eq. (2.2).

If (1) $\delta\rho$ is some local density, (2) ϕ assumes its present value ϕ_0 and (3) $a = constant = 1$ is the present size of the Universe, then eq. (A10) is the Poisson equation with an effective gravitational constant given by

$$G_{eff} = \phi_0^{-1}\left(\frac{4+2\omega}{3+2\omega}\right), \tag{A12}$$

which is precisely the expression obtained in BD theory. This result is expected since we are using the post-Newtonian parameters for STTs. However, the complete eq. (A10) tells us that the Poisson equation is not valid as we know it for any epoch in the Universe. There are time-dependent corrections to it, and perhaps even scale-dependent corrections embedded in $\delta\phi$ (if, for example, we integrate eq. (A9)).

In order to get rid of the extra terms in the Poisson equation (leaving it invariant for all times and all scales), we require:

$$\left[3\left(\frac{\dot{a}}{a}\right)^2 + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2\right]\left(\frac{2+\omega}{1+\omega}\right) = 0. \tag{A13}$$

We are not considering here the case $\omega = -2$ since that would imply a null gravitational constant. Therefore eq. (A13) is valid if

$$a = \mp m c_1 \phi^{\pm m}, \tag{A14}$$

where

$$m = \sqrt{-\frac{\omega}{6}} \tag{A15}$$

and c_1 is an integration constant.

We see immediately that eq. (A14) is of the sort assumed in order to find exact solutions in STTs (eq. (2.19)). Besides, eq. (A15) shows that the exponent m is related to ω in a rather simple way. Examination of eqs. (A14) and (A15) shows that in order to get a real value for m the coupling parameter ω must be negative. The power-law relation we have found is valid specifically in the BD case. A more general perturbation treatment would be required to access the validity of the relation for other STTs. It is possible that there are scale-dependent modifications to the Poisson equation arising from the term proportional to $\delta\phi$. Such scenario would be a variant of the scale-varying G cosmological models (Bertolami, Mourão & Pérez-Mercader 1993) and perhaps a more theoretically motivated approach to the problem.

REFERENCES

Albrecht, A. and Steinhardt, P. J., Phys. Rev. Lett., **48**, 1220 (1982)

Bardeen, J. M., Steinhardt, P. J. and Turner, M. S., Phys. Rev. D, **28**, 679 (1983)

Barrow, J. D. and Mimoso, J. P., Phys. Rev. D, **50**, 3746 (1994)

Batista, A. B., Fabris, J. C. and Ribeiro, R. S., gr-qc/0001055 (2000)

de Bernardis, P. *et al.*, Nature, **404**, 955 (2000)

Bertolami, O. and Martins, P. J., Phys. Rev. D, **61**, 064007 (2000)

Bertolami, O., Mourão, J. and Pérez-Mercader, J., Phys. Lett. B, **311**, 27 (1993)

Bonnor, W. B., MNRAS, **117**, 104 (1957)

Brans, C. and Dicke, R. H., Phys. Rev., **124**, 925 (1961)

Chen, X. and Kamionkowski, M., Phys. Rev. D, **60**, 104036 (1999)

Chiba, T., Sugiyama, N. and Yokoyama, J., Nucl. Phys. B, **530**, 304 (1998)

Coles, P. and Lucchin, F., *Cosmology: The Origin And Evolution of Cosmic Structures* (John Wiley & Son) (1995)

Dahia, F. and Romero, C., Phys. Rev. D, **60**, 104019 (1999)

Dehnen, H. and Obregón, O., ApSS, **14**, 454 (1971)

Dehnen, H. and Obregón, O., ApSS, **15**, 326 (1972)

Díaz-Rivera, L. M. and Pimentel, L. O., Phys. Rev. D, **60**, 123501 (1999)

Efstathiou, G., Bridle, S. L., Lasenby, A. N., Hobson, M. P. and Ellis, R. S., MNRAS, **303**, L47 (1999)

Esposito-Farèse, G. and Polarski, D., gr-qc/0009034 (2000)

Gaztañaga, E. and Lobo, J. A., ApJ (*accepted*) astro-ph/0003129 (2001)

Guth, A., Phys. Rev. D, **23**, 347 (1981)

Guth, A., *The Inflationary Universe: The Quest for A New Theory of Cosmic Origins* (Addison-Wesley) (1997)

Hanany, S. *et al.*, ApJ Lett., **545**, 5 (2000)

Helbig, P., A&A, **350**, 1 (1999)

Hoyle, F., MNRAS, **108**, 372 (1948)

Hoyle, F., Burbidge, G. and Narlikar, J. V., ApJ, **487**, 69 (1993)

Hu, W., Fukugita, M., Zaldarriaga, M. and Tegmark, M., ApJ (*accepted*) astro-ph/0006436 (2000)

Jeans, J. H., Phil. Trans. R. Soc., **199A**, 1 (1902)

Johri, V. B. and Desikan, K., Gen. Rel. Grav., **26**, 12 17 (1994)

Lange, A. E. *et al.*, Phys. Rev. D, (*submitted*) astro-ph/0005004 (2000)

Lifshitz, E. M., Sov. Phys. JETP., **10**, 116 (1946)

Linde, A. D., Phys. Lett B, **108**, 389 (1982)

Lineweaver, C. H., Science, **284**, 1503 (1999)

Mould, J. R. *et al.*, ApJ, **529**, 786 (2000)

Navarro, A., Serna, A. and Alimi, J. M., Phys. Rev. D, **59**, 124015 (1999)

Nordvedt, K., ApJ, **161**, 1059 (1970)

Overduin, J. M. and Cooperstock, F. I., Phys. Rev. D, **58**, 043506 (1998)

Peebles, P. J. E., *Principles of Physical Cosmology* (Princeton University Press, Princeton) (1993)

Penzias, A. A. and Wilson, R. W., ApJ, **142**, 419 (1965)

Ratra, B. and Peebles, P.J.E., Phys. Rev. D, **37**, 3406 (1988)

Reasenberg, R. D. *et al.*, ApJ Lett., **234**, 219 (1979)

Sahni, V. and Starobinsky, A., Int. J. Mod. Phys. D, **9**, 373 (2000)

Sen, A. A., Sen, S. and Sethi, S., gr-qc/0010030 (2000)

Sen, S. and Seshadri, T. R., gr-qc/0007079 (2000)

Seshadri, T. R., Phys. Lett. B, **277**, 67 (1992)

Smoot, G. F. *et al.*, ApJ Lett., **396**, 1 (1992)

Starobinsky, A. A. and Yokoyama, J., Proc. 4th Workshop on General Relativity And Gravitation, eds Nakao *et al.* (Yukawa Institute for Theoretical Physics) (1994)

Susperregi, M. and Mazumdar, A., Phys. Rev. D, **58**, 083512 (1998)

Tryon, E., Nature, **246**, 396 (1973)

Vilenkin, A., Phys. Lett. B, **117**, 25 (1982)

Wald, R. M., *General Relativity* (The University of Chigago Press, Chicago) (1984)

Weinberg, S., *Gravitation And Cosmology: Principles And Applications of The General Theory of Relativity* (Wiley, New York) (1972)

Will, C., *Theory And Experiment In Gravitational Physics* (Cambridge University Press, Cambridge) (1981)

Zel'dovich, Ya. B. & Novikov, I. D., *The Structure And Evolution of The Universe* (The University of Chicago Press, Chicago) (1983)

This manuscript was prepared with the AAS LaTeX macros v4.0.