Modeling CO\textsubscript{2} Emission Allowance Derivatives

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Master of Finance Research Project
Saint Mary`s University
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September 17th, 2013
Acknowledgement

I would like to express my gratitude to all those who helped me during the writing of this paper. A special acknowledgement should be shown to Dr. Dodds, from whose lectures I benefited greatly. I am particularly indebted to Christ, who gave me kind encouragement and useful instructions all through my writing. Finally I wish to extend my thanks to the library assistants who supplied me with reference materials of great value.
Abstract
Modeling CO2 Emission Allowance Derivatives
By Zhang, Jing Yu

Efficient pricing of carbon derivative product is the core of the EU-ETS system, which plays an important role in keeping stability and developing the carbon financial market. The volatility of the underlying assets is the essential factor, so how to get the features of the volatility effectively becomes a key problem. Although the B-S model gives a classic tool for option pricing, it is based on the assumption that the volatility is constant. Increasingly, the research finds that financial data exhibit fat-tail and high kurtosis, so the assumption of constant volatility is not suitable.

In this paper we first analyse the carbon emissions trading market, and then we use a GARCH model to appropriately reproduce the dynamics of the EUA futures’ returns. We obtain the conclusions that GARCH(1,1) model is appropriately to reproduce the futures dynamics.

Key Words: Kyoto Protocol; EU-ETS; carbon finance; GARCH
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Chapter 1. Introduction

Global warming and climate change are increasing considered a major threat to human health and the global economy due to their potential catastrophic consequences. A generally accepted theory is that the increasing concentration of greenhouse gases in the atmosphere is responsible for such change. One of the major international efforts to cope with this increasing environmental risk is the Kyoto Protocol to the United Nation Framework Convention on Climate Change (UNFCCC), which is an international treaty that sets binding obligations on member countries to reduce emissions of greenhouse gases. The major mechanism of greenhouse gases reduction dictated by the protocol was the trading of human-related emission allowances.

In January 2005, the EU-wide CO$_2$ greenhouse gas emissions trading system (EU ETS) formally entered into operation. Since then, the European Energy Exchange (EEX), European Climate Exchange (ECX), Powernext, Chicago Climate Exchange (CCX), and New York Mercantile Exchange (NYMEX) began trading EU allowance (EUA), certified emission reduction (CER), and their derivatives. The trading volumes in these markets are increasing as the EU ETS expands, therefore, the pricing of emission allowances and their derivatives is becoming an important issue.

The major participants in the emission allowance market include polluting
industries, mainly electricity producers, and the so called carbon investors. Under the scheme, the emission intensive firms would be given the right to emit a certain amount of greenhouse gases into the atmosphere. This allowance comes in the form of a new tradable asset—the European Union Allowance (EUA). One EUA gives the right to emit one ton of greenhouse gases. Surplus allowances can be stored for use during the following year or be traded in the market. In addition, the companies would have to surrender the missing allowances in the following year. In general, the amount of EUA stock of a company determines the degree of allowed plant utilization. Therefore, a shortage of EUA will result in a cut of the emission producing activity or the purchase of additional allowances.

The introduction of emission allowances is obviously modifying operating costs of those sectors covered. Electric utilities, for instance, operate their power plants based in part on the price of the power and the relative cost of coal and natural gas. In a carbon constrained economy, the right to pollute is a new factor which comes into play. Now that a price has been put on such emission allowances, the differences in carbon intensity for fuels—coal and gas—could potentially change the way companies run their power plants and lead to an increased amount of emission-rights trading.

The existence of mandatory emission trading schemes in Europe and the United States, and the increased liquidity of trading on futures contracts on CO2 emissions allowances, led naturally to the next step in the development
of these markets: These futures contracts are now used as underliers for a vibrant derivative market. In this paper, we will apply GARCH model to analyze the allowance futures prices, demonstrate its calibration to historical data, and show how to price European call options written on these contracts.
Chapter 2. Literature Review

When talking about solving the climate change problem, economists tend to prefer the use of economic means. However, there are also many papers doubting the effectiveness of using market to reduce greenhouse gas emission.

There are two kinds of economics means, either by introducing the so call “carbon tax” or by using market mechanism. While the carbon tax may be fairer, it faces more political difficulty since it concerns the interests of many parties. In practice, carbon trading is a more common solution and an example might be the EU ETS.

Since the emission allowance trading is at the core of the carbon market, research of the mechanism of this trading system would be a hot topic. In this field, researchers mainly investigate the market mechanism and the trading effectiveness. The major method for allowance distribution is through auction. Though auction is more effective, it creates room for corruption and speculation which makes the market unstable.

2.1 Whether the current carbon market mechanism is beneficial for emission reduction is still controversial

Croker(1966) discussed the possibility of applying Coase's theory of property in controlling greenhouse emission. According to Croker, since the external effect of greenhouse gas emission reduction crosses the country boundaries,
such approach is difficult to apply in practice. The main barrier is how to coordinate the different interests among different countries and groups.

When it comes to the effectiveness of the current mechanism, the controversy reflects mainly in two aspects. For one thing, whether the global trade helps to reduce greenhouse emission is still in doubt. For another, due to the different stages of economic development, the costs of emission reduction vary a lot. How to allocate the cost among different countries is a problem.

Some experts think that global trading is not a good thing in terms of environmental protection. Chichilnisky (1994) holds that global trading would do harm to environment protection. He thought that due to the difference in trading structure and status, developed countries can take advantage in free trade and reduce greenhouse emission while such trading has a negative impact on the environment in developing countries. As a whole, the international trading is bad for environment protection. Copeland and Taylor (1994) used a North-South trade model to analyze the relationship between global trading and environment protection. They found that the environment standards of developed countries are obviously stricter than that of developing countries. Such differences in standard help the developed countries improve their environments through trading while exacerbate environmental degradation of developing countries.

On the other hand, Grossman and Krueger (1991) build a model to describe the impact global trading has on the environment. They divide different factors
in the model into three effects: technology, structure and scale. Based on this model, Antweiler et al (2001) used data from more than forty countries to analyze the relationship between free trade and carbon emissions and found that the positive effect of technology is greater than the negative effect of scale. Thus, Antweiler holds that global trading is good for environmental protection.

Things get more complicated when it comes to the issue of emission reduction responsibility allocation. Developed countries insist that the responsibility should be allocated in terms of greenhouse gas production while the developing countries think those who consume the end products should be responsible for that.

A study by Kondo et al (1998) found that, before 1985 Japan was a carbon export country, but the situation changed after 1990 when it turned into a carbon import country. By calculating the carbon in 24 countries` trading goods, Ahmad and Wyckoff (2003) found that international industrial transfer has a significant impact on carbon emission. Peters and Hertwich (2008) found that in 2001 global trading accounted for 5.3 billion tons of carbon emission and the developed countries are all net carbon import countries.

China is the country that has the most carbon emissions. However, a study from Li and Hewitt (2008) argued that global trading might be part of the reason. According to them, the reason why the U.K. `s domestic carbon emission reduced about 11% in 2004 is that they import China`s goods
instead of producing themselves. A study by Wang and Watson (2007) had a similar conclusion. According to their work, in 2004, 23% of China`s domestic carbon emission resulted from exports.

Based on the above, Ferng (2003) thought, the end product consumers rather than the direct carbon emitters should bear the responsibility. Schelling (1992) insisted that the cost to reduce emission in developing countries is larger than in developed countries. Thus, Schelling holds that it is unfair to force developing countries to reduce emissions and developed countries should take more responsibility. According to Schelling, developed countries can help developing countries to reduce emissions by capital and technological transfers and support.

2.2 Which method is more effective, trading or tax?

In practice, there are two ways of market-based policy instruments: the first one is emission allowance trading, which is based on the Coase's theory of property. The second one is the so-called “carbon tax” or Pigouivain tax. It is still controversial that which approach is more effective.

A Pigouivain tax is not a new thing. It refers to a tax applied to a market activity that is generating negative externalities. The tax is intended to correct an inefficient market outcome, and does so by being set equal to the negative externalities. In practice, governments would levy and collect environmental taxes, including carbon tariffs. The U.S.A. is always an advocate for an
international environmental tax.

Views that for the carbon tax are as follows:

From the angle of social welfare, Nordhaus (2006) implied that carbon tax adds to the cost of using natural resource and reduces the emission of greenhouse gases. Moreover, the emission allowance distribution system is far from perfect and can make room for corruption while a carbon tax seems fairer in this sense.

From the angle of social equality, Ellerman,A.(2005) thought carbon tax is a better approach. Ellerman mentioned that by collecting a carbon tax, those who cause more negative externalities would pay more and this would be fairer. Ellerman also discussed the political challenge since the traditional industries with dense carbon emission are politically powerful. The introduction of such tax would be a tough task.

Views that are for the emission allowance trading give the following reasons:

In term of cost saving, Montgomery (1972) gave the conclusion that emission allowance trading is the least expensive approach. Stern (2007) thought, the implementation of emission allowance trading would be easier than other alternatives, and the most effective one too. Edenhofer et al (2008) mentioned that a short-term carbon tax would be a good choice, but in long-run, considering the serious outcome of climate change, carbon emission allowance trading would be better.

From the angle of long-term corporate operation, Murray et al (2009) thought,
if the banking of emission allowances is allowed, it would be beneficial for the
corporate development while carbon tax does not has such advantage.

2.3 Study of Carbon Emission Allowance Trading System

The first attempt to use market mechanism to solve pollution problem was
made by Dales (1968). Dales thought the best and cheapest way to control
pollution is to treat the right to pollute as property. This also became the
theoretical foundation of the emission allowance market. Based on that,
Montgomery (1972) tried to use a model to describe the effectiveness of
using market to solve the pollution problem.

Stavins (1995) pointed out that transaction costs could do harm to the market,
as they would reduce both supply and demand of the emission allowance
market, resulting in a decline of volume.

Egterten and Weber (1996) discussed the trading system using game theory.
According to their work, if the cost of emission allowance is higher than the
marginal fine, a moral hazard issue would occur.

2.4 The Distribution Method of Emission Allowance

There are mainly two kinds of emission allowance distribution method: free
distribution and auction.

Pizer (2003) thought auctions could be a more effective way, but the
disadvantage is that the implementation would have to face the pressure from
the greenhouse gas emitters. Cramton and Ken (2002) discussed the disadvantage of free distribution: First, it might be unfair for the public since within the free allowance amount, firms would not pay for the negative externality they caused. Second, such a free distribution would reduce the enterprise's competition consciousness.

Eventually, experts agree on auctions being more efficient. However, in practice, if we take EU ETS as an example, free distribution is the primary way. The reason is that it would be more feasible for the early implementation since auctions are unpopular among firms and would faces political challenges. But the implementation of an auction is to be the trend. An important amendment of the European commission in 2008 decided that the main way of emission allowance distribution would be transfer through auction and by the year 2020, 80% of the allowance will be distributed through auction.

2.5 Studies of Emission Allowance Market Efficiency

By 2008, the volume of the global emission market had reached about 100 billion U.S. dollar, and research of the emission allowance market efficiency has been a hot topic. So far, the focus is on the relationship between allowance price and related commodities prices, usually the price of electricity. Oberndorfer (2009) used empirical experiment to show that the price of EUA is related to the stock prices of electricity enterprises. Zachman and von
Hirschhausen (2008) found that the rise of EUA price has a greater impact on electricity price than the decline of EUA price do. They viewed this as a sign of lacking competition.

As the price of an EUA is related to the stock price of major emitter firms, the emission allowance market does help to build a low carbon emission economy.

2.6 Empirical Studies of Carbon Financial Products

Research on spot market:

Chesney and Taschini (2008) use an endogenous model to analyze the spot price of carbon emission market. Their study showed that asymmetric information problem existed, which might make the market less efficient.

Benz and truck (2007) observed the leptokurtosis and fat tail phenomenon in the dynamic change of EUA spot price.

The prices of EUA futures recorded a tremendous fluctuation between Phase I and Phase II. Uhrig-Homburg and Wagner (2007) thought that this was because the prohibition of any unused EUA in Phase I to be stored and used in Phase II. The fluctuation of future prices led to high volatility in the spot market. Daskalakis et al (2007) mentioned that one of the most important reasons for the rise of EUA price is the uncertainty of government policies, which adds to the risk of investors. The rise of prices reflects a risk premium demanded by the investors.
Research on the relationship between spot prices and future prices:

Theissen (2009) found that when there are arbitrage opportunities, the future markets can observe more market signals, meaning the future markets are more sensitive to market information. Thus, when there are arbitrage opportunities the spot prices would converge to future prices.

Research on the pricing of carbon derivatives:

As the carbon financial market grows, new types of derivatives are put into the market. They are playing an increasingly important role in the trading system and the demand for pricing accuracy is getting higher and higher. According to Daskalakis et al (2007), the volume of EUA futures market is about 5 times higher than the volume of the EUA spot market in 2006. Also, data from ECX (2008) shows that in the first season of 2008, there were 653502 units of EUA being traded while, during the same period, the volume of EUA futures being traded was 14391000 units.

Uhrig-Homburg and Wagner (2007) found that risk-neutral pricing theory can be applied to the study of carbon derivative. Benz, E. and Truck, S. (2007) thought this was due to the difference of fluctuation and prices in different phases. As a result, the return of carbon derivative can be better described using an AR-GARCH model rather than a Auto-Regressive model and Mean Reversion model.

Also, Paolella and Taschini (2008) suggested the use of a GARCH model to solve the heteroscedasticity problem observed in data when analyzing the
return of EU ETS CO2 emission allowance trading.
Chapter 3. Methodology

3.1 Futures Pricing Theory

Futures are one of the most fundamental derivative products. The most widely used pricing model is the cost-of-carry model as follows (Equation 3.1)

\[ F_t(T) = e^{(r-c)(T-t)}S_t \]  

where \( F_t(T) \) indicate the price of contract in time \( t \), \( S_t \) indicate the spot price, \( r \) indicate risk free rate and \( c \) is the cost of carry

3.2 Option Pricing Theory

3.2.1 Black–Scholes Model

The Black–Scholes or Black–Scholes–Merton model is a mathematical model of a financial market containing certain derivative investment instruments. From the model, one can deduce the Black–Scholes formula, which gives a theoretical estimate of the price of European-style options.

The Black–Scholes model of the market for a particular stock makes the following explicit assumptions:

1. There is no arbitrage opportunity (i.e., there is no way to make a riskless profit).
2. It is possible to borrow and lend cash at a known constant risk-free interest rate.
3. It is possible to buy and sell any amount of stock, even fractional (this
includes short selling).

4. The above transactions do not incur any fees or costs (i.e., frictionless market).

5. The stock price follows a geometric Brownian motion with constant drift and volatility.

6. The underlying security does not pay a dividend.

Let

$S, t$, be the price of the stock (please note inconsistencies as below).

$V(S, t)$, the price of a derivative as a function of time and stock price.

$C(S, t)$ the price of a European call option and $P(S, t)$ the price of a European put option.

$K$, the strike price of the option.

$r$, the annualized risk-free interest rate, continuously compounded (the force of interest).

$\mu$, the drift rate of $S$, annualized.

$\sigma$, the volatility of the stock's returns; this is the square root of the quadratic variation of the stock's log price process.

$t$, a time in years; we generally use: now=0, expiry=T.

$\Pi$, the value of a portfolio.

Finally we will use $N(x)$ which denotes the standard normal cumulative distribution function,

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}z^2} dz$$
$N'(x)$ which denotes the standard normal probability density function,

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$  \hspace{1cm} (3.3)

The value of a call option for a non-dividend-paying underlying stock in terms of the Black–Scholes parameters is:

$$C(S, t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{1}{\sigma \sqrt{T-t}} \left[ \ln \left( \frac{S}{K} \right) + \left( r + \frac{\sigma^2}{2} \right)(T-t) \right]$$

$$d_2 = \frac{1}{\sigma \sqrt{T-t}} \left[ \ln \left( \frac{S}{K} \right) + \left( r - \frac{\sigma^2}{2} \right)(T-t) \right]$$

$$= d_1 - \sigma \sqrt{T-t}$$  \hspace{1cm} (3.4)

The price of a corresponding put option based on put-call parity is:

$$P(S, t) = Ke^{-r(T-t)} - S + C(S, t)$$

$$= N(-d_2)Ke^{-r(T-t)} - N(-d_1)S$$  \hspace{1cm} (3.5)

For both, as above:

- $N(\cdot)$ is the cumulative distribution function of the standard normal distribution
- $T - t$ is the time to maturity
- $S$ is the spot price of the underlying asset
- $K$ is the strike price
- $r$ is the risk free rate (annual rate, expressed in terms of continuous compounding)
• $\sigma$ is the volatility of returns of the underlying asset

3.2 GARCH model

The traditional B-S option pricing model assumes that the returns of underlying assets have a constant volatility, which is not true. As a matter of fact, the return series usually have the following properties: 1. Volatility clustering, one of the most important features of financial time series data. 2. Heteroscedasticity, assets prices are usually volatile. 3. Leptokurtosis and fat tail, returns of assets usually do not follow the normal distribution and the distributions tend to have leptokurtosis and fat tails.

First introduced by Engle (1982), the ARCH (Autoregressive Conditional Heteroskedasticity) model can solve the very problems of return series data. Based on that, Bollerslev (1986) developed the GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) model. The advantage of GARCH model is that it considers the volatility clustering feature of time series data and can smooth the leptokurtosis. Since it can accurately catch the features of time series data, the GARCH model is widely used.

A GARCH(p,q) model is as follow:

$$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j}^2$$

$$x_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = h_t^2$$

3.6
Where $x_t$ is the return series, $\mu_t$ is conditional expected value, $\varepsilon_t$ is the residue, $h_t^2$ is the conditional variance. To keep the model stationary, the following requirement should be met:

$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$$
Chapter 4. Empirical Analysis of Carbon Derivative Pricing

4.1 Data Selection

This paper uses the daily closing prices of EUA future contracts whose maturity date is December 2012. The time period of these contracts is from January 2008 to August 2012. Figure 4.1 shows a plot of daily EUA future price for the period January 1st 2008 to August 27th 2012. The data were provided by the European Environment Agency.

Figure 4.1 EUA future contract price
4.2 Processing Data

4.2.1 Analyzing the Logreturns of EUA Futures

Figure 4.2 shows a plot of the EUA future logreturns $y_t = \log S_t - \log S_{t-1}$ for the whole considered period. Figure 4.3 shows the descriptive statistics of the logreturns. The logreturns distribution has a skewness of 0.197421 and a kurtosis of 11.38327. Obviously, the data show the properties of leptokurtosis, fat tail and volatility clustering.

Figure 4.2 Logreturns of EUA future

Figure 4.3 Descriptive statistics of logreturns
4.2.2 Unit Root Test

Table 4.1 shows the unit root test of the logreturns. We can see that the ADF statistic is $-33.34020 < -3.3965816$, meaning at a 1% confidence level we reject the hypothesis that there is a unit root. Thus, the time series is stationary.

**Table 4.1 ADF test of the logreturns**

<table>
<thead>
<tr>
<th>Null Hypothesis: LOGRET has a unit root</th>
<th>Exogenous: Constant, Linear Trend</th>
<th>Lag Length: 0 (Automatic - based on SIC, maxlag=22)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>t-Statistic</td>
<td>Prob.*</td>
</tr>
<tr>
<td>Test critical values:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-3.965816</td>
<td>0.0000</td>
</tr>
<tr>
<td>5% level</td>
<td>-3.413612</td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-3.128862</td>
<td></td>
</tr>
</tbody>
</table>

4.2.3 Autocorrelation Coefficient Testing and Partial Autocorrelation Coefficient Testing

Table 4.2 shows the ACF test and PACF test results. We can see that at 15th lags level there is autocorrelation. Thus, we can use the following equation to run a LS regression:

\[ r_t = c + ar_{t-15} + \epsilon_t \]

where \( r_t \) is the logreturns at time t.

Table 4.3 shows the result of the LS regression.

\[ r_t = -0.000903 - 0.033769 r_{t-15} + \epsilon_t \]

Table 4.2 ACF and PACF test result of logreturns

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
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<td><img src="image.png" alt="Table content" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3 LS regression of logreturns
Tables 4.4 and 4.5 show the ACF and PACF tests of the residual and residual-square respectively. Obviously, there is autocorrelation in residue-square series. Figure 4.4 shows the plot of residual-square. From this figure, we can conclude that the fluctuation of $\epsilon_t^2$ has the properties of time varying and clustering. Thus, we can use the GARCH model to analyze the behavior of the logreturns.
Table 4.5 ACF and PACF test of Residual-square

Date: 08/20/13  Time: 11:27
Sample: 1/24/2008 7/16/2012
Included observations: 1170

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.122</td>
<td>0.122</td>
<td>17.458</td>
<td>0.000</td>
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</tr>
<tr>
<td>2</td>
<td>0.086</td>
<td>0.072</td>
<td>26.116</td>
<td>0.000</td>
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<tr>
<td>3</td>
<td>0.073</td>
<td>0.055</td>
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<tr>
<td>4</td>
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<td>0.106</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>0.170</td>
<td>0.141</td>
<td>84.468</td>
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<tr>
<td>6</td>
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<td>0.042</td>
<td>93.743</td>
<td>0.000</td>
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<tr>
<td>7</td>
<td>0.203</td>
<td>0.264</td>
<td>194.66</td>
<td>0.000</td>
<td></td>
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<tr>
<td>8</td>
<td>0.052</td>
<td>-0.020</td>
<td>197.62</td>
<td>0.000</td>
<td></td>
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<tr>
<td>9</td>
<td>0.081</td>
<td>0.002</td>
<td>202.22</td>
<td>0.000</td>
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</tr>
<tr>
<td>10</td>
<td>0.090</td>
<td>0.037</td>
<td>211.50</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.4 Residual-square

4.2.4 GARCH(1,1) Modeling

GARCH(1,1) model:

\[ r_t = C + \varepsilon_t \]
\[ \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \]  \tag{4.3}

where \( r_t \) is the logreturn of time \( t \). According to the requirement of GARCH model, we have \( \alpha > 0, \beta > 0, \alpha + \beta < 1 \).

Table 4.6 presents the results of GARCH(1,1) modeling. The equation of the estimation is:

\[ r_t = 0.000725 + \varepsilon_t \]

\[ \sigma_t^2 = 2.50e-05 + 0.164437 \varepsilon_{t-1}^2 + 0.810531 \sigma_{t-1}^2 \]

\( \alpha + \beta = 0.97 < 1 \) Thus, the model is stationary.

Table 4.6 GARCH(1,1) modeling

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
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<tr>
<td>C</td>
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<td>0.000916</td>
<td>1.170313</td>
<td>0.2395</td>
</tr>
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<td>LOGRET(-15)</td>
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<td>0.020238</td>
<td>0.9839</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>RESID(-1)^2</td>
</tr>
<tr>
<td>GARCH(-1)</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
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<tr>
<td>S.E. of regression</td>
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<tr>
<td>Sum squared resid</td>
</tr>
<tr>
<td>Log likelihood</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Mean dependent var</td>
</tr>
<tr>
<td>S.D. dependent var</td>
</tr>
<tr>
<td>Akaike info criterion</td>
</tr>
<tr>
<td>Schwarz criterion</td>
</tr>
<tr>
<td>Hannan-Quinn criterial</td>
</tr>
</tbody>
</table>

Table 4.7 shows the ACF and PACF test result of the GARCH(1,1) model.

From the tables we can see that compared to the original logreturns series, the GARCH(1,1) model does not have the problem of autocorrelation.
Table 4.7 ACF and PACF test of GARCH(1,1) model

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.008</td>
<td>0.008</td>
<td>0.0762</td>
<td>0.793</td>
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<tr>
<td>2</td>
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<td>0.001</td>
<td>0.0768</td>
<td>0.922</td>
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<td>3</td>
<td>-0.030</td>
<td>-0.030</td>
<td>1.1598</td>
<td>0.783</td>
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<tr>
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<td>-0.007</td>
<td>1.2341</td>
<td>0.872</td>
</tr>
<tr>
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<td>5</td>
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<td>0.011</td>
<td>1.3733</td>
<td>0.927</td>
</tr>
<tr>
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<td>6</td>
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<td>-0.004</td>
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<td>0.957</td>
</tr>
<tr>
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<td>0.034</td>
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<tr>
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<td>-0.032</td>
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<td>0.858</td>
</tr>
<tr>
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<td>0.025</td>
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<td>10</td>
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<td>-0.021</td>
<td>5.3896</td>
<td>0.964</td>
</tr>
<tr>
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<td>11</td>
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<td>12</td>
<td>-0.013</td>
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<td>5.5807</td>
<td>0.936</td>
</tr>
<tr>
<td>13</td>
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<td>0.056</td>
<td>0.057</td>
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<td>-0.011</td>
<td>9.9648</td>
<td>0.822</td>
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</tbody>
</table>
Chapter 5. Conclusion

Derivative pricing has been the core issue of financial markets since the 1970’s at least. By investigating the time series price data of the EUA futures, we found that there is an obvious leptokurtosis and fat tail phenomenon. For a better description of the time series data, a GARCH(1,1) model is used to run the empirical experiments.

The experiment results show that a GARCH(1,1) model works well in capturing the volatility of EUA futures. It has a satisfactory descriptive power on the fluctuation of the EUA futures market.

In order to make a convincing conclusion, however, a large amount of data is needed for a GARCH model. Since the EUA spot market did not exist until 2005, not to mention the EUA future market, the amount of EUA future price data is limited. Moreover, the EUA market is a new market with lots of speculation and its governance is far from perfect. All these add to the uncertainty of our model.

In spite of these shortcomings, this study of applying the GARCH(1,1) model in EUA futures pricing can lay a foundation for further research.
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