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Solar temperature at 4 GHz: An undergraduate experiment

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An experiment is described using readily available satellite television receiving equipment to obtain an approximate value for the solar (quiet Sun) temperature at a frequency of 4 GHz (7.5 cm), using a comparison method. The procedure described here appears to be quite suitable for use at the undergraduate level inasmuch as it does not require any special electronic equipment. It is found that the observed value is within 10% of the accepted value (approximately 28 000 K). The results are considered to be satisfactory, given the substantial approximations made in the performance of the experiment and analysis of the data.

I. INTRODUCTION

The ready availability (at least in North America) of domestic satellite TV receiving systems costing less than \$1500 makes it practical to consider an experimental project at the undergraduate level in which the temperature of the quiet Sun in the microwave region of the electromagnetic spectrum may be obtained with relatively little effort. Once the satellite TV system—operating as a total power radiometer—is calibrated, then simple physical arguments provide the basis for a determination of the solar temperature within the passband of the receiver system.

In this particular version of the experiment, which uses a comparison technique, no knowledge of the system characteristics, such as bandwidth, noise-figure, etc., is required. In fact, only one calibration point approximately equivalent to a blackbody at a known temperature (for example, the roof or ground just in front of the antenna) is required, along with the beamwidth of the antenna.

Evidently, substantial approximations need to be made, but these appear to be conceptually clear and acceptable.

II. THEORY OF THE EXPERIMENT

The experiment depends on several basic ideas. First, it depends on the idea that an astronomical object can be radiating electromagnetic energy, and that the intensity of this radiation, usually called its brightness, can be expressed as a function of an equivalent blackbody temperature. In the low-frequency approximation, this relationship between the brightness $B(f)$ at a given frequency f and the equivalent blackbody temperature T is usually known as the Rayleigh-Jeans law,[1] and has the form

$$B(f) = 2kTf^2/c^2 \text{ W/m}^2 \text{ Hz rad}^2, \quad (1)$$

where k = Boltzmann's constant and c = speed of light. The low-frequency approximation implies that $hf/kT \ll 1$, where h = Planck's constant.

Also, in the spirit of a first approximation, the experiment depends on the associated idea that an astronomical object of finite angular size may be assigned a uniform equivalent brightness temperature distribution. In this experiment, it is being assumed, therefore, that the quiet Sun has an equivalent blackbody temperature of 28 000 K in the neighborhood of 4 GHz, uniformly distributed over a disk with a diameter of 0.5°.[2]

Second, the experiment depends on the idea that the total electromagnetic energy collected by the antenna per unit time within the system's passband appears as electrical power associated with the antenna's output port (point A in Fig. 1), and that this electrical power is then amplified by the satellite TV receiver system (points B–D in Fig. 1), assumed to be operating as a linear amplifier.[3] In addition, it is being assumed that there is one more source of electrical power being amplified by the receiver system; namely, the noise power generated by the receiver's input stage. Hence, the total electrical power P_{tot}, within the passband of the system, appearing at the input to the detector stage (point C in Fig. 1) in the receiver is assumed to be a superposition of three components; the power associated with the Sun P_{Sun}; the power associated with non-Sun sources $P_{\text{non-Sun}}$, and the power associated with the input stage of the receiver P_{rcvr}. This may be expressed symbolically as in (2):

$$P_{\text{tot}} = P_{\text{Sun}} + P_{\text{non-Sun}} + P_{\text{rcvr}}. \quad (2)$$

It is further assumed that the last two terms in Eq. (2) are constant throughout the experiment, and thus constitute the background power output of the receiver. Hence, any change in the total power appearing at the output of the receiver's radio frequency (rf) amplifying circuits (point C in Fig. 1) is associated with P_{Sun}. It should be noted that the rf amplifier in Fig. 1 represents, in an actual satellite TV system, the low-noise amplifier (LNA), the downconverter, and a possible intermediate frequency (IF) amplifier.

P_{tot} is applied to the detector, (block 2 in Fig. 1) which is

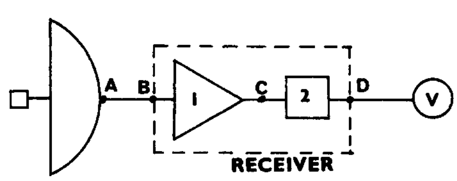

Fig. 1. Simplfied block diagram of the satellite TV system. The output of the antenna appears at point A. Block 1 includes the LNA, downconverter, and any additional amplifiers up to the square-law detector in block 2. The output of the detector goes to a voltmeter.

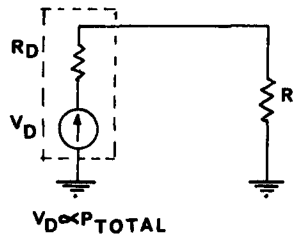

Fig. 2. Simplified equivalent circuit of the square-law detector circuit, consisting of an equivalent constant-voltage generator V_D, an equivalent series resistance R_D, and an external load resistance R. The voltmeter shown in Fig. 1 connects across R.

usually a semiconductor diode operating in the square-law mode,[4] This implies that the current in the diode and a series resistance R (usually called a load resistance) is linearly related to the power being rectified.[5] Hence, a voltmeter connected across R will, in fact, represent P_{tot}. Figure 2 shows a highly simplified Thevenin equivalent circuit of the diode and an associated load resistance R. If the antenna is not viewing the Sun, then the voltmeter deflection is proportional to a background power level only. When, however, the antenna is directly viewing the Sun or other object of interest, then the amount by which the voltmeter deflection changes represents P_{Sun} or the power associated with the object of interest.

From another point of view, since the detected power representing the Sun or any other object of interest is associated with an equivalent blackbody temperature on the basis of the Rayleigh–Jeans law mentioned above, then the voltmeter indication could be interpreted in terms of temperature rather than power. From this point of view, the

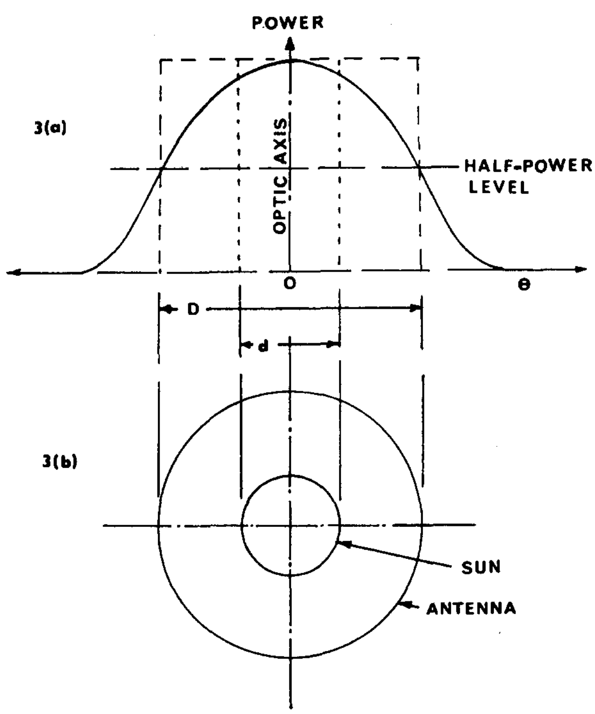

Fig. 3.(a) Schematic diagram of an idealized antenna beam pattern, showing power as a function of the angle θ on either side of the optic axis. The broken outline of diameter D represents the equivalent antenna beam pattern obtained from the half-power points, and the broken outline of diameter d represents the angle subtended by the Sun at the antenna. The antenna is assumed to have negligible side lobes. (b) Schematic diagram of the solar disk superposed on the disk defined by the antenna's equivalent beamwidth.

voltmeter deflection represents an equivalent blackbody temperature. But now the question is how is this blackbody temperature related to the equivalent blackbody temperature of the Sun or other object of interest?

The answer to this question is based on the idea that the equivalent blackbody temperature associated with the power being collected by the antenna—that is, the antenna temperature T_{ant}—will be numerically equal to the equivalent blackbody temperature of the object being viewed if the solid angle subtended by the object at the antenna is equal to or larger than the solid angle associated with the antenna's field of view. Otherwise, it is being assumed that T_{ant} will be some fraction of the object's equivalent blackbody temperature T_{obj}. This idea may be expressed symbolically in the form of an equation:

$$T_{ant} = T_{obj}\,(d/D)^2\ \mathrm{K}, \tag{3}$$

where d = angular diameter of the object and D = angular diameter of the antenna's field of view. The antenna's field of view is related to its beamwidth, defined as the angular separation between half-power points, as indicated in Fig. 3(a).[6] In the experiment, d is the angular diameter of the Sun (taken to be 0.5°), and D is obtained in a straightforward manner, as described in Sec. III.

III. EXPERIMENTAL PROCEDURE

First, it is necessary to have a means of measuring the satellite TV receiver's signal-strength voltage (point D of Fig. 1). This voltage is found at a point in the actual circuit ahead of the limiting amplifiers, where it varies with the total received power P_{tot}. It would appear that most satellite TV systems come equipped with a so-called "tuning meter" or "signal strength meter," corresponding to the voltmeter indicated in Fig. 1. In these particular systems, it suffices to use the meter already present in the system, or to connect a more convenient version—with an input resistance in the order of 1 MΩ or more—across the built-in meter. Typically, the background voltage, corresponding to $P_{non\text{-}Sun} + P_{rcvr}$, will be a fraction of a volt, whereas the "signal" voltage, corresponding to P_{Sun}, is 20 mV or so.[7] Hence, the voltmeter needs to resolve at least about 1 part in 10. If the receiver system does not have a built-in meter of the type mentioned above, then the external voltmeter is to be connected at a test point in the circuit associated with installation of the system. Information regarding the location of this test point is usually readily available from the vendor of the system.

Second, the beamwidth of the antenna is determined by obtaining the angular separation between the antenna positions associated with the half-power points, as indicated in Fig. 3(a).[8] Since the receiver is assumed to be operating as a square law detector, then the signal voltage is linearly related to the power incident on the antenna, and the half-power points are those two antenna positions at which the signal voltage has fallen off by 50% from its maximum value. It should be noted that this procedure for determining beamwidth presupposes that the angular diameter of the radiation source is small relative to the antenna beamwidth. For a typical satellite TV antenna, with a physical diameter of approximately 2 m, the ratio d/D is approximately 0.3, assuming that the effective diameter (which takes into account such things as illumination efficiency, etc.) is not much less than the physical diameter. This is considered to be small enough.

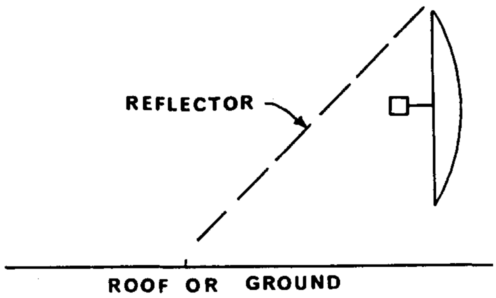

Fig. 4. Schematic diagram of a configuration to obtain the temperature of the roof or ground just in front of the antenna. This provides a calibration temperature, and is taken to have a nominal value of 300 K.

Table I. Results obtained on three consecutive days as indicated. The temperature of the roof T_{cal}, was taken to be 300 K; $D = 2.8°$; $d = 0.57°$ (see Ref. 2). Results rounded off to two significant figures.

Date (1984)	V_{Sun} (arbitrary units)	V_{cal} (arbitrary units)	T_{Sun} (K)
December 28	7.4	1.8	30 000
December 29	7.4	1.7	32 00
December 30	7.8	2.2	26 000
		Average	29 000

Third, the system is calibrated by noting the change in the output voltage as the antenna is moved from the sky, known to have a temperature in the order of only several degrees kelvin at this wavelength,[9] to the roof or the ground just in front of the antenna. It is being assumed that the roof or gound is at approximately 300 K. This measurement may be accomplished by providing a plane reflecting surface (constructed of metal screen, for example), in front of the antenna, as shown in Fig. 4. Presumably, the roof or ground should be dry to more closely approximate a blackbody. The screen material for the reflector should be such that the holes or openings are not much larger than 1/10 of a wavelength.

It appears plausible that a building clad with concrete, brick, etc. should also be an adequate equivalent to a blackbody for the purposes of this experiment. In particular, a building which would completely fill the antenna's field of view would be most convenient, since the signal thus obtained could be taken as directly representing the radiometer's response to a blackbody at 300 K, for example. In the event that such a building were not at hand, then one which only partly fills the antenna's field of view could be used, taking into account the fact that the antenna aperture is not being completely filled. However, care would need to be exercised to ensure that there were no extraneous signals reaching the antenna from the direction of the building, such as reflections of the Sun or artifical signals in this frequency band.

Taking into account the various steps outlined above, the temperature of the sun will be given by

$$T_{Sun} = Y\, T_{cal}\; V_{Sun}/V_{cal}\;\; \text{K}, \tag{4}$$

where $Y = (D/d)^2$, T_{cal} is the calibration temperature (ordinarily, 300 K), V_{Sun} is the signal voltage when the antenna is pointing directly at the Sun, and V_{cal} is the signal voltage when the antenna is pointing directly at the surface (roof or ground) taken to be the reference temperature.

IV. RESULTS

The data in Table I represent average values obtained from observations extending over several days. In fact, it is desirable to repeat the experiment over several days inasmuch as the solar temperature at these frequencies can increase noticeably for a duration of at least several minutes, whereas the objective of this experiment is to verify the temperature at 4 GHz for the "quiet" Sun. Inspection of the results in Table I indicates that the observed temperature of the Sun at 4 GHz, averaged over three days, is well within 10% of the published value; approximately 28 000 K for the quiet Sun.

It will be noticed that the observed equivalent temperature of the Sun at 4 GHz (29 000 K) is far larger than the equivalent temperature of the Sun at optical frequencies (approximately 6000 K), which illustrates that the Sun is not a simple blackbody source. In fact, the source of the solar microwave radiation (electrons in the corona) can be described as a source with an equivalent temperature as given above.

V. CONCLUSION

In view of the objective of this experiment, which is to perform a relatively simple experiment at the undergraduate level to measure the temperature of the quiet Sun at 4 GHz, and in view of the relatively good agreement between the observed and expected values for this temperature, it is concluded that the experiment as described does, indeed, constitute a useful undergraduate exercise.

ACKNOWLEDGMENT

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[1] John D. Kraus, *Radio Astronomy* (McGraw-Hill, New York, 1966), p. 85.

[2] W. Allen, *Astrophysical Quantities* (Athlone, University of London, London, 1973), 3rd ed., p. 192. The value of the solar temperature at 4 GHz for the quiet Sun is taken to be 28 000 K and was obtained by interpolating the data given in this reference.

[3] The Rayleigh–Jeans law, along with the bandwidth of the receiver system and the effective aperture of the antenna could be used to calculate the antenna temperature, and eventually, the equivalent temperature of the Sun in this part of the spectrum. However, the approach adopted in this experiment is to calibrate the system in terms of a known temperature and then to measure the temperature of the Sun.

[4] From a semiquantitative point of view, inspection of a typical current-voltage diagram for a diode, in the forward-bias region, suggests a parabolic curve, thereby providing a semiquantitative basis for calling the device a "square-law" detector. For a more quantitative discussion of square-law detection, see, for example, R. W. Landee, D. C. Davis, and A. P. Albrecht, in *Electronic Designer's Handbook* (McGraw-Hill, New York, 1957), pp. 7–104.

[5] This follows from the fact that the power associated with a resistance is proportional to the square of the voltage across that resistor. Hence, the voltage appearing at the input to the diode detector (point C in Fig. 1) is proportional to the square root of the power there. Since the square of the

current in a semiconductor diode is approximately proportional to the forward bias voltage (square-law detection), for relatively small values of forward bias voltage, and since this current appears in the load resistor, then the voltage across the load resistor is approximately proportional to the square of the voltage appearing at the input to the diode detector.

[6]For a relatively detailed discussion of the field distribution associated with a continuous aperture (applicable to a circular dish) including some discussion of the Reciprocity Theorem (enabling the dish to be considered either as a transmitting or receiving antenna), see Kraus, *Radio Astronomy, op.cit.* p. 165 ff; John D. Kraus and Keith R. Carver, *Electromagnetics* (McGraw-Hill, New York, 1973), 2nd ed., p. 668 ff. For a more explicit treatment of the circular dish, including an expression for the beamwidth, see Samuel Silver, Ed. *Microwave Antenna Theory and Design* (Boston Technical, Boston, MA, 1965), p. 192 ff.

[7]It should be noted that all three terms in Eq. (2) represent noise in the sense that each term represents a random temporal distribution. Hence, the terms are uncorrelated and are taken to combine as indicated. The noise power from the Sun is being called "signal" power on the basis that it is the significant quantity in the experiment.

[8]The beamwidth of the antenna could, in principle, be calculated, but would require a detailed knowledge of the antenna system, especially of the illumination of the dish by the feed horn. For further discussion of this point, see Kraus, *Radio Astronomy, op.cit.* p. 212 ff.

[9]See Kraus, *Radio Astronomy, op.cit.* p. 237. Note: this section of the book is by Martti E. Tiuri.

Zero angular momentum turns

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Simple explanations are given of three examples of zero angular momentum turns. Humans and cats free of external torques can use internal forces to move parts of their bodies to produce angular momenta $\mathbf{L}_1$ and $\mathbf{L}_2$ of the parts, whose vector sum may not be zero. Conservation of angular momentum then requires an $\mathbf{L}_3$ of the whole body so that $\mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3 = 0$. This can produce a rotation or partial rotation of the body in the absence of any external torque or initial angular momentum.

I. INTRODUCTION

There seems a curious lack of clear and simple explanations of zero angular momentum turns, or orientational changes, which can readily be produced, for example, by falling cats, or astronauts, or divers or trampolinists. Some physicists seem to insist, quite incorrectly, that such midair turns are not even possible! Kenneth Laws,[1] in a recent article in Physics Today on "The Physics of Dance" even says

"For example, no matter how much a dancer may wish to leap off the floor and *then* start turning (say for a *tour jeté*—that is, a turning leap), the law of conservation of angular momentum absolutely prevents such a movement."

Jearl Walker[2] also says

"In jeté en tournant, a turning leap, the dancer jumps into the air with no apparent spin about her vertical axis, yet near the top of the leap she begins to rotate. Impossible. One of the firm laws of physics is that the angular momentum of an object remains constant unless a torque acts on the object. If the dancer was not spinning when she left the floor, she cannot begin to spin in midair."

Walker claims

"The explanation of the illusion is that the dancer does have a small spin at the beginning of the leap ... too slight for an observer to notice it."

It seems as though Laws and Walker only believe angular momentum can be conserved when it is not zero!

Of course the photographic evidence proving that cats, gymnasts, divers, and astronauts, can change their orientation in midair even when they have no initial angular momentum, and no external torques, is readily available. The detailed explanations of such rotations or partial rotations merely require a consideration of the conservation of vector angular momentum at a value of zero, so that both $\mathbf{L} = 0$ and $d\,\mathbf{L}/dt = 0$.

Here are three such cases, where internal forces can produce orientational changes, or turns, while no external torques are applied.

II. HUMAN BODY ROTATION FORWARD OR BACKWARD

If a nonrotating person, free of external forces and torques with arms extended straight out to the sides, starts moving his arms in forward circles as if swimming the butterfly stroke, the arms acquire a vector angular momentum to his left. To conserve vector angular momentum at its original value of zero, the entire body must acquire an equal and opposite angular momentum to his right. This requires a head-backward feet-forward body rotation. This rotation of course only lasts as long as the arms are "paddled." If the sense of rotation of the arms is reversed the sense of rotation of the body will also reverse.[3] (So if a diver had enough time in the air, he could do backward somersaults followed by forward somersaults.)

