Possibility of enlarged core structure of $N = 15$
neutron-rich nuclei

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Abstract

The recently measured interaction cross section data for $^{22}\text{N}$, $^{23}\text{O}$, and $^{24}\text{F}$ in a $^{12}\text{C}$ target at relativistic energies have been analysed in a few body Glauber model approach. Conventional fixed core-plus-neutron model for halo nuclei is unable to explain the observed enhanced cross section for these nuclei by any selection of the neutron orbitals. Microscopic calculations like many-body Monte Carlo shell model, relativistic mean field theory and cluster model are also shown to fail in describing the large difference in interaction cross section between $^{22}\text{O}$ and $^{23}\text{O}$. A possibility of core enlargement is suggested in these nuclei. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Study of nuclei far from the stability line has revealed new structure features, thereby modifying our conventional concept of nuclear shell structure. The most striking discovery in this field was the existence of nuclear halo structure [1,2], a phenomenon quite different from the matter distribution of the stable nuclei. A halo nucleus is considered to be composed of a core with one or two loosely bound neutrons tunneling out at distances far away from the core [3]. The structure of halos are usually analysed by the “core-plus-halo neutron(s)” model. The core nucleus in this model is always assumed to have the same size as the bare nucleus that forms a core. Thus the core nucleus has a radius which follows the continuous increase following an isotopic chain.

The sudden abrupt increase in radius for the next isotope (which we refer to as the halo nucleus) in such a model is thus attributed to the large relative distance of the one or two valence neutrons from the core. Since, only one or two neutrons are included in a halo, the density outside the core nucleus is small in magnitude but stays for a long distance giving rise to a thin large diffused surface which is the origin of the halo characteristic.

Experimentally, such a structure manifests itself in several observables like the interaction cross section ($\sigma_I$), the longitudinal momentum distribution ($p_{\parallel}$), the one or two neutron removal cross section ($\sigma_{-n}$). An abrupt enhancement of the interaction cross section of a nucleus compared to its preceding isotope neighbours can be a signature of such a halo structure. The observation of large interaction cross section indicates a larger root-mean-square (r.m.s.) radius for these nuclei deviating from the usual $r_0A^{1/3}$ scaling. Particu-
lar examples of such isotopes include $^{11}\text{Be}$, $^{11}\text{Li}$, $^{19}\text{C}$, $^{22}\text{N}$, $^{23}\text{O}$, $^{24}\text{F}$, nuclei and are presented in Fig. 1.

The probability of this quantum tunneling of the valence nucleons is enhanced by the fact that the valence nucleons in these nuclei usually occupy low angular momentum states like the $2s_1/2$-orbital. Thus the absence of centrifugal barrier for the $s$-orbital facilitates the extension of the tail of the wavefunction compared to the usual occupation probability of the $d$- or $p$-orbital, coupled to the fact that usually for the nuclei near the drip line the one/two neutron separation energy is also quite small.

The usual core-plus-$n$ halo model has been found to be very successful in describing consistently the interaction cross section and momentum distribution of the core fragment for low $Z$ nuclei up to $Z = 5$ [4]. To illustrate this, the case of $^{11}\text{Be}$ ($^{10}\text{Be} + n$) is discussed later. This model can also explain the experimental cross sections for isotopes like $^8\text{He}$ which are expected to have more complex structure [5–7] as well.

Fig. 1. The experimental interaction cross section for Be, Li, C, N, O, F isotopes as a function of the mass number. The upward arrow marks on the X-axis indicate the nuclei having unusually large cross sections.
as neutron skin nuclei like the Na isotopes (see inset Fig. 2 of Ref. [8]). As seen in Fig. 1 the $N = 15$ nuclei, $^{22}\text{N}, ^{23}\text{O}, ^{24}\text{F}$, show a large increase in the interaction cross section. This observation leads us to consider the existence of core (usual size)–plus–n structure for these nuclei. However, beyond our expectation even $100\%$ occupancy of the neutrons in the $s$–orbital fails to explain the observed enhancement of $\sigma_I$ in these nuclei. This anomalous increase of $\sigma_I$ suggests the possibility of a new structure for high $Z$ neutron-rich nuclei. This Letter suggests, that the core nucleus surrounded by the valence nucleons is enlarged compared to the “free” core nucleus. Such a core enlargement may suppress the halo formation. It maybe mentioned here that the core polarisation phenomenon has been discussed for the $^{8,11}\text{Be}$ [2,9] and $^{17,19}\text{C}$ [10] and could cause an enlargement of the core. However, this is not necessarily true, as in Ref. [2] a shrinkage of $^7\text{Be}$ core in $^8\text{B}$ is suggested. The effect of core polarisation depends on the excitation energy, which is quite small for the $2^+$ states of $^{16,18}\text{C}$ but is rather high, $\sim 3.2$ MeV [11], for $^{22}\text{O}$. For similar reason quite expectedly the effect of core polarisation for $^{13,15}\text{C}$ is rather small. In this context it is pertinent to mention that the ground state configuration of $^{11}\text{Be}$ can be understood without consideration of any enlargement of the $^{10}\text{Be}$ core whose $2^+$ state is at 3.36 MeV. Thus, our suggested enlarged core structure may not be due to core polarisation only. Further experimental investigation may shed more light on the possible reason.

Densities of the $^{23}\text{O}$ nucleus derived from many body Monte Carlo calculations [12] are unable to explain the observed rise of the cross section as shown in Fig. 1. It is seen that in the even $Z$ case ($^{11}\text{Be}, ^{19}\text{C}, ^{23}\text{O}$), the rise of cross section is a rather abrupt increase, with the next isotope neighbour not showing much more increase in cross section. However, for the odd $Z$ case ($^{22}\text{N}, ^{24}\text{F}$) the cross section shows a continuously increasing trend even for the next isotope. This feature is probably a reflection of the effect of pairing interaction.

2. Analysis

The present work firstly analyses the available interaction cross section and momentum distribution data in a few body Glauber model approach [15]. In a “core–plus–neutron” model the total wavefunction of the nucleus is expressed as

$$\psi = \sum_{ij} \psi_{\text{core}}^i \phi_0^j,$$

where $i, j$ denote the different configurations for the core nucleus and the valence neutrons, respectively. The $\sigma_{\text{reac}}$ in such framework is given by

$$\sigma_{\text{reac}} = \int db \times \left[ 1 - |\langle \phi_0 | \exp \left[ i \chi_{FT}(b) + i \chi_{nT}(b + s_1) \right] |\phi_0 \rangle |^2 \right],$$

where $s_1$ and $t$ are the perpendicular component of the distances of the halo neutron and target, respectively, from the co-ordinate origin, $b = b - \frac{1}{\beta} s_1$, where $M$ is the mass of the nucleus, $\phi_0$ is the wavefunction of the valence neutron. The phase shift functions are given by:

$$i \chi_{FT}(b) = -\int ds dt T_F(s)T_T(t) \Gamma(b + s - t),$$

$$i \chi_{nT}(b) = -\int dt T_T(t) \Gamma(b - t);$$

$$\Gamma(b) = \frac{1 - i \alpha}{4\pi \beta^2} \sigma_{\text{tot}}^{(NN)} \exp \left( -\frac{b}{2\beta^2} \right)$$

is the profile function. The parameters $\alpha$ and $\sigma_{\text{tot}}^{(NN)}$ have been taken from Ref. [16]. We adopt a zero range calculation here, thus finite range parameter, $\beta = 0.0$. The thickness functions,

$$T_F(s) = \int \rho_F(s, z) dz,$$

$$T_T(t) = \int \rho_T(t, z) dz,$$

for the fragment and target, respectively, are defined through the respective densities, $\rho_F(s, z)$ and $\rho_T(t, z)$.
The longitudinal momentum distribution for one neutron halo nucleus is given by [17]

\[
\frac{d\sigma}{dp} = \frac{1}{2\pi} \int d\mathbf{r} \int d\mathbf{r}' \phi_0^*(\mathbf{r}_\perp, z') \phi_0(\mathbf{r}_\perp, z) \exp(ik_z(z - z')) \int d\mathbf{b} D(\mathbf{b}, \mathbf{r}_\perp)
\]

(2)

where,

\[
D(\mathbf{b}, \mathbf{r}_\perp) = \exp\left\{-2i\chi_{FT}\left(\mathbf{b} - \frac{m}{M + m}\mathbf{r}_\perp\right)\right\} \times \left[1 - \exp\left\{-2i\chi_{FT}\left(\mathbf{b} - \frac{m}{m + M}\mathbf{r}_\perp + \mathbf{r}_\perp\right)\right\}\right]
\]

(3)

is the distorting function expressing the reaction dynamics.

The ingredients required for evaluation of these quantities are the density distribution of the core and the wavefunction for the valence nucleon. The core is considered to have a harmonic oscillator type of density distribution. At first the oscillator width is adjusted to reproduce the experimental interaction cross section for the bare core nucleus. It should be noted that in doing so, one also includes possibility of that nucleus being deformed. This, however, assumes that the core size is same as the bare nucleus. The valence neutron wavefunction is obtained by solving the eigenvalue problem of the neutron bound in a potential which is constructed by folding the density distribution of the core with an effective nucleon–nucleon interaction. The modified Seyler–Blanchard interaction [18] having density and isospin dependence is chosen for this purpose. The depth of the potential is adjusted to reproduce the separation energy of the halo neutron.

2.1. Analysis of $^{11}$Be

In this work we focus our attention to the possible one neutron halo cases. $^{11}$Be, is a well established one-neutron halo nucleus. Thus, we first examine this nucleus within our framework to understand its structure and check the consistency from such analysis with other experimental observations.

The ground state spin parity of $^{11}$Be is experimentally known to be $1/2^+$, contrary to the normal expectation of $1/2^-$. This means that the last neutron is placed in the $2s_{1/2}$ orbital with the core $^{10}$Be in the $0^+$ ground state. There could however be a possibility that the core exists in the first $2^+$ state ($E_{\text{ex}} = 3.368$ MeV) with the neutron in the $1d_{5/2}$ orbital. Thus we consider a mixture of the $0^+; 2s_{1/2}$ and $2^+; 1d_{5/2}$ configurations with varying contribution of the $s$-wave for the halo neutron to evaluate the interaction cross section and momentum distribution. The neutron wavefunction with core in excited state is calculated so as to reproduce the relevant separation energy (i.e., the ground state $S_n^+$ excitation energy). The density of the core is considered to be same for the $0^+$ and the $2^+$ states.

Fig. 2(a), shows that the experimental $\sigma_I$ [4] for $^{11}$Be + $^{12}$C interaction at $E_{\text{lab}} = 790$ A MeV, favours 80% to 100% $s$-wave component for the ground-state within the usual core-plus-$n$ halo model. The analysis of the longitudinal momentum distribution data for $^{11}$Be + $^9$Be at $E_{\text{lab}} = 63$ MeV [19], see Fig. 2(b), also supports an 80% ground state structure for this nucleus, and is thus consistent with the observations from the interaction cross section. The conclusion from this analysis for the ground state configuration of $^{11}$Be is a dominant $0^+; 2s_{1/2}$ configuration with up to 20% core excited ($2^+; 1d_{5/2}$) admixture.

This is in agreement with the observation of spectroscopic factors from the transfer reaction studies of this nucleus by Fortier et al. [20], and also from the momentum distribution measurement by knockout reaction by Aumann et al. [21].

2.2. Analysis of N, O, F isotopes

Motivated by the success of the "core-plus-neutron" model for the light one neutron halo nucleus, we now explore the structure of the the $N = 15$ neutron rich N, O, F isotopes using the recently available experimental interaction cross section data [8,22] for a $^{12}$C target at incident energies of 967 A MeV, 961 A MeV, 1004 A MeV, respectively. As discussed earlier (Fig. 1), the experimental $\sigma_I$ data show an interesting enhancement for $^{22}$N, $^{23}$O, and $^{24}$F isotopes.

The previous experience with lighter neutron-rich nuclei, in such a situation leads us to consider a formation of halo structure in these nuclei. However, we would like to note here is that the single-neutron separation energy for these nuclei exceeds 1 MeV ($S_n = 1.27$ MeV for $^{22}$N, $S_n = 2.75$ MeV for $^{23}$O, $S_n = 3.72$ MeV for $^{24}$F).
The ground-state structure for these nuclei still awaits experimental investigation, we consider two possible location for the valence neutron, namely, the $2s_{1/2}$ orbital (expected from normal shell model ordering) and the $1d_{5/2}$ orbital.

The density of the core nuclei, $^{21}$N, $^{22}$O, $^{23}$F, are considered to be of harmonic oscillator type. As a starting point the r.m.s. radii of the core nuclei are considered to be the same as the bare $^{21}$N, $^{22}$O, $^{23}$F nuclei, which reproduces the experimental $\sigma_I$ data [8,22] for these nuclei. This consideration is in accordance with the usual core-plus-n model.

Fig. 3 represents the prediction of $\sigma_I$ by the few body Glauber model calculation with the above discussed possible neutron configurations. Results of the usual core-plus-n model that is represented as the \( \Delta r = 0 \) point in the figure, underpredict the cross section in all the possible configurations for the $^{22}$N, $^{23}$O, $^{24}$F. It thus shows that any combination of these configurations cannot reproduce the experimental cross section.

This deviation from the so far accepted picture of neutron-rich nuclei thus makes it additionally challenging to study the structure of these nuclei. Since s-wave contribution gives the largest cross section, we have no way to explain the observed interaction cross section with the present halo model. It may be mentioned here that even consideration of two neutrons occupying the s-orbital fails to explain the observed enhancement of $\sigma_I$.

Description of $^{23}$O with many-body Monte Carlo shell model densities considering $^{16}$O core + multi neutrons is unable to reproduce the observed $\sigma_I$ (Fig. 4(a)). Cross sections with densities derived from cluster-model calculations based on AMD [14] also fall below the experimental observations (Fig. 4(b)). The relativistic mean-field theory densities [13] with pairing also cannot explain the amount of rise of experimental cross section from $^{22}$O to $^{23}$O. The important aspect to note is that all these different microscopic models predict a smooth increase if $\sigma_I$ along the Oxygen isotopic chain showing no abrupt rise at $^{23}$O.

As a new possibility to explain the observed interaction cross section, we propose here the occurrence of an enlarged core structure. To examine the effect of core enlargement we increase the r.m.s. radius of the core by an amount $0 \leq \Delta r \leq 0.5$ fm by changing the harmonic oscillator width. In each case, the valence neutron wavefunction was constructed by folding the nucleon–nucleon interaction cross section with the relevant increased core density. The results of the calculation are shown in Fig. 3. From the figures one finds that a minimum increase of 0.2 fm in the core r.m.s. radius is capable of explaining the observed magnitude. One of the reasons for core enlargement could be the existence of the core in an excited state and related to core polarisation. However, the $2^+$ state of $^{22}$O is found to be at 3.2 MeV [11], which is quite high for core polar-
Fig. 3. (a) $^{22}\text{N} + ^{12}\text{C}$, (b) $^{23}\text{O} + ^{12}\text{C}$, and (c) $^{24}\text{F} + ^{12}\text{C}$ interaction cross section data [8,22] are represented by the horizontal lines. The shaded region shows the $1\sigma$ error of the experimental data. The solid (dashed) lines indicate the Glauber model calculation considering the valence neutron to be in the $2s_{1/2}$ ($1d_{5/2}$) orbital.

Fig. 4. The experimental interaction cross sections (squares) of the Oxygen isotopes [8,22]. Calculated cross sections with densities derived from (a) Monte Carlo shell model, where circles/triangles/rhombus represent cases with two neutrons in $2s_{1/2}$ orbital + ($x-2$) neutrons in $1d_{5/2}$ orbital/one neutron in $2s_{1/2}$ orbital + ($x-1$) neutrons in $1d_{5/2}$ orbital/x neutrons in $1d_{5/2}$ orbital; (b) cluster model based on AMD (circles); (c) relativistic mean field theory with pairing blocking (circles).
isation to affect the core so strongly. On the other hand a low lying monopole strength could exist, but is still just a conjecture. A similar situation of core enlargement with core in the excited state has been suggested for the $^{19}$C nucleus in Ref. [23].

3. Conclusion

In conclusion, the present analysis indicates that the usual core-plus-$n$ model for neutron rich nuclei with the core nucleus remaining same as the bare nucleus is successful in explaining structure of light neutron rich nuclei up to $Z = 5$. But as we enter the $s$–$d$ shell region the scenario possibly changes with a possibility of core enlargement coming up. The reason for the enlarged core has to be studied in more detail. The possibilities include excitation of the core nucleus, if some very low lying monopole or quadrupole excitation is present, or maybe some cluster structure which also needs further experimental and theoretical investigation.

References