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Identification of the $g_{\frac{9}{2}}$ proton and neutron band crossing in the $N = Z$ nucleus ^{76}Sr

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High-spin states in ^{76}Sr have been studied using Gammasphere plus Microball detector arrays. The known yrast band has been extended beyond the first band crossing, which involves the simultaneous alignment of pairs of $g_{\frac{9}{2}}$ protons and neutrons, to a tentative spin of $24\hbar$. The data are compared with the results of cranked relativistic mean-field (CRMF) and cranked relativistic Hartree-Bogoliubov (CRHB) calculations. The properties of the band, including the $g_{\frac{9}{2}}$ proton/neutron band crossing frequency and moments of inertia, are found to be well reproduced by the CRHB calculations. Furthermore, the unpaired CRMF calculations show quite good agreement with the data beyond the band crossing region, indicating that pairing is weak at these frequencies. The high spin results suggest that there is little evidence for an isoscalar ($t = 0$) *np* pair field. Moreover, a systematic study of the band crossings in even-even $N = Z$ nuclei for the first time reveals that there is no evidence to support the existence of the Coulomb antipairing effect caused by the Coulomb exchange term.

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Much debate has ensued in recent years over whether or not there is any significant delay observed in the $g_{\frac{9}{2}}$ proton and neutron band crossing frequencies in $N = Z$ nuclei. Early experimental observations (see Refs. [1–4]) and theoretical calculations [5–8] on medium-heavy even-even $N = Z$ nuclei suggested that a delayed alignment (paired band crossing) of the $g_{\frac{9}{2}}$ proton and neutron pairs, compared to their more neutron-rich neighbors, may be a signature of neutron-proton (*np*) pairing correlations. This resulted in a debate about whether or not the delayed alignments, reported at that time, were due entirely to an isovector ($t = 1$) *np*-pair field [5] or whether there was also a strong isoscalar ($t = 0$) component involved [6–8], and to what extent each of these components was responsible for the delays in crossing frequency that were reported in the literature. It is also important to recognize that all of the theoretical predictions in the papers quoted above ignore effects related to shape changes as a function of angular momentum and thus they have to be treated with considerable caution, since such changes are not uncommon [9].

The theoretical description of the $N \approx Z$ nuclei is extremely difficult: in the general case, the isovector and isoscalar *np* pairing as well as isospin symmetry conservation have to be taken into account (see discussion in Ref. [9]). On the mean-field level, the symmetry breaking in the case of *np* pairing and isospin can be small, which may require the exact methods of symmetry restoration by projection techniques. However, such symmetry-unrestricted mean-field calculations within an isospin-conserving formalism will not be available in the foreseeable future. In view of this situation it is essential to try and identify the most important correlations that are responsible for the observed features of the $N = Z$ nuclei. The isovector mean-field theory of Ref. [5] represents an attractive choice for such a study. This theory assumes that there is no isoscalar *np* pairing, but takes into account isovector *np* pairing and isospin symmetry conservation. Thus, this theory eliminates the problem of including the isoscalar *np* pairing channel from the description of the $N = Z$ rotating nuclei. This is important since in contrast to the isovector *np* pairing, the strength of which is defined by the isospin

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conservation [5], there is no well justified procedure on how to define the strength of the effective isoscalar *np* pairing interaction [9]. The fact that isovector mean-field theory takes into account the isospin symmetry conservation is its clear advantage because this aspect is ignored in other studies.

A major advantage of the isovector mean-field theory of Ref. [5] lies in the fact that standard mean-field models with only $t = 1$ like-particle pairing can be employed. The fundamental modification of these theories lies in adding the isorotational energy term $T(T+1)/2J_{\mathrm{iso}}$ to the total energy. However, because all low-lying rotational bands in even-even $N = Z$ nuclei have isospin $T = 0$, this term vanishes in the excitation energies of such nuclei. A systematic investigation of rotational bands in the $N \approx Z$ nuclei of the $A = 58$–80 mass region using this approach shows that it provides a successful description of the rotational properties of these nuclei (see Ref. [9] and references quoted therein).

One of the strongest experimental cases for the observation of a delayed alignment of the $g_{\frac{9}{2}}$ protons and neutrons in an even-even nucleus was believed to be ^{72}Kr [2,3], where a significant delay was initially reported. However, the latest experimental work on this nucleus has revealed that there are in fact two parallel even spin bands at high spin [10–12]. The new band shows a much lower frequency for the paired band crossing, suggesting that in fact there is little or no delay in the alignment frequency compared to both heavier even Kr isotopes and to calculations for this nucleus which include standard $t = 1$ neutron and proton pairing [9]. Thus, at the present time there are only two nuclei, ^{60}Zn and ^{72}Kr [9,12,13], where a reliable interpretation of the $g_{\frac{9}{2}}$ proton and neutron band crossing is possible because the effects of shape coexistence are not that pronounced in these cases. The present study of ^{76}Sr provides only the third such case, furthermore, it is the heaviest $N = Z$ system to be studied and as such yields an important additional test of isovector mean-field theory.

The investigation of excited states in medium-heavy $N = Z$ nuclei in the $A \sim 80$ region presents a major experimental challenge both because of the very low production cross-sections and because of the necessity to use ^{40}Ca targets, which readily oxidize and therefore yield very high γ-ray count rates from this contaminant. The identification of the highest spin states in such nuclei is therefore extremely difficult and requires very sensitive detection capabilities. Experimentally, excited states in ^{76}Sr have been previously observed up to spin 14 and evidence for a modest delayed alignment of the $g_{\frac{9}{2}}$ proton and neutron pairs was suggested [2]. However, the previous data do not extend beyond the band-crossing region hence they do not allow the extraction of the crossing frequency with any degree of certainty. In the present work we have studied the high spin states in ^{76}Sr and been able to extend the yrast levels up to spin $22\hbar$ (tentatively $24\hbar$), which goes beyond the $g_{\frac{9}{2}}$ band-crossing region for the first time. The new data has allowed the band crossing frequency to be extracted and the results (crossing frequency and moments of inertia) to be compared with the cranked relativistic Hartree-Bogoliubov calculations [9]. The data are found to agree very well with the calculations.

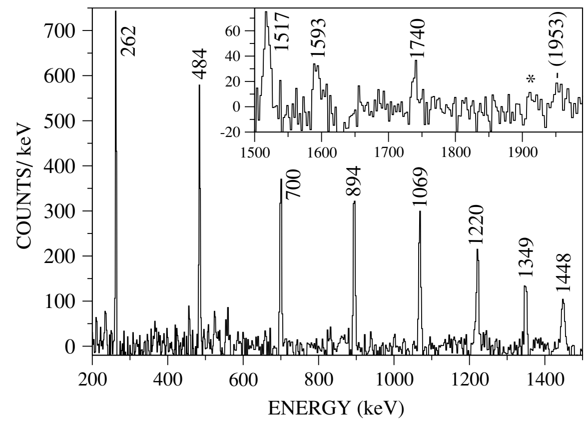

FIG. 1. Partial γ-ray spectrum showing the yrast band in ^{76}Sr. The spectrum was produced from a E_γ-E_γ-E_γ cube by adding together five spectra created using (a) 262 keV in coincidence with all higher transitions, with the exception of 894 and 1349 keV (b) 484 and 700 keV transitions in coincidence with all higher transitions with the exception of 894 and 1349 keV (c) 894 and 1069 keV in coincidence with all higher transitions (d) 1517 keV in coincidence with all higher transitions and (e) 1593 keV in coincidence with all higher transitions. The weak γ ray marked with an asterisk at 1915 keV is a contaminant from ^{73}Br.

Excited states in ^{76}Sr were populated via the ^{40}Ca(^{40}Ca, $2p2n$) reaction. The 165 MeV beam of ^{40}Ca, provided by the ATLAS accelerator system at Argonne National Laboratory, was incident upon a 350 μg/cm^2 enriched ^{40}Ca target, which was flashed with 150 μg/cm^2 of Au on both sides in order to inhibit oxidation. The event trigger required the presence of four or more Germanium gamma-ray detectors of the Gammasphere array [16], in prompt coincidence with each other and in anticoincidence with their Compton supressor shields. This array contained 101 detectors. Light charged particles from the reaction, including protons, were also detected using the 4π CsI-array Microball [17].

In the off-line analysis the high-fold Ge data were unpacked into triples events and an E_γ-E_γ-E_γ "cube" was produced using those triples events that were in coincidence with either one or two protons. RADWARE analysis software [18] was used to produce spectra from the cube. Figure 1 shows a representative γ-ray spectrum of the yrast cascade in ^{76}Sr, the energies and intensities of the γ rays are presented in Table I. The gate combination used to produce this spectrum

TABLE I. Measured γ ray energies and intensities from the $1p + 2p$ gated E_γ-E_γ-E_γ cube. The intensities have been obtained from triples data using the 262 and 484 keV γ rays as gates, and hence are normalised to the $6 \rightarrow 4$ transition.

Energy (keV)	I_γ	Energy (keV)	I_γ
262.3(2)	...	1349(1)	36(3)
484.4(2)	...	1448(1)	12(2)
699.6(2)	100	1517(1)	<10
894.2(2)	70(9)	1593(1)	<10
1069(1)	55(6)	1740(1)	<10
1220(1)	42(3)	1953(1)	<10

(see caption to Fig. 1) minimizes the contamination introduced by the strong $\alpha 3p$ channel leading to ^{73}Br, since some of the transitions in ^{76}Sr and ^{73}Br have similar energies.

The yrast band of ^{76}Sr extended by Fischer *et al.* [2] was confirmed up to $I^{\pi} = 14^{+}$. However, the γ ray previously assigned to the decay from the $16\hbar$ spin state, with an energy of 1498 keV, was not observed in the current work. It is possible that the level from which this γ ray emerges is not as well populated under present reaction conditions. In particular, we note that Fischer *et al.* [12] carried out their experiment using a 180-MeV beam and the reaction ^{24}Mg(^{54}Fe, $2n$) to populate ^{76}Sr. This combination of beam energy and exit channel would produce a lower value for the excitation energy and maximum angular momentum in the compound system, thereby providing a different entry point into the yrast sequence; a feature which could explain the discrepancy between their work and the present results. In the present work four new γ rays are clearly observed at the energies 1448, 1517, 1593, and 1740 keV which, assuming the transitions are electric quadrupoles, correspond to decays from states with tentative spins of $16\hbar$, $18\hbar$, $20\hbar$, and $22\hbar$, respectively. A fifth tentative γ ray at 1953 keV is observed and believed to result from the decay of a spin $24\hbar$ level at the top of the yrast band. The transition at 1915 keV, observed just below this γ ray in Fig. 1, is found to be in coincidence with γ rays that are in ^{73}Br. Hence this weak transition is thought to be a contaminant.

Figure 2 shows the dynamic and kinematic moments of inertia for the yrast band in ^{76}Sr. The results clearly show that the band now extends beyond the $g_{\frac{9}{2}}$ proton and neutron band crossing region and that the crossing frequency is $\hbar\omega = 0.74 \pm 0.01$ MeV. This is $\sim$0.12 MeV higher than that suggested by the data of Fischer *et al.* [2]. Figure 2 also shows the results of cranked relativistic Hartree-Bogoliubov (CRHB) and cranked relativistic mean-field (CRMF) calculations. These calculations were first published, before the current data became available, in a recent theoretical paper which discussed band crossings and configurations of rotational bands as well as np-pairing in this mass region [9]. The NL3 parametrization of the relativistic mean-field Lagrangian [19] is used in both calculations and the pairing correlations are neglected in the CRMF calculations [20]. The D1S Gogny force [21] is used in the pairing channel of the CRHB theory [22]. In addition, approximate particle number projection is performed by means of the Lipkin-Nogami (LN) method in the CRHB framework, so the abbreviation CRHB+LN is used below for this type of calculation.

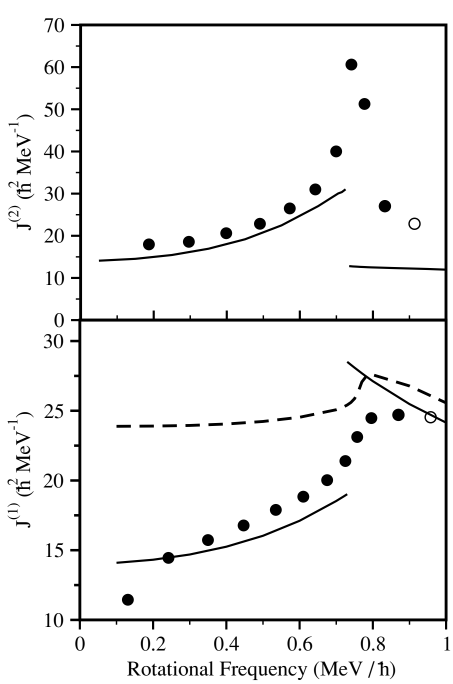

FIG. 2. The upper (lower) panels show the dynamic $J^{(2)}$ (kinematic $J^{(1)}$) moments of inertia, respectively. Circles are used for the ^{76}Sr data (solid for firmly established transitions and open circle for the point incorporating the tentative 1953 keV transition). The solid and dashed lines show theoretical values obtained in the CRHB+LN and CRMF calculations, respectively, which are taken from Ref. [9].

The calculations show very good agreement with the experimental data. In particular, the experimental crossing frequency of $\hbar\omega = 0.74$ MeV and the theoretical CRHB+LN crossing frequency, $\hbar\omega = 0.73$ MeV, are in excellent agreement, see Fig. 2. Furthermore, if our tentative γ ray transition at 1953 keV is correct then the experimental results for the kinematic and dynamic moments of inertia appear to tend towards the theoretical CRHB+LN values after the band crossing. It is also worth noting that the high spin data (above the band crossing where the pairing is expected to be of little importance) and band-crossing frequency seems to be well reproduced by the CRMF calculations. In addition, the present data also show good agreement with the predicted crossing frequency of 0.75 MeV obtained using the standard total Routhian surface (TRS) model (see Fig. 2 of Ref. [14]). Similar to the CRHB+LN calculations, the TRS calculations include only $t = 1$ neutron and proton pairing (i.e, no np pairing). However, we note that the TRS calculations were performed with a rotational frequency step of 0.1 MeV and that they suggest that the band crossing should be rather smooth, while the present data indicate a somewhat sharper crossing in agreement with the CRHB+LN calculations. It would be interesting to see if the sharpness of the band crossing and the calculated crossing frequency change if TRS calculations with a finer mesh step size are performed.

One way to check if an appreciable delay in band crossing takes place in the $N = Z$ nucleus as compared with its neighbors with higher neutron numbers is to compare the crossing frequencies in the $N = Z$, $N = Z + 2$, $N = Z + 4$ systems. Such an approach was employed in the study of the Kr isotopes, where the ground-state rotational bands in $N = Z + 2$ ^{74}Kr and $N = Z + 4$ ^{76}Kr nuclei show almost the same frequency of band crossing (see, for example, Fig. 1 in Ref. [9]) indicating that the nuclear mean-field does not vary so much with neutron number. However, this approach cannot be used in the case of Sr isotopes, since (i) the crossing frequency in the ground-state band of ^{78}Sr is not well defined (see Sec. IV C in Ref. [9] for details), and (ii) large variations in the crossing frequencies are observed in more neutron rich ^{80,82,84}Sr nuclei [23] suggesting considerable changes in nuclear mean-field as a function of neutron number.

Comparison with the neighboring odd-neutron ^{77}Sr nucleus, where several bands are observed to undergo paired band crossings, is also not fully conclusive with respect of the presence of additional neutron-proton pairing correlations in ^{76}Sr, since minor differences in the band-crossing frequencies of the bands in ^{76,77}Sr may be caused by the polarizations both of the deformed mean-field and of the $t = 1$ pair field induced by the additional neutron in ^{77}Sr.

Recent investigations of the impact of the Coulomb exchange term on the pairing field, within the framework of the Hartree-Fock-Bogoliubov approach based on the Gogny force, found a considerable decrease of the proton pairing energies due to a Coulomb antipairing effect [24]. So far, this effect has not been investigated experimentally due to the difficulties of disentangling this phenomenon from other effects in the $N \neq Z$ systems. However, the even-even $N = Z$ systems provide an excellent laboratory to test this prediction. The results from the present work, when combined with the limited results from previous high spin studies of other $N = Z$ nuclei in this region, provides us with the first real opportunity to test these predictions.

The similarity of the proton and neutron single-particle spectra (apart from some constant shift in absolute energies by the Coulomb energy) leads to the fact that proton and neutron pairing energies are almost the same for proton and neutron subsystems in calculations which do not contain a Coulomb exchange term (as is the case with CRHB+LN calculations). As a consequence, the alignment (paired band crossing) of proton and neutron pairs takes place at the same rotational frequency in such calculations, which in turn leads to only one bump in the dynamic moment of inertia. However, if the predictions of Ref. [24] are correct then the proton pairing energy should be considerably smaller than that due to the neutrons, and should result in an alignment of proton and neutron pairs at different frequencies, which would manifest itself in a double peaked shape for the dynamic moments of inertia.

The $g_{\frac{9}{2}}$ proton/ neutron paired band crossing has now been observed in the even-even $N = Z$ nuclei ^{60}Zn [13], ^{68}Se [12], ^{72}Kr [11] and ^{76}Sr (current work). Figure 3 shows the kinematic moments of inertia for all four nuclei and the dynamic moments of inertia for ^{60}Zn and ^{76}Sr. The ground and $I^{\pi} = 2^{+}$ states of ^{68}Se and ^{72}Kr are believed to be oblate [9,12] which leads to low values for the kinematic moments of inertia at low frequencies (see Fig. 3). With increasing spin highly-triaxial (^{68}Se) or near-prolate (^{72}Kr) structures become yrast. Thus, the first irregularity seen in the kinematic moments of inertia of these nuclei at a rotational frequency $\hbar\omega \sim 0.4$ MeV is due to this shape coexistence. However such shape coexistence is not present in ^{60}Zn and ^{76}Sr at low spin. These nuclei are characterized by gradually increasing kinematic moments of inertia at low rotational frequency (see Fig. 3). In ^{68}Se the band crossing seen at $\hbar\omega \sim 0.7$ MeV is not related to the standard change from the ground ($g-$) band to the $g_{\frac{9}{2}}$ aligned proton/ neutron S-band [9], and, thus, can be excluded from consideration. It is clear, however, that for the other three nuclei the proton and neutron $g_{\frac{9}{2}}$ paired band crossings take place simultaneously at $\hbar\omega = 0.6$–1.0 MeV. The experimental dynamic moments of inertia in ^{60}Zn and ^{76}Sr only show evidence for one peak at these band crossings. As a result, the currently available experimental data in even-even $N = Z$ nuclei do not support the existence of the Coulomb antipairing effect caused by the Coulomb exchange term.

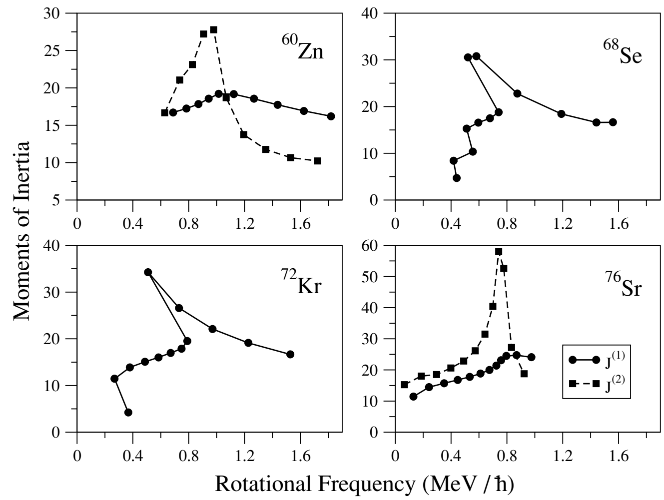

FIG. 3. Kinematic (solid lines) moments of inertia [and dynamic (dashed lines) moments of inertia for ^{60}Zn, ^{76}Sr] as a function of rotational frequency for a superdeformed band in ^{60}Zn and the yrast bands in ^{68}Se, ^{72}Kr, and ^{76}Sr.

Previous theoretical investigations within simplistic models [6,7] suggested that isoscalar ($t = 0$) neutron-proton pairing can cause delays in band crossings in the $N = Z$ systems. Moreover, the isoscalar neutron-proton pair field carries angular momentum and is significantly less affected by the Coriolis force than the isovector pair field. Hence, the presence of an isoscalar pair field might well be expected to show up as a systematic increase, i.e., being larger than expected at high rotational frequencies, in the experimental moments of inertia compared to calculated values obtained in the models with no $t = 0$ neutron-proton pairing. These features do not apparently show up when the ^{76}Sr data are compared with CRHB+LN and CRMF calculations (see Fig. 2). Thus, the present result together with others discussed in Ref. [9] suggest that isoscalar np pairing is not important in describing the yrast band properties of even-even $N = Z$ nuclei.

In conclusion, the yrast cascade of ^{76}Sr has been observed to a tentative spin of $I^{\pi} = 24^{+}$, thus extending the sequence beyond the $g_{\frac{9}{2}}$ proton/neutron band crossing region for the first time. The alignment of the $g_{\frac{9}{2}}$ proton/ neutron pairs has been observed to occur at a rotational frequency of 0.74 MeV. Furthermore, the rotational properties of the observed band are found to be in very good agreement with earlier predictions obtained within the CRHB+LN and CRMF frameworks. These results provide additional support for isovector mean-field theory, which is based on the assumption of the existence of an isovector neutron-proton pair field, the strength of which is defined by isospin conservation, and assumes that there is no isoscalar neutron-proton pair field. The present investigation does not find any evidence for the existence of the latter field. In addition, the systematics of the band crossings in even-even $N = Z$ nuclei do not support the existence of the Coulomb antipairing effect caused by the Coulomb exchange term.

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