

A MATHEMATICS CURRICULUM PACKAGE

THE STUDY OF THE DEVELOPMENT IMPLEMENTATION  
AND EVALUATION FOR GRADE NINE AT EASTERN  
PASSAGE JUNIOR HIGH SCHOOL HALIFAX COUNTY

NOVA SCOTIA

AN ACTION RESEARCH THESIS PRESENTED TO THE  
DEPARTMENT OF EDUCATION SAINT MARY'S  
UNIVERSITY

IN PARTIAL FULFILLMENT OF ~~ONE~~ REQUIREMENTS  
FOR THE DEGREE OF MASTER OF ARTS (EDUCATION)

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Introduction

This study traces the development, implementation, and evaluation of a mathematics curriculum package at Eastern Passage Junior High School, Halifax County, Nova Scotia. The program, implemented at the grade nine level, covers two units of study and is based on the use of behavioral objectives and a contractual approach. Evaluative aids include control groups, standardized t-tests and an attitudinal questionnaire.

While this study in no way claims to be the last word on the use of behavioral objectives or a contractual approach, it is hoped that the study does lend credence to an increasingly popular area of educational thought.

....there is no royal freeway to pedagogical success, no painless solution to complex instructional problems, and no future in our persistent effort to describe 'best teaching practice'. What is emphasized is the wide range of options the teacher may adopt and adapt to his unique situation. (Joyce & Weil, 1972 p. xi)

Chapter IPurpose and Justification

Within any efficient organization there is a set of pre-established objectives by which the organization is run. These clearly defined objectives provide members of the organization with direction and purpose to their tasks and allow for differences in speed, skill, and technical competence of individual members within the organization.

(Alfonso, Firth & Neville, 1975)

Since every classroom is an organization on a small scale, to expect students to learn, to develop, and to perform specific tasks without first knowing precisely what these tasks or objectives are, is impractical.

A teacher in order to be effective, should plan and organize the material to be covered, thus establishing a set of course objectives. If these objectives are written so as to communicate the outcome of learning as something the student will be able to do following instruction that he was unable to do before, then they are written in behavioral terms and are called behavioral objectives. (Flager, 1962)

The main purpose of the study is to determine whether there is a significant difference

in results, between teaching a given unit of mathematics (a) by a traditional method (at the board and to the whole class) or (b) by the use of an individualized instruction method utilizing behavioral objectives (called the contract method).

Following the first experience of teaching a grade nine mathematics course, a search began for a more flexible method, designed to more clearly state the objectives of the course and to allow for more individualized instruction. Partly through experience with them at university, and partly through continued research, behavioral objectives were looked upon as one possible solution to the problem at hand.

The advantage of using behavioral objectives in the instructional process is that they describe the behaviors the student should demonstrate at the end of the process. Since evaluation is based on whether or not the student can perform these behaviors, the exact areas of failure can be located and communicated to the student.

(Sund & Picard, 1972 p. 5)

As one author suggests, educational units must be prepared in response to the questions:

- i) What is it that we must teach?
- ii) How will we know when we have taught it?
- iii) What materials and procedures will work best to teach what we wish to teach? (Lager, 1962, p. v)

Behavioral objectives, then, offer the teacher an almost scientific method for stating course objectives in behavioral terms. Each objective can be stated in terms of a desired perceivable outcome with acceptable performance criteria to be demonstrated by the student upon completion of the objective. With the aims of the course thus written, the teacher's attention may then be directed towards more economical means of instruction.

One method of obtaining more individualized instruction is through the use of "contract teaching", which for our purposes will be defined as that method by which the material to be learned, and the resources available, are prepared and presented to the student as a contract or package. The work is completed at the students' pace using any of the resources available including individual assistance or instruction from the teacher.

Through the blending of the contract method and the use of behavioral objectives, a secondary purpose of the study arose. To determine whether there was a significant difference when teaching a given unit of mathematics traditionally between (a) stating the objectives in writing and (b) not stating the objectives in writing.

A detailed and chronological description of the development and implementation of the Program follows.

Chapter IIDescription of the Development and Implementation

An examination of the Department of Education Program of Studies, presented the general parameters of content within which the curriculum package must fall. (See Appendix A). But it was the supplemental guide, Mathematics Programs Grades 7 - 12, that elaborated on these general guidelines, and listed by topic, those areas to be covered at the grade nine level. (See Appendix B).

With these suggestions in mind, an examination began to determine the topics normally covered at the grade nine level in our school. Through discussions with the school administration and fellow staff members, a continuum of math topics to be covered at the grades six to nine level was devised. At a subsequent in-service day, the mathematics department categorized these selected topics into grade levels. Some topics from the Department of Education guide for the grade nine level were deleted or relegated to future units, while others were added.

From the list of topics to be taught at the grade nine level at our school, those which corresponded to the first two chapters of the school text were used as the basis for the study.

With the help of such notable authors as Robert Mager and Norman Gronlund, each topic (e.g. symbols) was broken down and stated in behavioral terms. Each was called an objective. (See Appendix C objectives 1 - 4)

Upon completion, the objectives were divided into sections, based on topics and those which corresponded to chapter one of the school text were prepared as package or contract one, while those in chapter two became contract two. The total result was two contracts with five sections each and a total of 51 specific objectives. (See Appendices C & F).

Evaluative procedures occupied the next step towards implementation. Since each objective was to be tested, the criterion checks or tests were made to correspond with the existing objectives of each section. All objectives were tested. An unsuccessful objective was repeated before going on to the next section. (See Appendices E & H).

The next step was to prepare materials and procedures to teach the specific objectives. The school text, readily available, acted as a resource, but was unsuited for general usage because of its sometimes technical nature. A study of additional texts and instructional aids led to a set of class notes for each

section, that in many cases were self-explanatory, thus minimizing the need for teacher instruction. (See appendices D & G).

To implement and evaluate the curriculum package, it was necessary to first establish homogeneous groupings among the test and control groups. To this end, chapter one was taught traditionally to the class as a whole in all three grade nine classes. The unit was based on identical teaching procedures, resources and tests. Standardized t-tests showed no significant differences among groups. (See page 9 and Appendix K).

Through random selection the three classes assumed their respective roles in the study. Chapter two was taught using the research study procedures. Class 9 - 1 was given the complete contract behavioral objectives package, utilizing individual instruction. (See Appendices F, G & H) Class 9 - 2 was taught the same material as 9 - 1, traditionally as a group, from the front of the room, but were given the objectives in writing, on the board, as they were taught. Class 9 - 3 was taught the same material as 9 - 1 and 9 - 2, traditionally as a group, from the front of the room, but were not given the objectives in writing.

Between class 9 - 1 and 9 - 2, the only

variable was the teaching method—individualized versus class instruction. Between 9 - 2 and 9 - 3, the variable of stating or not stating the objectives was able to be tested. And between 9 - 1 and 9 - 3, the contract behavioral objectives method and a traditional method were to be tested. (See chapter three).

All three classes began their programs at the same time with no emphasis being made on the differences between classes. Class 9 - 1 was given a one-class instruction/discussion period on the contract procedure. (See Appendix H). Classes 9 - 2 and 9 - 3 began instruction immediately.

Each class covered the same material and was given the same tests. The average mark of the five tests was calculated for the students in each class and the results were applied to a standardized t-test. (See Appendix L).

An analysis of data and research study conclusions follows.

Chapter IIIAnalysis of Data and Conclusions

A series of five pre-tests, used to establish the extent of homogeneous groupings among the three classes, produced the following results.

## PRE-TEST

## Average Score

9 - 1	75.73
9 - 2	73.78
9 - 3	75.67

## PRE-TEST

## T-Score

9 - 1 and 9 - 2	.568637
9 - 2 and 9 - 3	.5054559
9 - 1 and 9 - 3	.0164792

Statistically, no significant difference was found among the groups. (See Appendix K).

Having thus established homogeneous groupings, the curriculum package was implemented and control groups created as described in chapter two, pages 7, 8, and 9.

As a section of the curriculum was completed, each class was given the same post-test. In all, the series of five post-tests, produced the following results.

POST-TEST

Average Score

9 - 1	76.27
9 - 2	69.00
9 - 3	72.46

POST-TEST

T - Scores

9 - 1 and 9 - 2	2.1264059
9 - 2 and 9 - 3	.900967
9 - 3 and 9 - 1	1.0742177

(See Appendix L for raw scores and calculations).

The t-scores between 9 - 1 and 9 - 3 revealed no significant difference between the classes. The indication was that there is a slight increase in results using the contract method, but it was not so significant as to prove the effectiveness of this method over the more traditional approach.

The results do show, however, that one method is as effective as the other. With a larger sample and an extended test period, a more significant difference may appear.

In those statistics involving class 9 - 2, unexpected results were obtained. Upon beginning the study it was thought that the final scores would range from 9 - 1 as the highest, 9 - 2 second, and finally 9 - 3. The t-scores between 9 - 2 and 9 - 3, however, show that there is a slight difference in favour of 9 - 3. In addition, those scores between 9 - 1 and 9 - 2 show a very nearly significant difference in favour of 9 - 1. The results were very perplexing at the time, and, through discussion with class 9 - 2, one possible explanation arose. Some students felt that they weren't sure what to do in class. On the one hand, they were given written objectives to complete and yet were still working as a class through a traditional method. If we can agree that the methods used in instructing classes 9 - 1 and 9 - 3 were relatively unambiguous in their tasks, then this may in part explain the poorer performance of class 9 - 2.

As a result of the non-significant differences among the groups on the post-tests, an attitudinal questionnaire was devised and administered. (See Appendix I).

## Questionnaire

## Average Score

9 - 1	5.4
9 - 2	4.9
9 - 3	4.67

## Questionnaire

## T - Score

9 - 1 and 9 - 2	1.0969565
9 - 2 and 9 - 3	.587484
9 - 3 and 9 - 1	1.5766738

The scores from the questionnaire indicate that those students on the contract method preferred chapter two using their new method in comparison to chapter one, than did those students in the other two classes. However, once again the difference was considered to be statistically non-significant.

While statistically there is no significant difference between the teaching methods, non-statistical indications, such as attitude towards instruction, degree of confusion in the classroom and teacher satisfaction, offer encouragement that the contract method is an asset within a teacher's repertoire.

In future studies of this type two test groups and a single control group are recommended. Emphasis would be placed, if possible, on larger groups, longer test periods, and a single variable for testing.

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APPENDIX A

Extract from Department of Education  
Program of Studies

27

drawing, percent, discount, tax, interest, mappings, transformations - symmetry, congruence, similarity, parallel and perpendicular planes, Pythagorean Theorem, collecting and displaying data, bar, circle graphs, histograms, median mode, probability \*, irrational numbers \*, right angle trigonometry \*.

### GRADE 9 - Algebra

{ Extension of previous work with integers, rational and irrational numbers, exponents; relations, function and solution sets of equations and inequalities, graphing on the number line and in the Cartesian plane, laws of exponents, operations with polynomials, factoring, linear functions, slope intercept form, solutions of systems of linear equations, operations with rational algebraic expressions, review of geometry. *SEE "MATHEMATICS PROGRAMS GRADES 7-12"*  
TOPICS RECOMMENDED FOR STUDY.  
or

### GRADE 9 - Mathematics

This course is intended for students who need more time to cover the basic topics as listed under Grade 7 and Grade 8. The needs and abilities of these students are varied and they should be given every opportunity to develop the mathematical understanding and skills that will benefit them in later studies. These students require patient assistance and an opportunity to experience success in mathematics.

#### Adjusted Program 7-9

Teaching Guide: *Adjusted Program, Junior High School*

For the type of slow learner described in the teaching guide, core material for a program in basic mathematical skills has been selected. Teachers are urged to use the guide for the topics to be covered.

---

\*Optional topics

## APPENDIX B

Extract from Mathematics Programs Grades 7 - 12

# Grade 9 - Algebra

\* TOPICS MARKED CORRESPONDING

## Unit 1. Introduction

- \*(a) Use of symbols
- (b) The language of sets
- \*(c) Use of parentheses
- \*(d) Order of operations
- \*(e) Variables and open sentences
- \*(f) The number line
- \*(g) Operations and properties of the set of integers
- (h) Order and Inequalities
- (i) Relations, functions, and mappings
- (j) Cartesian products

TO THOSE TOPICS IN UNITS  
1 AND 2 OF THE TEXT USED  
AT EASTERN PASSAGE JUNIOR HIGH.

## Unit 2. The Set of Rationals - Q

- \*(a) Signs of quality and signs of operation
- \*(b) Addition in  $Q$
- \*(c) Subtraction in  $Q$
- \*(d) Multiplication in  $Q$
- \*(e) Division in  $Q$
- \*(f) Parentheses and signed numbers
- \*(g) Commutative Properties for Addition and Multiplication
- \*(h) Associative Properties for Addition and Multiplication
- \*(i) The Distributive Property
- \*(j) Axioms of equality
- \*(k) Solution of simple linear equations in one unknown.
  - (l) Solution of simple linear inequations in one unknown.
  - (m) Introduction to irrational numbers and approximate or estimated answers.
  - (n) Solution of word problems.

## Unit 3. Exponents

- (a) Meaning of exponent
- (b) Multiplication of powers
- (c) Division of powers
- (d) Zero and negative exponents
- (e) Power of a power
- (f) Multiplication, division, and powers of monomials
- (g) Scientific notation

## Unit 4. Polynomials

- (a) Definition and degree of a polynomial (monomial, binomial, trinomial)
- (b) Addition of polynomials
- (c) Subtraction of polynomials
- (d) Multiplication of polynomials.
  1. Multiplication of a polynomial by a monomial
  2. Multiplication of two binomials
  3. Squaring a binomial
  4. Multiplication of a polynomial by a binomial
  5. Multiplication of polynomials with more than two terms.
- (e) Division of:
  1. a polynomial by a monomial
  2. a polynomial by a binomial

- (f) Solving equations containing polynomials
- (g) Applications using word problems; e.g. in area, perimeter, volume.

#### Unit 5. Factoring

- (a) Prime factors, and writing monomial as a product of prime factors
- (b) Common factors
- (c) Factoring by grouping
- (d) Factoring trinomials of the type  $x^2+bx+c$
- (e) Factoring trinomials of the type  $ax^2+bx+c$
- (f) Difference of squares

#### Unit 6. Graphing

- (a) Introduction. Cartesian product
- (b) The rectangular coordinate system; points as ordered pairs
- (c) Review the concept of a function
- (d) The graph of a linear function
- (e) Intercepts and how to find them
- (f) Slope - intercept form of graphing a linear equation
- (g) Graphs of linear inequalities
- (h) Solution of systems of linear equations by graphing

#### Unit 7. Rational Expressions - Algebraic Fractions

- (a) Review of fundamental operations with common fractions and decimal fractions
- (b) Ratio and proportion
- (c) Simplifying algebraic fractions
- (d) Multiplying algebraic fractions
- (e) Dividing algebraic fractions
- (f) Adding and subtracting algebraic fractions and whole numbers
- (g) Solution of equations containing fractions
- (h) Solving a formula for one variable in terms of another variable

(If time does not permit, this topic can be handled in Grade 10)

#### Unit 8. Systems of Linear Equations

- (a) Solution by addition and subtraction
- (b) Solution by substitution

Applications in the form of word problems should be dealt with wherever possible.

APPENDIX C  
Contract or Package One

Instructions

The following is a contract on Chapter One of your Math text. Complete all parts and follow directions carefully. Each student is to work at his/her own rate and all questions or problems are to be directed towards the teacher. Below you will find a series of objectives which each student is to complete. If problems arise, refer to the appropriate pages of the text for guidance. A prepared test is available for each section and will cover all objectives in that section. When you have finished each section, ask to write the test for that section. These tests will comprise the greatest percentage of your mark.

Section One: Symbols

Objective One: To be able to state in your own words and demonstrate with the use of five examples what a symbol is in (a) everyday life and (b) Mathematics.

Objective Two: To be able to write the mathematical symbols for plus, minus, times, divided by, is equal to, is not equal to, is greater than, is less than.

Objective Three: To be able to state in your own words and demonstrate with two examples the meaning of: product, sum, difference and quotient.

Objective Four: To be able to use objectives one, two and three to change mathematical expressions involving words into expressions using symbols and vice-versa, of the type:

- (a) the product of five and seven
- (b) X times Y times Z
- (c) a plus b
- (d) the sum of a and b
- (e) r is not equal to the difference of b and c
- (f) 4 divided by 3
- (g) eight more than X
- (h) 5 more than two times a
- (i) fifteen minus X
- (j) nineteen divided by X
- (k) X minus 15
- (l)  $\underline{ab}$
- (m)  $\frac{r}{t}$
- (n)  $\underline{3c-5a}$
- (o)  $\frac{7x}{5} - (3+y)$

Resources: If you are having problems with any of the objectives in this section or the related questions, then refer to the text and your class notes. Text pages 1 - 3.

## SECTION TWO: AXIOMS

Objective Five: To be able to state in your own words what is meant by an Axiom.

Objective Six: To be able to state in your own words and give two examples of: (1) the Addition Axiom, (2) the Subtraction Axiom, (3) the Multiplication Axiom, (4) the Division Axiom.

Objective Seven: To be able to recognize, use and label the Addition, Subtraction, Multiplication, Division, Replacement, Reflexive, Symmetry and Transitive Axioms in changing one equation to another.

Exercise - Name the Axiom or Axioms used to change the equations in column one into the equations in column two.

I.

- (a)  $2x = 16$
- (b)  $x + 3 = 7$
- (c)  $9 - 2 = 7$
- (d)  $x = \frac{7}{4}$
- (e)  $a = b$
- (f)  $y = y$
- (g) If  $a = b$  in  
 $x + a = c$
- (h)  $c = 2d$  and  
 $2d = 15$

II.

- $x = 8$
- $x = 4$
- $9 = 9$
- $x = 28$
- $b = a$
- $y = y$
- then  
 $x + b = c$
- $c = 15$

Objective Eight: To be able to solve a given equation for the variable by using the Axioms to transform the original equation into the solution -  
Sample problem -

$$\begin{aligned} 2x - 7 &= 7 && \text{(original equation)} \\ 2x - 7 + 7 &= 7 + 7 && \text{(addition axiom)} \\ 2x &= 14 \\ \frac{2x}{2} &= \frac{14}{2} && \text{(division axiom)} \\ x &= 7 && \text{(solution)} \end{aligned}$$

Exercise - Solve the following equations for the variable, by applying the axioms. Label each use of the axioms as in the above example. Show all of your work.

- (a)  $y + 2 = 4$
- (c)  $x - 2 = 6$
- (e)  $3x - 6 = 6$
- (g)  $\frac{5x}{5} = 5$
- (i)  $\frac{x}{9} - 3 = 2$

- (b)  $2y = 6$
- (d)  $\frac{x}{4} = 10$
- (f)  $5x + 6 = 26$
- (h)  $\frac{x}{5} + 6 = 16$
- (j)  $\frac{2x}{4} - 17 = 23$

Contract #1Chapter #1Section Three - Properties

Objective Nine: To be able to state in your own words and give three examples of the following properties.

- (1) the closure property
- (2) the zero property
- (3) the unity property
- (4) the negative property
- (5) the reciprocal property
- (6) the commutative property
- (7) the associative property
- (8) the distributive property
- (9) the zero product theorem

Objective Ten: To be able to recognize and name the properties of objective nine in use. Sample questions below:

Exercise: Name the principle or property which is illustrated in each of the following:

- |                                |                                 |
|--------------------------------|---------------------------------|
| (a) $3 + 7 = 7 + 3$            | (g) $6(2 + 7) = 6 \times 2 + 7$ |
| (b) $52 + 13 = 65$             | (h) $0 \times 4 = 0$            |
| (c) $12 + (-12) = 0$           | (i) $4 \times 6 = 6 \times 4$   |
| (d) $32 \times 1 = 32$         | (j) $7 \times 15 = 105$         |
| (e) $2 \times (5 \times 8)$    | (k) $0 + 8 = 8$                 |
| (f) $\frac{1}{2} \times 2 = 1$ | (l) $4 + (3 + 9) = (4 + 3) + 9$ |

Objective Eleven: To be able to state in your own words the two rules for the order of operations.

Objective Twelve: To be able to perform computations using the rules of operations, of the type:

SHOW EACH STEP!

- |                                       |   |
|---------------------------------------|---|
| (a) $4 + 12 \times 3$                 | (k) $2(2 \times 5 - 1)$   |
| (b) $60 - 8 \div 8$                   | (l) $6(5 - 3)$  |
| (c) $20 \div 4 \times 3 + 16 \div 8$  | (m) $4(3 \times 5 - 2 \times 3)$  |
| (d) $5 + 4(6 - 2)$                    | (n) $2 \times 6(2 \times 5 - 3 \times 3)$                                       |
| (e) $12 \div 3 \times 4 \div 8$       | (o) $3 + 2(6 - 5)$  |
| (f) $5 + 4 \times 3 \div 2$           | (p) $5 \times 6 - 3(2 \times 3 + 1)$  |
| (g) $18 - 6 \div 2 \times 3$          | (q) $3(2 \times 6 - 5 \times 1)$  |
| (h) $5 \times 6 + 8 \times 4 \div 16$ | (r) $2 + 3 - 12 \div 2$   |
| (i) $5 \cdot 6 \div 3 + 7$            | (s) $\underline{\underline{\frac{5 + 7}{6}}}$                                   |
| (j) $(6 - 1)(6 - 3) + 6(6 - 5)$       | (t) $\underline{\underline{\frac{(6 - 1)(3 + 1)(5 - 3)}{5 \times 3 \times 6}}}$ |

Objective Thirteen: To be able to state in your own words the meaning of inverse.

Objective Fourteen: To be able to state the inverse of the following:

- |                 |                    |
|-----------------|--------------------|
| (a) addition    | (c) multiplication |
| (c) subtraction | (d) division       |

Objective Fifteen: To be able to state the inverse of specific mathematical operations, of the type:

- |                           |                        |
|---------------------------|------------------------|
| (a) adding 7              | (e) adding -7          |
| (b) subtracting $(6 + X)$ | (f) subtracting $-X$   |
| (c) dividing 9            | (g) multiplying by -12 |
| (d) multiplying 14        | (h) dividing by -2     |

RESOURCES: If you are having problems with any of the objectives in this section or the related questions, then refer to the text and your class notes. Text pages 13 - 19.

80% Mastery Level.

Contract #1Chapter #1Section Four - Equations

Objective Sixteen: to be able to label and recognize the following:

- |                          |              |
|--------------------------|--------------|
| (a) an expression        | (e) identity |
| (b) algebraic sentence   | (f) variable |
| (c) open sentence        | (g) constant |
| (d) conditional equation |              |

Read pages 20 - 21 of the text and complete questions 1 - 18 on page 22.

Objective Seventeen: to be able to solve simple equations for a variable, state the axioms or principles used in solving it, and to be able to check your answer.

Read pages 24 - 26 in your text and complete Exercise p. 26 - 27, questions 1 - 20.

Objective Eighteen: To be able to state in your own words what is meant by a root and solution set of an equation.

Objective Nineteen: To be able to identify the root of an equation. Read pages 22 - 23 and complete questions 1 - 10 on page 23.

NOTE: When you are satisfied that you have mastered the material well enough to perform each objective, ask the teacher for a test on this section.

---

Contract #1Chapter #1Section Five - Words You Should Know

Objective Twenty: To be able to state in your own words the meaning of:

- |                          |                |
|--------------------------|----------------|
| (a) algebraic expression | (e) binomial   |
| (b) term                 | (f) trinomial  |
| (c) factor               | (g) polynomial |
| (d) monomial             |                |

Objective Twenty-one:

To be able to recognize and label what is meant by:

- |              |                            |
|--------------|----------------------------|
| (a) power    | (d) coefficient            |
| (b) exponent | (e) like and unlike terms; |
| (c) base     |                            |

by completing questions of the type 1 - 5, page 38 - 39 of the text.

Objective Twenty-two: To be able to state and use the principle of like and unlike terms.

Exercise: Simplify the following:

- (a)  $3X + 5X + 7X + 6 + 3 + 21$
- (b)  $9X + 7Y - 6Y - 5X + 17$
- (c)  $10X + 7Y + 12X + 19Y + 6 + 12$
- (d)  $12 + 9 + 12X + 17Y + 12Y + 17X$
- (e)  $19 + 6 + 12X + 3X + 5X + 25Y$

Objective Twenty-three: To be able to evaluate an expression by replacing variables with specified numerical values and performing the indicated operations. Do questions 1 - 10, page 40.

Resources Pages 34 - 40 in text.

NOTE: Before writing the test on Section Five to complete Contract One on Chapter #1 - do questions 1 - 5, page 42 (in your notebook).

APPENDIX D  
Contract One Notes

- Contract One ) 1. Algebra is the study of the properties,  
Section One ) and relationships of numbers and the  
                  symbols used to represent those  
                  numbers.
2. A symbol is something used to  
represent something else in a shorter  
form.
3. In arithmetic we use the symbols  $6 \times 2$   
to represent what we mean by six  
multiplied by two. We could also  
write  $6 \cdot 2$  where the  $\cdot$  represents  
"multiply". In algebra, to represent  
a multiplied by b we could use any of  
the following methods:

a x b
a . b
(a)(b)
a b

The sum is the answer you get when you add.

The difference is the answer you get when you subtract.

The product is the answer you get when you multiply.

The quotient is the answer you get when you divide.

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GRADE NINE MATH NOTES

Mr. Creamer

CONTRACT ONE - SECTION TWO

1. An AXIOM, very simply, is a rule used in Math.
2. ADDITION AXIOM: When equals are added to equals, the result is equal. If  $a = b$  then  ~~$a + 5 = b + 5$~~ .  $a + 5 = b + 5$

SUBTRACTION AXIOM: When equals are subtracted from equals, the results are equal. If  ~~$x = y$~~  then  $x - 5 = y - 5$ .

MULTIPLICATION AXIOM: When equals are multiplied to equals, the results are equal. If  $g = h$  then  $g \cdot 3 = h \cdot 3$ .

$$\text{or } 3g = 3h$$

DIVISION AXIOM: When equals are divided by equals, the results are equal.

$$\text{If } v = w \text{ then } \frac{v}{6} = \frac{w}{6} \quad \text{or} \quad \cancel{v = w} \quad v \div 6 = w \div 6$$

REPLACEMENT AXIOMS: When equals are replaced by equals, the results are equal:

$$\text{If } x = y \text{ then } 5 + x = 5 + y$$

POWER AXIOM: Equal powers of equal numbers are equal.

$$\text{If } a = b \text{ then } a^3 = b^3$$

REFLEXIVE AXIOM: Any number equals itself.  $a = a$

SYMMETRY AXIOM: If a first number equals a second, then the second must equal the first.

$$\text{If } a = b \text{ then } b = a$$

TRANSITIVE AXIOM: If a <sup>first</sup> number equals a second and a second equals a third, then the first must equal the third.

$$\text{If } a = b$$

$$\text{and } b = c$$

$$\text{then } a = c$$

3. CLOSURE PROPERTY: The sum or product of any two numbers is a number.  $(a + b)$  is a number  $(a \cdot b)$  is a number.

ZERO PROPERTY: Zero added to any number will give that number.  $8 + 0 = 8$ ;  $a + 0 = a$ .

UNITY PROPERTY: One multiplied to any number will give that number.  $8 \times 1 = 8$ ;  $a \cdot 1 = a$ .

NEGATIVE PROPERTY: For every number, there is another number (called its NEGATIVE) such that their sum is zero.

$$-x + x = 0$$

RECIPROCAL PROPERTY: For every number, there is another number (called its RECIPROCAL) such that their product is one.  $(1/2 \cdot 2) = 1$ ;  $(a \cdot 1/a) = 1$

COMMUTATIVE PROPERTY: The sum or product of two numbers does not depend on their order.

$$a + b = b + a \text{ and } a \cdot b = b \cdot a$$

ASSOCIATIVE PROPERTY: The sum or product of three numbers does not depend on their grouping:

$$(a + b) + c = a + (b + c) \text{ and } (a \cdot b) \cdot c = a \cdot (b \cdot c)$$

DISTRIBUTIVE PROPERTY:  $a \cdot (b + c) = ab + ac$

ZERO PRODUCT THEOREM: Zero times any number is equal to 0

$$a \cdot 0 = 0 \quad x \cdot 0 = 0$$

GRADE NINE MATH NOTESMr. CreamerCONTRACT ONE - SECTION THREE

1. ORDER OF OPERATIONS: When performing operations, including adding and subtracting and multiplying and dividing when no parentheses occur always perform the multiplying and dividing first in the order they occur and then the adding and subtracting.

When parentheses do occur, perform those first and then follow Rule One.

2. Inverse means opposite.

Addition is the inverse operation of subtraction.  
Multiplication is the inverse operation of division.

3. Equation Solving Procedure

- (1) Copy down the question.
- (2) Ask yourself what operations were performed to the variable or letter.
- (3) Perform the inverse operations by eliminating the adding and subtracting first, then multiplication and division.  
(Remember the opposite order from the order of operations. WE ARE UNDOING).
- (4) Remember what is done to one side must be done to the other.
- (5) Solve for the variable.
- (6) Label the use of Axioms.

Contract One - Section Four

1. An EXPRESSION in algebra is any combination of symbols representing a number or operation.

Ex.  $2x - 3$        $\frac{5x}{2y}$

2. An ALGEBRAIC SENTENCE occurs when we join two or more expressions together by the use of the symbols:

$= \neq > <$       Ex.  $2x - 3 = 5$

3. There are three types of algebraic sentences:

TRUE ---  $4 + 5 = 9$

FALSE ---  $4 + 5 = 14$

OPEN ---  $a + 5 = 9$

An open sentence is any sentence that contains a letter.

Ex.  $a + 5 = 9$

4. There are two types of open sentences:

CONDITIONAL - True for some replacements

IDENTITY - True for all replacements.

5. A variable is any letter representing a number.

A constant is a symbol which represents only one number.

6. The root of an equation is the answer which makes the equation true.

7. The solution set is the set of all numbers that make the equation true.

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GRADE NINE MATH NOTES

Mr. Creamer

CONTRACT ONE - SECTION FIVE

1. An ALGEBRAIC EXPRESSION is any combination of symbols representing a number and operation.
2. When one number is considered alone or two or more numbers are considered together, the resulting expression is called a TERM.

Ex. 5                     $12x$                      $15xyz$

3. Each number in a term is called a FACTOR.

Ex. in the term  $15xyz$       15      x      y      z  
are all factors.

4. A POLYNOMIAL is the resulting expression when one or more terms are united by addition and subtraction.

Ex.  $5x$ ;       $5x + e$ ;       $x^2 + 2x + 3$

There are special names given to the many types of polynomials:

A polynomial with one term is called a MONOMIAL.

A polynomial with two terms is called a BINOMIAL.

A polynomial with three terms is called a TRINOMIAL.

5. A COEFFICIENT is defined as the product of the remaining factors.

Ex. In  $12xyz$  the coefficient of  $z$  is  $12xy$  - the coefficient of  $x$  is  $12yz$ .

6. POWER {      5 ← EXONENT  
                  {      x ← BASE

7. LIKE TERMS CAN BE COMBINED BY ADDITION AND SUBTRACTION.  
UNLIKE TERMS CANNOT.

APPENDIX E  
Contract One Tests  
Used as Pre-Tests

Answer all questions. Read directions carefully.

UE 15

1. State in your own words and demonstrate with the use of five examples, what is meant by symbol (a) in everyday life (b) in Mathematics.

UE 20

2. State in your own words and demonstrate with the use of two examples the meaning of:

product	difference
sum	quotient

UE 9

3. Express the following in symbols.

- (a) one plus two
- (b) two minus seven
- (c) a times (b divided by c)
- (d) X is equal to the sum of a and b
- (e) Y is not equal to the difference of a and b
- (f) seven is greater than five
- (g) nine is less than twelve
- (h) seven more than Y is not equal to 15 minus X
- (i) seven more than two times Y

4. Express in words:

UE 6

- (a)  $5 + 2$
- (b)  $X - Y$
- (c)  $3X = (1/2) h (a - b)$
- (d)  $3b + 5 X Y$
- (e)  $\frac{3f - 5d}{9a}$
- (f)  $\frac{ab}{p}$

Answer all questions. Read directions carefully.

UE 4 1. State in your own words what is meant by an Axiom.

2. State in your own words and give two examples of each of the following:

- (a) the addition axiom
- (b) the subtraction axiom
- (c) the multiplication axiom
- (d) the division axiom

3. Name the axiom or axioms used in changing the equations in column I to the equations in column II.

I.

II.

(i) $3a = 24$	$a = 8$
(ii) $a + 7 = 14$	$a = 7$
(iii) $X - 6 = 12$	$X = 18$
(iv) $a = 7$	$a = 21$
(v) $\frac{a}{3}$	
(vi) $Z = Z$	$Z = Z$
(vii) If $X = 7$ in $X + 3 = 10$	then $7 + \underline{\quad} = 10$
(viii) $a = 5d$ and $5d = 10$	then $a = 10$

4. Solve the following equations for the variable by applying the axioms. Label each use of the axioms. Show all of your work.

E 20 (a)  $X = 7 = 10$

(b)  $a - 5 = a 12$

(c)  $2Y = 13$

(d)  $\frac{X}{7} = 2$

(e)  $5X - 7 = 3$

(f)  $3a = 1 = 9$

(g)  $\frac{7a}{5} = 7$

(h)  $\frac{a}{2} = 9 = 10$

(i)  $\frac{X}{6} - 5 = 2$

(j)  $\frac{3a}{7} - 16 = 2$

34  
SECTION THREE

Chapter I

Test

Answer all questions, read all directions carefully.

1. State in your own words and give three examples of the following properties:

- (a) the closure property      (f) the commutative property  
(b) the zero property      (g) the associative property  
(c) the unity property      (h) the distributive property  
(d) the negative property      (i) the zero product theorem  
(e) the reciprocal property

2. Name the principle which is illustrated in each of the following:

- (a)  $3 + 7 = 7 + 3$       (g)  $X(a + b) = X \cdot a + X \cdot b$   
(b)  $9 + 21 = 30$       (h)  $0 \cdot X = 0$   
(c)  $a + (-a) = 0$       (i)  $a \cdot b = b \cdot a$   
(d)  $53 \cdot 1 = 53$       (j)  $9 \cdot 8 = 72$   
(e)  $X + (Y + Z) = (X + Y) + Z$       (k)  $0 + S = S$   
(f)  $a \cdot \frac{1}{a} = 1$       (l)  $(4 \cdot 3) \cdot 9 = 4 \cdot (3 \cdot 9)$

3. State in your own words the two rules for the order of operations.

4. State in your own words the meaning of inverse.

5. Perform the following operations. SHOW EACH STEP.

- (a)  $60 - 8 \div 2 + 3 \cdot 7$   
(b)  $20 \div 4 \cdot 3 + 16 \div 8$   
(c)  $5 \cdot 6 + 8 \cdot 4 \div 16$   
(d)  $(6 - 1)(6 - 3) + 6(6 - 5)$   
(e)  $2 \cdot 6(2 \cdot 5 - 3 \cdot 3)$   
(f)  $5 \cdot 6 - 3(2 \cdot 3 + 1)$   
(g)  $3(2 \cdot 6 - 5 \cdot 1)$   
(h)  $2 + 3 - 12 \div 2$   
(i)  $5 \cdot 6 \div 3 + 7$   
(j)  $5 + 4(6 - 2)$

6. State the inverse of the following operations:

- (a) adding 6      (e) adding  $-a$   
(b) subtracting 9      (f) subtracting  $-x$   
(c) dividing by  $-6$       (g) multiplying by  $-7$

M A T HContract #1Chapter #1Section Four - Test

1. Label the following sentences as TRUE, FALSE or OPEN!! In those sentences which are open, determine which are identities and conditional equations:

<u>WE 18</u>	(a) $5 + 12 = 17$	(e) $14 + 39 = 75 - 12$
	(b) $19 - 6 = 14$	(f) $x + 19 = 19 + x$
	(c) $12 + x = 24$	(g) $12x = 24$
	(d) $a - 5 = 5 - a$	

2. Solve the following equations for the variable. Show all of your work. Name the axioms used and check your answers.

<u>WE 70</u>	(a) $x + 4 = 13$	(d) $\frac{5x}{6} - 1 = 4$
	(b) $3x - 1 = 8$	(e) $\frac{3x}{4} + 7 = 10$
	(c) $\frac{5x}{3} - 9 = 16$	(f) $\frac{9x}{5} = 45$

3. State in your own words what is meant by (a) root, (b) solution set of an equation.

4. Tell which of the numbers (in brackets) is a root of the equation.

<u>WE 6</u>	(a) $x + 4 = 13$ (5, 7, 9, 17)
	(b) $2m = 24$ (6, 9, 12, 24)
	(c) $3x - 1 = 8$ (1, 2, 3, 4)
	(d) $4x - 7 = 1$ (0, 1, 2, 9)
	(e) $x - 7 = 0$ (5, 7, 12, 0)

M A T HContract #1Chapter #1Section Five - Test

1. State in your own words the meaning of:

- UE 14
- (a) algebraic expression
  - (b) term
  - (c) factor
  - (d) monomial
  - (e) binomial
  - (f) trinomial
  - (g) polynomial

2. Write in symbols:

- UE 6
- (a) the sixth power of M
  - (b) the second power of X
  - (c) the ninth power of C

3. Give the exponent and coefficient of X in each of the following:

- UE 10
- |                 |           |
|-----------------|-----------|
| (a) $5x^7$      | (d) $5ax$ |
| (b) $a^9 x^6$   | (e) $x$   |
| (c) $x^3 y^5 z$ | (f) $x^0$ |

4. State the principle of like and unlike terms.

Exercise: Simplify the following:

- UE 10
- (a)  $9x + 6x + 5x + 6 + 5$
  - (b)  $7y + 9x + 12y + 17x + 6 + 9$
  - (c)  $8x + 5y + 6y + 9x + 12$
  - (d)  $9x + 12y + 18x + 15y + 2x + 27$
  - (e)  $9 + 12 + 11x + 10y + 6x + 4x + 9y + 2y$

5. Evaluate the following; given that  $a = 5$

$$b = 7 \quad c = 4 \quad x = 12$$

- UE 10
- (i)  $2a + 6b$
  - (ii)  $5ax - 2b$
  - (iii)  $\frac{bx}{c}$
  - (iv)  $5a + 3b + 4c + x$
  - (v)  $9x + 5c - 12a$

APPENDIX F  
Contract or Package Two

Section OneCOPY CLASS NOTES INTO NOTEBOOK

Objective One: To be able to state the meaning of and give five examples of the following:

- (a) set of positive integers
- (b) counting numbers
- (c) set of whole numbers
- (d) digits
- (e) signed numbers
- (f) rational numbers

Objective Two: To be able to draw a number line and place a mark on that number line to represent a given numerical value. Complete questions 1-5, p.64 and 65 of the text.

Objective Three: To be able to state the meaning of and give five examples of the following:

- (a) sign of operation
- (b) sign of quality
- (c) absolute value

Objective Four: To be able to complete questions utilizing the above concepts, such as questions 1,2,3,4,5, and 7 to 24 on pages 67-68 of the text.

MASTERY LEVEL.....80%

REFERENCES:

- 1) class notes
- 2) class instruction
- 3) teacher
- 4) text-pages 58-68.

Section TwoObjective Five:

To be able to state and give two examples of the rules for the ADDITION of signed numbers.

Objective Six:

To be able to perform the addition of signed numbers.

Complete exercises p. 71 - 73 questions 1 - 24 and questions 1 - 30.

Objective Seven:

To be able to state and give two examples of the rules for the SUBTRACTION of signed numbers.

Objective Eight:

To be able to perform the subtraction of signed numbers.

1. Complete exercise p. 75 questions (2, 4, 6, 8....36)

2. Complete exercise p. 75 - 76 questions (1, 3, 5, 7,...35)

Objective Nine:

To be able to simplify questions containing both addition and subtraction of signed numbers. Complete questions 1 - 20 p. 76.

Mastery Level ..... 90%

Resources:

- 1) class notes
- 2) class instruction
- 3) teacher
- 4) text - pages 68 - 76

I A M UContract TwoChapter TwoSection ThreeCOPY CLASS NOTES INTO NOTEBOOKObjective Ten:

To be able to state and give two examples of the rule for the MULTIPLICATION OF SIGNED NUMBERS.

Objective Eleven:

To be able to perform the MULTIPLICATION OF SIGNED NUMBERS.  
Complete questions 1 - 34, p. 79.

Objective Twelve:

To be able to state and give two examples of the rule for the DIVISION OF SIGNED NUMBERS.

Objective Thirteen:

To be able to perform the DIVISION OF SIGNED NUMBERS.  
Complete questions 1 - 30 p. 80 - 81.

Objective Fourteen:

To be able to perform the ADDITION, SUBTRACTION, MULTIPLICATION and DIVISION OF SIGNED NUMBERS as they appear simultaneously in a question. Complete questions 1 - 25 p. 81 - 82.

Mastery Level ..... 90%

Resources:

- 1) class notes
- 2) class instruction
- 3) teacher
- 4) text - pages 76 - 82

MATHContract TwoChapter TwoSection FourCOPY CLASS NOTES INTO NOTEBOOK

Objective Fifteen: To be able to solve equations with negative roots and be able to CHECK your answer. Complete Exercise p. 84, questions 1 - 20. SHOW ALL OF YOUR WORK.

Objective Sixteen: To be able to state the two rules and give two examples each for the removal of parentheses.

Objective Seventeen: Complete questions 1a- 26, p. 86.

Objective Eighteen: To be able to perform the addition and subtraction of Powers by completing the following questions:

$$(a) 3x^2 + 5x^2 + x + x^3 - 5x + 4x = 3x^3 + 7x^2$$

$$(b) 7y^2 + 2y^3 - 6y^2 - 5y^3 + 3y^3 + 8y + 4y^2$$

$$(c) 7x + 5y + 6x + 3y^2 - 3y - 2x - 7x^2 + 4y^2$$

Objective Nineteen: To be able to state and give two examples for the law of exponents for multiplication.

Objective Twenty: Complete questions 1 - 24, p. 80.

Objective Twenty-one: To be able to state and give two examples of the law of exponents for division.

Objective Twenty-two: To be able to state the law of zero and negative exponents.

Objective Twenty-three: Complete questions 1 - 24, p. 91 - 92.

Objective Twenty-four: To be able to state and give two examples of the law of exponents of a Power.

Objective Twenty-five: Complete questions 1 - 15 AND 1 - 32, p. 93.

Mastery Level ..... 75%

Resources:

- 1) class notes
- 2) class instruction
- 3) teacher
- 4) text - pages 82 - 93

Contract TwoChapter TwoSection FiveCOPY CLASS NOTES INTO NOTEBOOKObjective Twenty-six:

To be able to perform operations involving the product of monomials.  
Complete questions 1 - 24, p. 94 - 95.

Objective Twenty-seven:

To be able to perform operations involving the quotient of monomials.  
Complete questions 1 - 23, p. 96.

Objective Twenty-eight:

To be able to solve equations with parentheses. Text - complete questions 1 - 18 p. 97 - 98.

Mastery Level ..... 80%

Resources:

- 1) class notes
- 2) class instruction
- 3) teacher
- 4) test - pages 92 - 97

**APPENDIX G**  
**Contract Two Notes**

Contract TwoM A T HChapter TwoClass Notes

The set of whole numbers is the set of all numbers that are not fractions or decimals. Ex. 21, 8, 12, 16, 19.

The set of positive integers is the set of all whole numbers greater than 0. Ex. 1, 2, 3, 4, 5.....

The set of counting numbers is the set of all positive integers plus 0. Ex. 0, 1, 2, 3, 4, 5.....

Digits are the ten symbols from which we can make any other number. Ex. (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

Signed numbers are numbers which are written with a (+) or a (-) sign. Ex. +5; -6; +3; -12.

The set of rational numbers is the set of all numbers, fractions, whole, signed, etc. Ex.  $\frac{1}{2}$ ; 3; +5.

A sign of operation results when the (+) and (-) signs are used to tell you to perform an operation such as adding and subtraction. Ex. 3 + 4; 6 - 5.

A sign of quality results when the (+) and (-) signs are used to tell you whether the number is positive or negative. Ex. +5; -6; -7.

The absolute value of a number is shown by using the symbol || around a number and means the value of the number without regards to its sign.

Ex.  $|+3| = 3; |-6| = 6$

ADDITION RULE FOR SIGNED NUMBERS

1. When adding signed numbers with LIKE SIGNS simply add the numbers and keep the sign.
2. When adding signed numbers with UNLIKE SIGNS subtract the smallest absolute value FROM the largest absolute value and keep the sign of the largest.

Ex.  $(+6) + (+8) = +14$   
 $(-8) + (+6) = -2$

SUBTRACTION RULE FOR SIGNED NUMBERS

To subtract one signed number from another, ALWAYS

- (1) change the sign of the number BEING SUBTRACTED;
- (2) change the subtraction sign to an addition sign;
- (3) THEN follow the RULES FOR ADDITION.

Ex. $+12 - +3$	$+12 - -3$
$= +12 + -3$	$= +12 + +3$
$= +9$	$= 15$

M A T HContract TwoChapter TwoClass NotesMULTIPLICATION RULE FOR SIGNED NUMBERS

- When multiplying numbers with LIKE SIGNS ALWAYS multiply the absolute values and use a (+) sign with the product.

$$\text{Ex. } (+12) \cdot (+3) = +36$$

$$(-12) \cdot (-3) = +36$$

- When multiplying numbers with UNLIKE SIGNS ALWAYS multiply the absolute values and use a (-) sign with the product.

$$\text{Ex. } (+12) \cdot (-3) = -36$$

$$(-12) \cdot (+3) = -36$$

DIVISION RULE FOR SIGNED NUMBERS

- When dividing numbers with LIKE SIGNS ALWAYS divide the absolute values and use a (+) sign with the quotient.

$$\text{Ex. } (+12) \div (+3) = +4$$

$$(-12) \div (-3) = +4$$

- When dividing numbers UNLIKE SIGNS ALWAYS divide the absolute values and use a (-) sign with the quotient.

$$\text{Ex. } (+12) \div (-3) = -4$$

$$(-12) \div (+3) = -4$$

CLASS NOTESContract TwoChapter TwoPRODUCT OF MONOMIALS

When multiplying monomials there are certain steps that we can follow to obtain the correct product.

1. Always multiply the numerical coefficients together.
2. Find the product of the powers of the variables by the Law of Exponents for multiplication.
3. Multiply the answer of Step 1 and Step 2 together.

Example: Find the product of  $(-3a^2 x)$  and  $5a^4 x^3$

1.  $(-3) \cdot (5) = -15$
2.  $(a^2) \cdot (a^4) = a^6$  and  $(x) \cdot (x^3) = x^4$
3.  $(-15) (a^6) (x^4) = -15 a^6 x^4$

QUOTIENT OF MONOMIALS

1. Divide the numerical coefficients together.
2. Find the quotients of the powers of the variables by the Law of Exponents for Division.
3. Multiply the two answers together.

Example: Find the quotient of  $(15 x^3 y^4)$  and  $(-3xy^2)$

1.  $(15) \div (-3) = -5$
2.  $(x^3) \div (x) = x^2$  and  $(y^4) \div (y^2) = y^2$
3.  $(-5) \cdot (x^2) \cdot (y^2) = 5 x^2 y^2$

M A T HContract TwoEQUATION SOLVINGNOTESChapter Two

- (a) In this section the roots (or answers) will be negative.  
 (b) When the original equation has the variable on both sides of the equation, it must be reduced to the form  
variables = numbers.

$$\text{Ex. } 5x + 3 = 3x - 17$$

$$5x + 3 - 3 = 3x - 17 - 3$$

$$5x = 3x - 20$$

$$5x - 3x = 3x - 20 - 3x$$

$2x = -20$  must be reduced to this form.

$$\frac{2x}{2} = \frac{-20}{2}$$

$$x = -10$$

REMOVAL OF PARENTHESES

When removing parentheses with a (+) in front you must remove both the (+) and the parentheses and change nothing.

$$\text{Ex. } + (a - b) = a - b$$

When removing parentheses with a (-) in front you must remove both the (-) and the parentheses and change the sign of each number inside.

$$\text{Ex. } - (a - b) = -a + b$$

EXPONENTS AND POWERS

1. In the examples  $5^2$  and  $x^2$  the two's are called exponents, the 5 and x are called the base and the whole thing is called power.
2. The rule for Addition and Subtraction of Powers states that only powers with the same base and exponent may be combined by addition and subtraction.  
Ex.  $x^2 + 2x^2 = 3x^2$  and  $7x^4 - 3x^4 = 4x^4$
3. The Law of Exponents for Multiplication states: to multiply powers of the same base, keep the base and add the exponents. It is written:

$$a^x \cdot a^y = a^{x+y}$$

4. The Law of Exponents for Division states: to divide powers of the same base and subtract the exponents. It is written:

$$a^x \div a^y = a^{x-y}$$

5. The law of zero exponent states that any number raised to the 0 power equals 1. Ex.  $a^0 = 1$

Zero Exponent Proof

$\frac{a^3}{a^3} = 1$  we know this because any number (except 0) divided by itself is always equal to 1

$\frac{a^3}{a^3} = a^{3-3} = a^0$  according to the law of exponents for division.

Therefore  $a^0 = 1$  Replacement

6. The law of negative exponents state that any number with a negative exponent is equal to one over that number with a positive exponent. Ex.  $a^{-2} = \frac{1}{a^2}$

Negative Exponent Proof

$$\frac{a^3}{a^5} = a^{-2}$$

$$\frac{a^3}{a^5} = \frac{a \cdot a \cdot a}{a \cdot a \cdot a \cdot a} \cdot a \cdot a = \frac{1}{a \cdot a} = \frac{1}{a^2}$$

Therefore  $a^{-2} = \frac{1}{a^2}$  by replacement.

APPENDIX H  
Contract Two tests  
Used as Post-Tests

M A T HContract #2Chapter 2Test - Section 1

1. State the meaning and give five examples of:
  - (i) positive integers
  - (ii) counting numbers
  - (iii) digits
  - (iv) signed numbers
  - (v) rational numbers
  
2. Draw a number line representing the set of positive rational numbers from 0 to 15. Mark the points with an X to represent each of the following:
 

(i) $\frac{3}{5}$	(iv) $9\frac{1}{2}$
(ii) $\frac{5}{13} + \frac{14}{2}$	
(iii) $10 - 3$	
  
3. Draw a number line representing all of the rational numbers between -8 and +8. Mark the points with an X to represent each of the following:
 

(i) -7	(iv) $12 - 15$
(ii) +6	(v) $17 - 13$
(iii) 0	
  
4. State the meaning of and give five examples of:
  - (a) sign of operation
  - (b) sign of quality
  - (c) absolute value.
  
5. Name five rational numbers that are less than:
 

(a) $-5$	(b) $+5$
----------	----------
  
6. Name five rational numbers that are greater than:
 

(a) $-3$	(b) $+8$
----------	----------
  
7. Give the absolute values of the numbers indicated:
 

(a) $ 5 $	(b) $ -5 $	(c) $ 15-12 $
(d) $ 19 - 11 $	(e) $ X $	(f) $ -X $
(g) $ 12 - 27 $	(h) $ 19 - 31 $	(i) $ 15 - 2 $
(j) 0		

Contract TwoTest - Section TwoChapter Two

1. State and give two examples of the rule for the ADDITION of SIGNED NUMBERS.

2. Find the sums:

- |                                 |                                 |
|---------------------------------|---------------------------------|
| (a) $(11) + (-15) + 7$          | (f) $(-2a) + (5a) + (-9a)$      |
| (b) $(12) + (-19) + (-23)$      | (g) $(-3ab) + (5ab) + (-7ab)$   |
| (c) $(-12) + (8) + (14) + (-2)$ | (h) $(3xy) + (7xy) + (12xy)$    |
| (d) $(19) + (23) + (-47)$       | (i) $(-7x) + (-16x) + (-21x)$   |
| (e) $(-19) + (-17) + (-22)$     | (j) $(-9ac) + (-16ac) + (22ac)$ |

3. State and give two examples of the rule for the SUBTRACTION of SIGNED NUMBERS.

- |                                 |                                 |
|---------------------------------|---------------------------------|
| 4. (a) $(11) - (-15) - (7)$     | (f) $(-2a) - (5a) - (-9a)$      |
| (b) $(12) - (-19) - (-23)$      | (g) $(-3ab) - (5ab) - (-7ab)$   |
| (c) $(-12) - (8) - (14) - (-2)$ | (h) $(3xy) - (7xy) - (12xy)$    |
| (d) $(-19) - (23) - (-47)$      | (i) $(-7x) - (-16x) - (21x)$    |
| (e) $(-19) - (-17) - (-22)$     | (j) $(-9ac) - (-16ac) - (22ac)$ |

5. Simplify the following using the rules studied in this section.

- |  |
|--|
| (a) $(7) + (-9) - (-7) + (-8) - (-9) - (-7) - (2)$   |
| (b) $(7) - (-7) - (6) - (3) + (-27) - (-17) - (-19)$ |
| (c) $19a + 16a - (17a) - (-19a) + (12a) - (-16a)$    |
| (d) $(12x) + (5x) + (7x) + (-12x) - (-16x) - (37x)$  |
| (e) $(17ab) + (-19) + (7) - (5ab) - (6) - (-6ab)$    |

Contract TwoChapter TwoTest - Section Three

1. State and give two examples of the rule for the MULTIPLICATION OF SIGNED NUMBERS.
2. Find the products:
- (-7) • (+15)
  - (-9) • (-5)
  - (+8) • (+16)
  - (8) • (-5)
  - (-7a) • (-5)
  - (f) (-1) • (+1)
  - (g) (-5) • (-7)
  - (h) (+2) • (-1) • (-5) • (+2)
  - (i) (-2) • (4) • (-1) • (3) • (-1)
  - (j) (-8) • (-3) • (-2)
3. State and give two examples of the rule for the DIVISION OF SIGNED NUMBERS.
4. Find the quotients:
- (+36) ÷ (+4)
  - (72) ÷ (-9)
  - (-12) ÷ (-3)
  - (-12) ÷ (-4) ÷ -2
  - (20) ÷ (-5) ÷ (-2)
5. Simplify the following using the rules for the ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF SIGNED NUMBERS.
- $\left[ (+3) + (-18) \right] \cdot (-2) \div (-5)$
  - $(-1) \times (0) + (-6) - (+4)$
  - $(18) \div [(-3) + (-3)]$
  - $\left[ (10a) - (-3a) \right] \cdot 2$
  - $\left[ (7x) + (-5x) \right] \div 2$

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M A T H

Contract Two

Chapter Two

Test - Section Four

1. Solve the following equations for X using ~~any~~ of the techniques we have studied this year. Check all answers. SHOW ALL WORK.
  - (a)  $X + 12 = 4$
  - (b)  $4 = 2x + 19$
  - (c)  $5x + 6 = 3x + 4$
  - (d)  $mx + n = p$
2. State the rules and give two examples of each for the removal of parentheses.
3. Simplify the following using the rules for the removal of parentheses.
  - (a)  $3 + (2 - 4)$
  - (b)  $6 + (-7 + 6)$
  - (c)  $7a - (3a + 2a - 5a)$
  - (d)  $(-2x + y) - (2y + x)$
  - (e)  $(2a + 9b) - (3b - 7a) - (10a + b)$
4. ~~Explain~~ State the law and give two examples for each of the following:
  - (a) the law of exponents for multiplication
  - (b) the law of exponents for division
  - (c) the law of exponents for power of a power
  - (d) the law of zero exponents
  - (e) the law of negative exponents.

2. Use the laws of exponents to simplify the following:  
Simplify as far as possible.

(a)  $3a^2 + 5a^2 + a + a^3 - 5a + 4a - 3a^3 + 7a^3$

(b)  $7x + 5y + 6x + 3y^2 - 3y - 2y - 7y^2 + 4y^2$

(c)  $a^{12} \cdot a^6 \cdot a^3$

(d)  $y^2 \cdot y^n$

(e)  $d^{2x} \cdot d^{3x}$

(f)  $a^{2n} \cdot a^{3-2n}$

(g)  $c^7 \div c^5$

(h)  $x^a \div x^b$

(i)  $a^{-2x} \div a^{-3x}$

(j)  $(a^2)^3$

(k)  $(a^2 b^3)^3$

(l)  $x^7 \div x^7$

(m)  $a^{10} \div a^{14}$

Contract FiveChapter FiveSection Five - Cont.

Find the products:

1. (a)  $(-3x)(+12x^2)$   
 (b)  $(-25x^4)(8y)$   
 (c)  $(-1)(-4x^2)(2a^2)$   
 (d)  $(-2)(5x^2) + (-3y)(-6y) = 8x^2$   
 (e)  $(a^2)(-5b) + (8b)(2a^2) = (-a)(9ab)$

2. Find the quotients:

- (a)  $-7xz^3 \div 7z^2$
- (b)  $21x^2yz \div 3xz$
- (c)  $4\pi r^2 \div -2\pi$
- (d)  $-35x^5 z^6 y^7 \div 11 xy^3 z^5$
- (e)  $-12a^2 b^2 \div 6 ab$

3. Solve the following equations and check your answer.  
Label the use of each Axiom.

- (a)  $7x - (2x + 9) = 16$
- (b)  $15x - (9-6x) = 5$
- (c)  $(7x-8) + (2x+5) = x + 24$
- (d)  $15w - (11w - 17) = 37 - w$
- (e)  $(-w+2) - (3w-7) = 6w$

APPENDIX I  
Attitudinal Questionnaire

At the beginning of the year we covered Chapter One of the grade nine Algebra Course from Sept. 7 to Oct. 31. We then covered Chapter Two from Nov. 1 to Dec. 15. Keeping in mind the way in which the chapters were taught, please indicate your level of enjoyment by circling that item below which best suits your own personal feelings. Do not put your name or any other identifying marks on this paper.

1	2	3	4	5	6	7
strongly preferred chapter one	preferred chapter one	slightly preferred chapter one	no preference	slightly preferred chapter two	preferred chapter two	strongly preferred chapte. two

APPENDIX J  
Contract Procedure

# CONTRACT PROCEDURE

- (1) Each contract will consist of several sections.
- (2) Each section will cover some specific aspect of the course and is to be completed in full in your notebooks.
- (3) To help complete each section, there are certain page references given for the text. If you are having difficulty in completing the objectives, refer to the text BEFORE asking for help.
- (4) Upon completing each section, a required test will be written.
- (5) On this test, every objective from the preceding section will be covered. To successfully complete the test, a predetermined mastery level must be met.
- (6) On a successful test, ALL necessary corrections are to be made and handed in to the teacher... before continuing.
- (7) On an unsuccessful test, the subject matter is to be RESTUDIED and the test REWRITTEN BEFORE going on to the next section.
- (8) Students are to work at their own speed and are expected to AUTOMATICALLY continue from one section to another without worrying about the "whereabouts" of other students. You will be assessed on YOUR performance in relation to the required number of objectives NOT in relation to the work of other students in the class.

**APPENDIX K**  
**Pre-Test Scores**

APPENDIX KPre-Test Scores and Calculations

9 - 1	9 - 2	9 - 3
1. 50	71	85
2. 75	87	66
3. 67	83	55
4. 52	88	50
5. 78	79	79
6. 86	88	68
7. 82	69	86
8. 75	72	89
9. 76	92	90
10. 75	69	85
11. 74	39	80
12. 62	68	73
13. 94	73	78
14. 90	50	50
15. 95	69	71
16. 77	83	57
17. 76	90	73
18. 76	69	85
19. 82	82	88
20. 62	69	67
21. 46	83	76
22. 98	74	89
23. 49	62	88
24. 79	87	83

APPENDIX K (CONTINUED)

25. 78 90

26. 75 50

27. 81 56

28. 89

29. 93

30. 80

Mean Score 75.73 73.78 75.67

$(x - \bar{x})^2$	9 - 1 5413.8670	9 - 2 4916.6683	9 - 3 3777.336
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## T - Scores

9 - 1 and 9 - 2	.568637
9 - 2 and 9 - 3	.5054559
9 - 3 and 9 - 1	.0164792

APPENDIX L  
Post Test Scores

APPENDIX LPost Test Scores and Calculations

	9 - 1	9 - 2	9 - 3
1.	67	64	77
2.	60	75	55
3.	71	77	43
4.	63	75	58
5.	85	70	71
6.	83	89	78
7.	64	67	82
8.	70	66	92
9.	77	87	35
10.	83	55	85
11.	71	41	57
12.	75	64	50
13.	93	72	83
14.	90	47	59
15.	94	59	61
16.	81	83	73
17.	89	75	58
18.	58	75	85
19.	85	76	66
20.	63	73	84
21.	54	89	69
22.	91	64	86

APPENDIX L (CONTINUED)

23.	56	61	91
24.	86	74	86
25.	76	91	
26.	56	46	
27.	76	48	
28.	84		
29.	93		
30.	74		

Mean  
Score      76.27                          69.00                          72.46

$(\bar{X} - \bar{X})^2$	9 - 1	9 - 2	9 - 3
	4337.3670	4798	4383.958

T - Scores

9 - 1 and 9 - 2	2. 1264059
9 - 2 and 9 - 3	. 900967
9 - 3 and 9 - 1	1. 0742177