THE DYNAMIC EFFECTS OF SHOCKS IN A CLEAN SURPLUS MODEL

Granville Ansong
Saint Mary's University
Robie Street, Halifax, NS, Canada, B3H 3C3
Tel: (902) 420-5618
Fax: (902) 420-5011
e-mail: gansong@shark.stmarys.ca

Andrews Oppong
School of Business Administration
Dalhousie University
6152 Coburg Road
Halifax, NS Canda B3H 3J5
Tel: (902) 494-1809
Fax: (902) 494-1107
e-mail: andrews.oppong@dal.ca

July 1999

Financial assistance for this study was provided by the Senate Research Committee and the Office of the Vice President, Academic and Research of Saint Mary's Unversity, Halifax.

Abstract

The paper derives some implications for the price-book value model of Ohlson [1995] and Feltham and Ohlson [1995] when earnings and dividends are subject to shocks (random disturbances). The paper's main conclusion is that a one-time autoregressive earnings shock has a permanent effect on earnings, book value and dividends. Under a particular parameterization, a positive earnings shock increases book value only to the extent that its effect on productivity is transitory.

1. Introduction

Without exaggeration, the two papers by Ohlson [1995] and Feltham and Ohlson [1995] constitute the most important development in capital market research in the last decade. By developing a theoretical model of firm valuation based only on financial statements, the papers provide a framework (or prototype) for conducting capital market research in financial accounting.

The model relies on three basic assumptions: first, the present value of expected future dividends determines value; second, there is a clean surplus relationship among book value, earnings, and dividends; and third, an autoregressive model describes the stochastic time-series behaviour of abnormal earnings. Using these three assumptions, Ohlson and Feltham [1995] show that the market value of the firm equals book value plus the present value of expected future abnormal earnings. Consequently, the model provides a cohesive theory of firm valuation and a distinct role for each of the fundamental variables of valuation, namely, earnings, book value, and dividends.

The objective of this paper is to study some of the implications for the price-book value model of Feltham and Ohlson [1995] when earnings and dividends are subject to shocks (random disturbances). The paper makes three analytically straightforward assumptions. First, a linear technology produces earnings where book value is the sole technological input (that is, a constant returns-to-scale function); second, clean surplus relation applies as in Feltham and Ohlson [1995]; and third, a random walk model characterizes the stochastic time-series behaviour of dividends. This setup is used to study the two types of shocks (to earnings production and the dividend series). For each type of shock, the paper derives closed-form solutions to the resulting dynamic equations. The main conclusion of the paper is that a positive (favourable) shock to earnings increases book value (and therefore price) only to the extent that its effect on productivity is transitory. If the effect on productivity is permanent, the increase in earnings is matched by an equal increase in dividends, resulting in an unchanged book value (and price). On the other hand, a shock to dividends has only a very limited dynamic effect on both earnings and price.

2.The Model

2.1 Preliminary ideas

It is a well-known fact that the problem of valuation is very simple if future cash flows are certain and there is one (risk-free) interest rate in the economy. First, the market value of the firm equals the present value of its future cash flows (otherwise, there will be intertemporal arbitrage opportunities). Second, the only relevant concept of earnings is Hicksian (or what Scott [1997] refers to as the accretion of discount). That is, earnings equal the risk-free interest rate times the present value of cash flows (that is, beginning-of-period value of the firm). Third, market value equals book value. This last conclusion is readily apparent in the context of Feltham and Ohlson [1995] valuation model which shows that market value equals book value plus the present value of abnormal earnings. If one defines abnormal earnings as the difference between actual and expected cash flows, then there are no abnormal earnings under certainty and market value equals book value. Book value alone suffices to determine both earnings and market value. In a steady state (under no-growth), dividends equal earnings and the clean surplus relation reduces to a constant, that is, the steady-state book value.

To make the above ideas more concrete and in anticipation of the uncertainty model to be developed momentarily, define the following:

P_t = market value of the firm's equity at date t
x_t = earnings for the period (t-1, t)
y_t = book value at date t
d_t = dividends paid at date t

Then the following equations can be used to characterize the certainty model:

$x_t = ky_t$, where k is the risk-free rate;
$y_{t+1} = y_t + x_t - d_t$,
$= y_t$ (clean surplus relation[1] under no growth, that is $x_t = d_t$.).
$P_t = y_t$, market value *equals* book value under no arbitrage.

From the perspective of this paper, the essential information provided by the certainty model is that book value alone determines earnings, dividends, and market value.

2.2 The model under earnings shock

To maintain the neoclassical framework, the following three equations are used to characterize the present value model under uncertainty:

$$x_t = ky_t + e_t \tag{1}$$
$$y_{t+1} = y_t + x_t - d_t \tag{2}$$
$$E_t(d_{t+1}) = d_t \tag{3}$$

Equation (1) is the earnings equation with a stochastic term, $e_t = \lambda e_{t-1} + \varepsilon_t$, modeled as an autoregressive process with an autoregressive parameter, λ, where $0 \le \lambda \le 1$ and ε_t is a white noise process. Earnings are produced by a linear technology of one input (that is, book value) and bombarded by an additive autoregressive shock. The earnings parameter, k, is equal to the risk-free interest rate in the economy. In the context of Ohlson [1995], book value suffices to determine earnings under constant returns to scale, with k being the marginal productivity of book value[2]. The only uncertainty in the model is characterized by the earnings shock, e_t. Because e_t enters into the earnings equation additively, it does not affect k. Therefore, e_t can be interpreted as an exogenous endowment in each period[3]. In the context of econometrics, the earnings stochastic term includes all other variables that might impinge upon the production of earnings. In particular, it includes all non-accounting variables. The autoregressive model as the appropriate stochastic process to describe e_t is motivated by Assumption 3 (A3) of Feltham and Ohlson [1995], which models non-accounting information (defined as v_t in Feltham and Ohlson [1995]) as an autoregressive process.

Equation (2) is the clean surplus relation, which states that the change in book value equals earnings minus dividends (net of capital contributions). The relation means that all changes in assets/liabilities unrelated to dividends must pass through the income statement. Obviously, the clean surplus relation does not hold under GAAP since current accounting standards permit some gains and losses to be charged directly to equity. But it is a good approximation that permits earnings and book value to be tied together. Furthermore, with good disclosure, earnings implied by clean surplus (a comprehensive income) can always be reconstructed.

Equation (3) is a random walk model of dividends. The random walk property of dividends is very well known among financial economists and its invocation here is without any loss of generality. However, since the analysis relies heavily on it, we must point out to the reader that equation (3) can be derived from neoclassical assumptions, but to do so will take us further afield.[4]

The modeling strategy is to determine dividends as a function of book value and the stochastic term since earnings are a function of the same variables. The linearity in the specification of the earnings function leads to a linear specification of the dividends equation as:

$$d_t = \alpha + \beta y_t + \tau e_t \tag{4}$$

The next step is to solve for the optimal values of the parameters of the dividend function, α and β of equation (4). A simple solution technique that is very useful is the method of undetermined coefficients. To employ this solution technique, substitute (4) and (1) into (2) to obtain:

$$y_{t+1} = -\alpha + (1 + k - \beta) y_t + (1 - \tau) e_t \tag{5}$$

Substitute (4) into (3) to get

$$E_t[\alpha + \beta y_{t+1} + \tau e_{t+1}] = \alpha + \beta y_t + \tau e_t \tag{6}$$

Now, substituting (5) into (6) (and noting that $E_t(e_{t+1}) = \lambda e_t$ and $E_t(\varepsilon_{t+1}) = 0$) yields

$$\alpha(1-\beta) + \beta(1 + k - \beta) y_t + (\beta(1-\tau) + \lambda\tau) e_t = \alpha + \beta y_t + \tau e_t \tag{7}$$

Equating coefficients in (7) yields

$$\alpha = \alpha(1-\beta); \quad \beta(1 + k - \beta) = \beta; \qquad \beta(1-\tau) + \lambda\tau = \lambda \tag{8}$$

Solve (8) to get

$$\alpha = 0; \quad \beta = k; \quad \tau = \frac{k}{1 - \lambda + k} \tag{9}$$

Substitute (9) into the dividends and book value equations, (4) and (5) respectively, to obtain

$$d_t = k y_t + \frac{k}{1 - \lambda + k} e_t \tag{10}$$

$$y_{t+1} = y_t + \frac{1-\lambda}{1-\lambda+k} e_t \tag{11}$$

Equations (1), (10), and (11) constitute the dynamic equations of the model. They are the closed-form solutions of earnings, book value, and dividends to a stochastic earnings disturbance that can be used to analyze the optimal responses of these three variables following a realization of e_t. To analyze the optimal responses, consider a one time positive realization of e_t such that $e_t = 1$, $e_{t+i} = 0$ for all $i, i \neq 0$. That is, the stochastic earnings term is zero before and after date t.

For $\lambda = 0$ so that the earnings shock is purely transitory, Equations (10) and (11) simplify, respectively, to:

$$d_t = ky_t + \frac{k}{1+k} e_t \tag{12}$$

$$y_{t+1} = y_t + \frac{1}{1+k} e_t \tag{13}$$

Clearly, from Equation (13), a transitory earnings shock increases book value.

For $\lambda = 1$ so that the earnings shock is permanent, Equations (10) and (11) simplify to:

$$d_t = ky_t + e_t \tag{14}$$

$$y_{t+1} = y_t \tag{15}$$

From Equation (15), a permanent earnings shock has no effect on book value!

Thus, a positive earnings shock increases book value only to the extent that its effect on productivity is transitory. If on the other hand the effect of the shock on productivity is permanent, the increase in earnings is matched by an equal increase in dividends, resulting in no change in book value.

Next, consider a one-time effect of earnings shock for the usual case where the autoregressive parameter is between zero and one, i.e., $0 \le \lambda \le 1$.

At date t

At date t, y_t is unaffected since from (11) y_t is determined by y_{t-1}, which is unaffected by the realization of the shock at date t. From (1), x_t is higher by 1 since $e_t = 1$ and from (10), d_t is higher by $\frac{k}{1-\lambda+k}$

At date t+1

At date t+1, $e_{t+1} = \lambda$ and from (11), y_{t+1} is higher by $\frac{1-\lambda}{1-\lambda+k}$. This is because between dates t and t+1 earnings increased by 1 while dividends increased by $\frac{k}{1-\lambda+k}$.
So the rest of the increase [1 - ($\frac{k}{1-\lambda+k}$) = $\frac{1-\lambda}{1-\lambda+k}$] was absorbed by book value, y_{t+1}. From (1), earnings, x_{t+1}, are higher by $k[\frac{1-\lambda}{1-\lambda+k}]+\lambda$. Finally from (10), dividends, d_{t+1}, are higher by $k[\frac{1}{1-\lambda+k}]$.

At date t+2

At date t+2, $e_{t+2} = \lambda^2$ and from (11) y_{t+2} is higher by $\frac{1-\lambda}{1-\lambda+k}[1+\lambda]$. From (1), x_{t+2} is higher by $k\{\frac{1-\lambda}{1-\lambda+k}[1+\lambda]\}+\lambda^2$ and from (10) d_{t+2} is higher by $k[\frac{1}{1-\lambda+k}]$.

At date t+s

By recursive substitutions, after s periods, at date t+s,

y_{t+s} is increased by $\quad \frac{1-\lambda}{1-\lambda+k}[1+\lambda+...+\lambda^{s-1}]$

x_{t+s} is increased by $\quad \frac{1-\lambda}{1-\lambda+k}k[1+\lambda+...+\lambda^{s-1}]+\lambda^s$

d_{t+s} is increased by $\quad k[\frac{1}{1-\lambda+k}]$.

These optimal dynamic responses indicate that a *one-time* autoregressive earnings shock will have a persistent effect on earnings, book value and dividends that will only be reached asymptotically. Consequently, for an economy that is constantly bombarded by shocks to earnings, the dynamic effects can be very large, even for a *transitory* earnings shock.

2.3 The model under dividends shock

Under dividends shock, the only uncertainty emanates from dividends; in particular, there is no productivity shock and earnings are simply an accretion of discount. Under these conditions, the three equations describing the model are:

$$x_t = ky_t \tag{1A}$$
$$y_{t+1} = y_t + x_t - d_t \tag{2}$$
$$E_t d_{t+1} = d_t + u_t \tag{3A}$$

Equation (1A) is the earnings equation without the stochastic term. Equation (2) is the clean surplus relation and equation (3A) is the random walk model of dividends with a mean-zero, i.i.d. stochastic term, u_t [5].

Again, the method of undetermined coefficients is used to determine the dynamic response equations. To do so, assume that dividends are a linear function of book value and the stochastic term:

$$d_t = \alpha + \beta y_t + \phi u_t \tag{16}$$

Substitute (16) and (1A) into (2) to obtain:

$$y_{t+1} = -\alpha + (1 + k - \beta) y_t - \phi u_t \tag{17}$$

Substituting (17) into (3A) (and noting that $E_t u_{t+1} = 0$) yields:

$$\alpha + \beta E_t (y_{t+1}) = \alpha + \beta y_t + (\phi + 1) u_t \tag{18}$$

Now substitute (17) into (18) and simplify to obtain:

$$\alpha(1-\beta) + \beta(1 + k - \beta) y_t - \phi\beta u_t = \alpha + \beta y_t + (\phi + 1) u_t \tag{19}$$

Equating coefficients in (19) yields

$$\alpha = \alpha(1-\beta); \quad \beta(1 + k - \beta) = \beta; \quad -\phi\beta = \phi + 1 \tag{20}$$

Solve (20) to get

$$\alpha = 0; \quad \beta = k; \quad \phi = -\frac{1}{1+k} \tag{21}$$

Substitute (21) into the dividends and book value equations, (16) and (17), respectively, to obtain:

$$d_t = ky_t - \frac{1}{1+k} u_t \tag{22}$$

$$y_{t+1} = y_t + \frac{1}{1+k} u_t \tag{23}$$

To determine the dynamic effects, consider a one-time positive realization of dividends shock at date t such that $u_t = 1$ and $u_{t+i} = 0$ for all $i, i \neq 0$. That is, the stochastic dividends term is zero before and after date t.

At date t

From (22), d_t is lower by $\frac{1}{1+k}$ and from (23) y_{t+1} is higher by the same amount, $\frac{1}{1+k}$. The reason is that the positive realization of $u_t = 1$ does not affect x_t and y_t so that the entire drop in dividends of $\frac{1}{1+k}$ is matched by an increase in book value by the same amount.

At date t+1

At date t+1, with y_{t+1} higher by $\frac{1}{1+k}$, earnings, x_{t+1}, are higher by $k(\frac{1}{1+k})$ and dividends, d_{t+1}, are higher by the same amount, $k(\frac{1}{1+k})$.

At date t+2 and beyond

From date t+2 onward, there will be no further dynamics. Book value stays at its new higher level ($\frac{1}{1+k}$ higher than at date t-1), earnings stay at their new higher level ($k(\frac{1}{1+k})$ higher than from date t-1 to date t) and dividends paid stay at their new higher level ($k(\frac{1}{1+k})$ higher than at date t-1).

Suppose an autoregressive process is used to model the dividends shock such as $u_t = \varsigma u_{t-1} + \varepsilon_t$, where ς is an autoregressive parameter and ε_t is white noise, then equations (22) and (23) become[6], respectively,

$$d_t = ky_t - \frac{1}{1+k-\varsigma} u_t \tag{24}$$

$$y_{t+1} = y_t + \frac{1}{1+k-\varsigma} u_t \tag{25}$$

Once again, the effect of a one-time dividends shock on book value, earnings, and dividends, after *s* periods (at date *t+s*), is derived by repetitive substitutions to yield, respectively:

$$y_{t+s}: \qquad \frac{1}{1+k}[1+\varsigma+\ldots+\varsigma^{s}]$$

$$x_{t+s} \qquad \frac{k}{1+k}[1+\varsigma+\ldots+\varsigma^{s-1}]$$

$$d_{t+s} \qquad \frac{k}{1+k}[1+\varsigma+\ldots+\varsigma^{s-1}] - \frac{1}{1+k}\varsigma^{s}$$

Thus, a one-time autoregressive dividends shock will also have permanent effect on earnings, book value and dividends that will only be reached asymptotically.

2.4 Further discussion of the model

From the perspective of accounting, earnings shocks are potentially more valuation-relevant than dividends shock since under clean surplus, valuation relates to book value and expected future abnormal earnings. For this reason, earnings shocks are explored further to gain more insights into their propagation mechanisms.

In Equation (1), earnings shock, e_t , is modeled as an autoregressive process. For a more general result, let us simply say that e_t follows a given (but unspecified) stochastic process. Then Equations (1), (2), and (3) can be solved (by repetitive substitutions) to yield:

$$d_t = k\{y_t + \frac{1}{1+k}\sum_{s=0}^{\infty}\frac{1}{(1+k)^s}E_t e_{t+s}\} \tag{26}$$

$$y_{t+1} = y_t + \{e_t - \sum_{s=0}^{\infty}\frac{k}{(1+k)^{s+1}}E_t e_{t+s}\} \tag{27}$$

From Equation (26), dividends depend on current book value and the expected present value of earnings shocks. The coefficient associated with e_t is $\frac{k}{1+k}$. This means that e_t can be treated as an annuity from which the firm draws down the interest for dividend each period. Consequently, if movements in e_t are transitory, there is dividend smoothing. From (27), the change in book value is equal to the difference between the current value of earnings shock and the expected present value of future earnings shocks[7].

The analysis suggests two possible explanations of the dynamic effect of earnings shock. One is that positive technological shocks generally lead to a higher book value that then amplifies the initial effect of these shocks.[8] The other is dividend smoothing. That is, when there is an earnings shock, an attempt to smooth out dividends may lead to serial correlation in earnings.

3. Concluding remarks

Using a neoclassical model of security valuation with clean surplus, this paper shows that positive earnings shocks are able to generate serial correlation in earnings. Therefore, the paper provides a possible explanation for why there is persistence in earnings. Persistence in earnings is the result of the optimal response of earnings to technological shocks to the economy. The paper also derives the optimal dynamic responses of book value and dividends to a one time technological shock.

Because firm valuation in a clean surplus model reduces, essentially, to the ability to forecast earnings and book value, knowledge of the optimal dynamic structure of earnings and book value is potentially useful in the forecasting effort.

The setup in the paper is embryonic, only intended to explain the basic dynamic mechanisms at work. Even so, the paper has succeeded in offering a cohesive framework for analyzing earnings persistence and other shocks without assuming much.

REFERENCES

Ansong, G. Dividend Policy in a Clean Surplus Model. Mimeo. 1999.

Beaver, W.H., R. Lambert, and D. Morse. The Information Content of Security Prices. *Journal of Accounting and Economics.* (June 1980), 3-28.

Bernard, V.L. Capital Markets Research in Accounting During The 1980s: A Critical Review. University of Michigan, 1989.

Bernard, V.L. The Feltham_Ohlson Framework: Implications for Empiricists. *Contemporary Accounting Research,* (Spring 1995), 733-747.

Black, F. General Equilibrium and Business Cycles. *NBER Working Paper 950,* 1982

Campbell, I. R. *Business Valuation: A Non-Technical Guide for Business People.* 1984.

Campbell, J.Y. Inspecting the Mechanism: An analytical Approach to the Stochastic Growth Model. *Journal of Monetary Economics,* (June 1994), 463-506.

Campbell, J.Y., A.W. Lo, and A.C.MacKinlay, *The Econometrics of Financial Markets* Princeton University Press, 1997.

Chiang, A.C., *Fundamental Methods of Mathematical Economics.* (Third edition), McGraw-Hill, Inc., 1984.

Feltham, G. A. and J. A. Ohlson. Valuation and Clean Surplus Accounting for Operating and Financial Activities. *Contemporary Accounting Research,* (Spring 1995), 689-731.

Komendi, R.C., and R.Lipe. Earnings Innovations, Earnings Persistence, and Stock Returns. *Journal of Business,* (July 1987), 323-346.

Lucas, Jr., R.E. Asset Prices in an Exchange Economy. *Econometrica,* (November 1978), 1429-1445.

Ohlson, J.A. Accounting Earnings, Book Value, and Dividends: The Theory of the Clean Surplus equation (Part 1). Working Paper, Columbia University, 1989.

Ohlson, J. A. A Synthesis of Security Valuation Theory and the Role of Dividends, Cashflows, and Earnings. *Contemporary Accounting Research*, (Spring 1990), 611-676.

Sargent, T. *Dynamic Macroeconomic Theory.* Harvard University Press, 1987.

Scott, W.R. *Financial Accounting Theory.* Prentice Hall Canada Inc., 1997.

Stewart, G.B. *The Quest for Value.* Harper-Collins, 1991.

Solow, R. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*, (February, 1956), 65-94.

---

[1] Clean surplus relation is discussed in detail in the next section.
[2] k is both the risk-free interest rate and the marginal productivity of the technological input.
[3] Such as manna from heaven, if you are biblically inclined.
[4] The interested reader is referred to Ansong (1999).
[5] The simple i.i.d assumption is used because it is more realistic than an autoregressive dividends shock.
[6] That is, equations (1A), (2), and (3A) can be solved to yield equations (24) and (25).
[7] By assuming an autoregressive model for e_t, (26) and (27) simplify to (10) and (11).
[8] Recall that book value is the only technological input of earnings production.