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## LA THÈSEE A ÉTÉ M̀icrofilmée telle que NOUS L'AVONS RECUE

A study of the double galaxy IN ABEL CLUSTER 1775

By Jeffrey J. E. Hayes

1
Submitted as "a Partial Requirement for the Degree of Master of Science
in Astronomy from St. Mary's University
(c) Halifax, October A.D. 1981

Photographic and photoelectric photometry are used to obtain intensity profiles, of the Al775 double galaxy. The intensity profiles are fitted to the standard ( $c=2.25$ ) King models, from which composite model.s are constructed. The apparent core radil of the $S E$ and, NW components are found to be 2.4 kpc and 3.9 kpc respectively. ( $\mathrm{H}_{\mathrm{O}}=$ $75 \mathrm{~km} / \mathrm{sec} / \mathrm{Mpc}$ ). The composite models consist of isophotal contour maps and a modelled surface brightness profile along the line joining the two components. The models constructed do not fit the data very well in the outer portions of the brightness profile, in that they overestimate the amount of light coming from these regions. On the basis. of the models however, it seems unlikely that the Al775 system is a superposition of two unrelated galaxies, but is more likely to be some sort of interacting system. The nature of the interaction, however, cannot be answered, nor can the existance of a large diffuse halo be -demonstrated on the basis of the present data.

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## SECTION I: INTRODUCTION

The double galaxy in Abell Cluster 1775 has been studied by Chincarini et al (1977) and by Hintzen (1979). The conclusions reached. in these two papers are quite different. Chincarini et al determined that the Al775 system forms a bound, closed system of extremely large mass, while Hintzen found that the system was not bound, but the superposition of two elliptical galaxies. The aim of this study is to use photographic and photoelectric photometry of the Al.775 system as a starting point for the construction of composite models for the light distribution of the system.

The thodel's chosen for the fitting process are those of King (1966), and it is upon the goodness of the fit of these composite models "to the observations that it may be possible to determine which hypothesis (either Chincarini et al's or Hintzen's) most accurately describes the Al775 system.

Section II of this study will deal with the observations and conclusions of earlier investigations of the Al775 system. Section III will be concerned with the present data and its reduction to a point where model fitting is possible. Section IV will be concerned with the King models and the procedure used to fit the modely fo the observations. Section $V$ will discuss the construction of the composite model, of the Al775 system, while Section VI will discuss the concluṣions which may be drawn from the modelling of the Al775 double, and suggestions for further study.

## SECTION II: PREVIOUS'STUDIES

The double galaxy in A1775 has been studjed before by two separate investigating teams. Chincarini et al in $1971^{*}$ concluded that the Al775 system is a bound one, where the two components are in mutual orbit. The mass of the system was estimated to be $\sim 2 \times 10^{13} \mathrm{M}_{9}$, making it one of the most massive known.

Hintzen's (1979) study concluded that the A1775 system was not bound, but a superposition of possibly unrelated elliptical galaxies.

The evidence cited by each of the above studies will be recapitulated. The evidence that Chincarini et al cite will be discussed first.

On the basis of photometry and spectroscopy, Chincarini et al concluded that the A17775 system is single bound system. They base $t \rightarrow+i r$ claim on several pieces of evidence.

Firstly, the isophotes of the system show that the two components appear to be immersed in a common envelope. Secondly, microdensitometry indicates a deficiency of surface brightness at the minimum between the two components when compared to the sum of the surface brightnesses at an equal distance on either side.

- According to Chincarini et al the deficiency of surface brightness is expected if the two components are bound, for the following reason. An optical dust-free double system would give no such deficiency of surface brightness. Consider two identical point masses. The gravitationial potential midway between them is weaker than the sum at identical distances on either side. If it is assumed that the mass distribution within the bound double system tends to be located at a position with stronger potential, and the surface brightness is a mass-density indicator, then the observed deficiency can be plausibly accounted for.

In addition to the above, Chincarini et. al. also cite the colour and late-type absorption-line spectrum of the system. Both give no indication of any violent activity, which might be expected if one were observing a colliding pair of gataxies. .

Also, on 48 -inch Schmidt plates, the two components of this system (see Figure la) are the brightest objects within a circular area whose radius is at least 25 times the individual angular diameter of each nucleus. If one considers the probability of such a chance superposition of galaxies within the cluster, the odds aré approximately 1 in 1000.

Using these arguments, Chincarini et. al. Conclude that the system in Al775 is one in which the two components are bound. If one assumes that the components are in circular orbits, and with the observed difference in radial velocities of $1720 \mathrm{~km} / \mathrm{sec}$ (see Table I), the lower Iimit to the mass is $-2 \times 10^{13} \mathrm{M}_{0}$. This would make it one of the most massive systems known.

Hintzen's (1979) paper is considered next, along with the evidence f he cites to support his conclusion that the system in Al775 is not a bound system.

First, Hintzen cites the velocity dispersion of the cluster of galaxies. He finds the mean velocity of nine galaxies in the cluster, including the two components of the system to be $20859 \mathrm{~km} / \mathrm{sec}$, with a very large velocity dispersion of $1522 \mathrm{~km} / \mathrm{sec}$. Hintzen says there are two ways to explain this dispersion (one of the largest measured).

Either the cluster itself has a very large intrinsic velocity dispersion, or there are two clusters of galaxies superimposed along the line of sight. Due to the fact that the two components of the system are the brightest and largest galaxies in the central region of the Al775

Figure la. The Al775 system orientation:


103aD. plate $20^{\mathrm{min}}$ exposure; scale $\simeq 1 \mathrm{~cm}=25 .{ }^{\text {.. }} 6$.

cluster, Hintzen concludes the the probability of the superposition of two large galaxies, one each in a cluster of high and low velocity dispersion respectively; to be small. Hintzen, therefore, considers the single.cluster model. The difference between the models of Chincarini et. al. and Hintzen is that the former conclude that the system is bound, while the latter does not. Thocounter-arguments that Hintzen uses against Chincarini et. af.'s conclusions follow.

- The velocities of the individual components of the Al775 system are $797 \mathrm{~km} / \mathrm{sec}$ and $853 \mathrm{~km} / \mathrm{sec}$ below and above the mean velocity of the cluster for the SE and NW components respectively. This indicates that it is not necessary for the two components to be bound, as the difference between their velocities are comparable to the velocity dispersion of the cluster and can be attributed to the individual random motions of the - galaxies within the cluster.

Next., considering the fact that the region midway between the two components is hot as bright'as one would expect from the addition of the intensities of two separate galaxies, Hintzen states that the argument upon which it is based is physically incorrect. The argument is that the material of the halo would be preferentially located near regions of strong gravitational potential. He continues his argument by saying that the halo or envelope material need not be bound to either of the components but could still be found to the system as a whote. Such material would be distributed within an equipotential surface containing both components of the system, and the volume between them. Therefore, if the Al775 system is bound, and if the halo structure is a common envelope of both components, one would"expect the surface brightness to be greater than-or equal to the superposition of two separate

As to the cause of the observed effect, Hintzen suggests that the -Kostinsky effect (Kodak 1967) could be the cause. The Kostinsky effect is photographic in nature and is one in which the heavily.exposed images, such as the components on the plates used by Chincarrini et. al:, would exhaust the developer in the neighbourhood between the two components. The inter-component region would therefore be comparatively underdeveloped. This effect must be guarded against in any exposure of extended objects, such as galaxies. Hintzen notes that while rodking the plates during developing would reduce the ffect, it might not eliminate it entirely.

Hintzen continues his critique of the conclusions drawn by Chincarini ${ }^{r}$ et. al. by answering their argument about the colour and spectrum not showing any signs of violent activity. As both components have the appearance of elliptical galaxies, their colours should be dominated by older pöpulation objects and contain very little inter-stelliar gas. He therefore concludes that even in a high speed collision, one would not expect to find a difference in either the colour or the spectral characteristics. Replying to the argument that the chance superposition. of two large galaxies found at the centre of a cluster is very low, Hintzen points out that numerous other investigators have noted the tendency of large galaxies 'to be located in the central regions of clusters of galaxies. He adds that White (1976), in his $N$-body models of clusters; found that mass concentrations develop as the number of galaxy-galaxy encounters increases. This leaves the most massive gafaxies at the centre of the cluster. Therefore, in Hintzen's opinion; the two components of the Al775 system could be interacting, but not necessarily bound gravitationally.

Perhaps the most damaging piece of evidence against the idea that the Al775 double is bound is that the SE component has been found to be a tailed radio source by Miley and Harris (1977). These authors state that unless our ideas about the formation of tailed radio sources are wrong, there is no way that the A1775 system could be bound. This. is because it is hard to see how two massive galaxies orbiting each . other at a separation of only $30 \mathrm{kpc}\left(\mathrm{H}_{0}=75 \mathrm{~km} / \mathrm{sec} / \mathrm{Mpc}\right)$ could produce a straight 300 kpc long radio tail. Even if there were some intercluster medium or buoyancy, it would still be difficult to model the radio morphology (see Figure. lb). In the opinion of Miley and Harris, the existance of this long, straight radiotail indicates that the Al775
double is not bound.
$\qquad$ $\therefore$

Figure 1b. Radio contours of the A1775 system from Miley \& Harris (1977). Orientation is;

$(+)$ marks the position of the optical components.

pare two points to be made. First, one would expect the components to (have a small angular separation. Second, assuming that a King model is appropriate for interacting objects, then the halo may not be either as extensive as in the bound case, orffontribute as much light to the overall surface brightness distribution. .In view of the fact that King's. quasi-isothermal spheres may not be appropriate to model tida子-interactions, the most important factor in determining an interacting, but unbound system would be the, angular separation between the components.

By the construction of composite models, one may be able to differentiate between the bounded and unbounded hypotheses outlined above. If a simple superposition of two scaled King models accurately describes the light distribution, then one may conclude that the Al775 system is unbounded; whereas if a more complex model is necessary to reprosent the contribution to the surface brightness by the balo, then one may say that the Al775 system is bound.

The composite models will be of two parts, consisting of an isophotal contour map, and an intensity profile along the line joining the two centres of light (called hereafter the system major-axis). A more completehand detailed discussion about the composite models will be found in Sections $V$ and VI.



[^0]
## SECTION III: OBSERVATIONS AND DATA REDUCTION

### 3.1 Data <br> 1

There are two forms of observations which were used in this study. There are photoelectric photometry and direct photography of the A1775 system. The direct photography will be discussed first.

### 3.2 Direct Photography

Five 48 -inch Palomar Schmidt plätes are used in this study. Four of the plates are in the $V$ bandpass, with the fifth piate in the $J$ bandpass. The four V plates were obtained by Dr. G. A. Welch in May 1970, and the $J$ plate was obtained also by Dr: Welch in June 1971.

All five plates were calibrated in density by means of spot sensitometry. The sensitometry was applied immediately after exposyre at the telescope, and the exposure time was one-half of the exposire time at the telescope. Filtres were used to approximate the spectral range-covered by the exposure at the telescope. -

Four of the plates (three $V$ bandpass and the $J$ bandpass) were traced at St. Mary's University using the Department of Astronomy's Joyce-Lobel microdensitometer (Model MK III C.S.).

These plates were traced using a 2.4 density wedge. The fifth plate was traced previously by Dr. Welch using a 3.0 density wedge.

Table II lists the plates used in this study, along with the bandpass, filtre, emulsion type and exposure times.

### 3.3 Photoelectric Photometry

The photoelectric data was obtained by Dr. Welch in February and March 1970 at Kitt Peak National obseryatory. It was obtained using a 1 P21 photocathode and DC electronics in conjunction with the \#1 36-inch

-

TABLE II
Plate Material Used in This Study

telescope and the 84 -inch telescope. Table III lists the photoelectric magnitudes, along with the diaphram diameters, the filters used and the telescopes. Most of the photoelectric observations appear, in an averaged form in Ctincarini et. al.'s paper (1971) (see their Table IIf, for each component of the Al775 system.

### 3.4 Data Reduction

A number of density tracings were obtained from each of thê plates. The first series of tracings taken from each plate were those of the spot sensitometry. All 12 of the spots were traced, along with the clearplate fog level. From the spot sensitometry, characteristic curves for each of the plates could be constructed.

The densities of the spots were determined by drawing a straight line through the clear plate level and the tops of the tracings of the spots. Measuring the difference (in the sense spot minus clear plate level) in densities gave the density co-ordinate of the characteristic curve. The intensity co-ordinate for each of the spots was provided. by Dr..J. Kormendy. In addition to the sensitometry tracings, the plates were traced in 3 directions at various magnifications.

Each of the plates were traced on a line joining the two components referred to throughout the rest of this study as the system major axis, and through each of the components in a direction perpendicular to the system major axis. These latter two tracings are called the SE añ NW minor axis; as the designation suggests, the minor axis were named for the positions of the components with respect to a point on the system major axis between them.

All of the plates were traced initially at a magnification of 10 in order to measure the density of the sky. The galaxy was centred on

TABLE III
Photoelectric Photometry of A1775 System

*The formal standard deviation between settings of the diaphram on the component in question.
yo.
these tracings. The plates were then retraced at higher magnifications ( $>.50$ ), and then retraced at the low magnification. Such a procedure was carried out for all three axes.

The reason for retracing the axes at the low (10) magnification is two-fold. The first reason was to check that the plate did not move when the lever arm was changed in order to change the magnification. The second reason was to check for machine drift in the zero-point of the density level. It was found, by comparing the first and second low magnification tracings by eye, that there was no evidence for either plate movement or zero-point drift for all five plates.

The measured values of the magnification were found to be different than the nominal values engraved on the lever arms of the micro-densitometer. The measured values of the magnifications are needed as it is essential to know the precise value at which a density tracing was traced when a series of such tracings are averaged together.

The measured values of the magnifications were found by tracing a standard grid of lines incised on a piece of glass. A reticle from an eyepiece marked in 0.1 millimetre intervals was used as the standard grid. The reticle was traced using a given lever arm, and the distance between successive peaks on the tracing measured using a centimetre rule and estimating to the nearest 0.1 millimetre. These distances were measured peak to peak. The grid lines were measured on the Department of. Astronomy's measuring engine. Table IV lists both the nominal and measured values of all the lever arms used in this investigation. In the remainder of this study the nominal values of the magnification will be quoted for ease of notation; however, it should be understood that the measured values of the magnification, as given in Table IV, were used in all the calculations.

## TABLE IV

# Nominal and Measure Value of Magnification of Microdensitometer Arm 



The aparature which admits the light to the photocathode of the microdensitometer is used as a projection screen. Projected upon this aperature (or slit) is the portion of the plate which is being traced. The size of the aperature could be changed and its size was determined by the exposure time of the plate being traced. The slit width was choosen, as a compromise between the best resolution of the object being traced and yet minimize the noise due to the individual grains within the emulsion. As the need for high resolution is greater in the short exposures, since the central regions are overexposed in the longer exposures, a small slit width was chosen. Conversely, the need to suppress the noise is greater in the longer exposures and one would not expect to find much fine detail in either the overexposed central regions or the faint outer regions. Table $V$ lists the 'plates, and exposures, along with the projected slit width (both in microns and arcseconds) and the magnification at which it was traced.

After the density tracings'of the A1775 system were made, these tracings were digitized. The digitization process involved setting up a co-ordinate axis on each one of the tracings and noting the position of an origin of this co-ordinate system. Whil the precise origin of the co-ordinate axis was arbitrary, it was always in the lower lefthand corner of the microdensitometer tracings below the level of the sky.

Digitization consisted of measuring at one millimetre intervals, points along the length of the density tracings of the galaxy, with respect to the origin of the co-ordinate axis. The digitization process was carried out by a computer programme called GALAXY, written by Mr. T. J. Deveau, on the St. Mary's University PDP $11 / 70$ in the BASIC language: The programme used a Textronix 4662 plotter in the data transmission mode.

TABLE V
Projected Slit Widths Used for
Microdensitometry

Plate Number

PS 5719 ~ 70
PS 5720
PS 5724 70

PS 6864
$\downarrow$

35
140
Projected Slit Width
$\mu m \geqslant \operatorname{arc}-s e c$

70
35
.

Data files of ( $x, y$ ) points were formed by the programme. Each of. the ( $x, y$ ) pairs correspond to a point on the density tracing of the system. The high magnification tracings and one of the low magnification tracings were digitized for each axis of the system (i.e. the system major axis and the NW and ${ }^{\text {SE }}$ minor axes). This was done for all five plates.

- It should be pointed out that stars and other galaxies were omitted where digitizating the low magnification tracings, as were regions surrounding the double galaxy itself in order to reduce the possible effects of a faint extended halo on the determination of the sky level.

The data files of ( $x, y$ ) pairs were transmitted to the Dalhousie University Cyber 170/720 computer, where the remaining steps in the data reduction were carried out. More information on the programe GALAXY may be found in Appendix 1.
$+0^{\circ}$
The next step in the data reduction was the conversion of the relative density tracings from the plates into plots of relative intensity versus radius with the galaxy. This process is performed by a FORTRAN computer programme called PROFİE. This computer drograme uses as input the digitized data from the microdensitometer tracings and the characteristic curves. The programme also performs a linear regression on the data input from one of the low magnification tracings, to obtain the level of the sky. Standard deviations of the scatter about this line are printed out in the form of $\sigma_{\text {sky }}$. This standard deviation is important in determining the boundary of the matching region when a series of microdensitometer tracings are averaged (or stacked) together. As each density tracing is reduced, the computer programme produces two graphs. These graphs are in the form $\log _{10} r$ versus $\log _{10} I$ (I is the relative intensity with 3.00 being the brightest point and $r$ is the
radial position in arc-seconds). The first plot shows the average of all the previous profiles and displays the current profile to be averaged in, shifted in $\log _{10}$ I to give the best fit with the previous average. The second plot shows the new average obtained. :-

During the averaging process the programme chooses a segment of each of the profiles to serve as a matching interval. The inner boundary of this interval is defined as the first point of the tracing to be averaged-in which is farther than the projected slit-width (used to trace the plate on the microdensitometer) from the centre of the system. The outer boundary is chosen as.the distance at which I = $5 x \dot{\sigma}_{s k y}$. These boundary criteria for the matching intervals were used for the two short exposures and one of the 20 minute exposures. For the remaining 20 minute exposúre and the $J$ plate, the outer boundary was tightened up some-what to be the point at which $I=10 \times \sigma$ sky. There was no preference in using one 20 minute exposure over another. It was necessary to raise the outer boundary criterion for the longer. exposures because any values of intensity outside the range of the progranme's capability (for example points found to have negative intensities within the matching region) would cause the programme to fail to run correctly for all the plates...

It should be pointed out that the plates used in this study are of small scale $\left(67.2 \mathrm{~mm}^{-1}\right)$; all the exposures are limited in the radial direction. This is in the sense that the semi-axes of the profiles have less than 25 points in the matching region, and this presented some difficulties as the programme will not average in a profile with less than 25 points in the matching region. However, using the boundary criteria outlined above, stacked profiles of all the semi-axes of the

Al775 system, using all five plates were obtained, For a more complete discussion of the programme PROFIL, the reader is urged to consult the study of English (1979). A sample rum of the programe profile is to be found in Appendix 2 of this study.

Another form of data used for this study are isophotometric tracings of the plate PS5724. There are two such tracings, and one of them is found in this study; Figure 2.

This form of data is important d's it allows one to see more clearly the various regions of the system under study. By inspection of Figure $\dot{2}$, one notes that the Al77.5 system is composed of 2 components surrounded by, and possibly enclosed by, a halo. The halo is notable in two ways. First, the halo is very 'boxy' in shape. Second, the halo may not be as extensive: a's those of cD galaxies studied by 0emper (1976) and Dressler (1979). 'The angular extent of the halo of Al775 from the centre (i.e. the radius of the halo from one of the components) is some 45 to 50 arc seconds. The isophotometric tracings of the Al775 system were obtained by Dr. G. Welch in 1971 using a 2-dimensional scanning Joyce-Lobel microdensitometer at Michigan State University.

### 3.5 Errors in the Data Reduction

The most critical step in the data reduction process is the construction of the characteristic curve. Indeed the accuracy with which one can construct the characteristic curve will determine the errors in the intensity at a given position within the galaxy.

The plates used in this study were stacked together to form an average intensity-radius profile. The stacking process was carried out in an attempt to minimize errors incorporated when plotting the characteristic curves or errors in interpolating intensities using the curves.

Figure 2. Isoplates of Al775 system. Orientation


Scale $\simeq 1 \mathrm{~cm}=7.45$ from a 102ad $20^{\mathrm{m}}$ plate.


Stacking of the tracings also help minimize the errors and uncertainties involved slit-smearing and seeing effects which would be recorded when : the plates were traced. Therefore, by stacking the plates one forid see, plate by plate, the run of intensity, in comparison with other profiles as well as any deviations in the relative intensity which might appear in one of the profiles. For example, if one were to see an increase in relative intensity at a given, radial position on only one of the five profiles, one would be inclined to disbelieve the increment in brightness as being an intrinsic property of the galaxy, but ascribe its nature to either an incorrectly plotted point on the characteristic curve or an inaccurate interpolation in the computer programme PROFIL.

Naturally, each of the plates will have a slightly different sensitivity and will have been traced on the microdensitometer under slightly different conditions, and the averaging process is performed to try to minimize these differences as much as possible.

By the process of stacking, one could see the differences in relative intensity caused by the differences in plate sensitivity, tracing conditions (slit smearing) and in interpolation of intensities using the different characteristic curves. Using the deviations in relative intensity from plate to plate allows one to attempt to quantify the uncertainties caused by all the above mentioned processes. This quantifying process was done in the following manner. A root mean square (RMS) deviation for the intensity was obtained by comparing the average of the 5 stacked plates with each separate profile at 12 radial positions for 2 typical semi-axes; namely, the NW semi-axes of the NW component (i.e. along the system major axis) and along the NE semi axes of the SE component. These RMS deviations are found in Table VI. One notes that the

TABLE VI
Deviations in the Stacking of Intensity Profiles
NW Component, NW Semi-axis

Radius
$r($ in $\operatorname{arc}-\sec ) \quad \log _{10} r$
$1.00 \quad 0.000 \cdots 20$
1.46
0.167
0.250
0.333
0.417 ,
0.500
. $4.650 .667^{\cdots} 13$
.5 .62
6.81
0.750
10.00
0.833
17.78
20.00
$1.301^{\circ}$
1.250

16

16
28
42:
60

55
RMS Deviation
Intensity (\%-age)

22
21
19
17

13

Surface Brightness
mag/arc sec
-0.19
$+0.24$
-0.22
$+0.27$
-0.21
$+0.26$
-0.19
$+0.23$
-0.17
$+0.20$
$-0.16$
$+0.19$
$-0.13$
$+0.15$
$-0.16$
$+0.19$
-0.27
$+0.36$
F0. 38
$+4059$
$-0.51$
$+0.94$
$-0.48$
$+0.87$

TABLE VI (cont.)

SE Component, NE Semi-axis

| Radius |  | RMS Deviation |  |
| :---: | :---: | :---: | :---: |
| $r$ (in arc-sec) | $\log _{10} \dot{r}$ | $\begin{gathered} \text { Intensity } \\ (\% \text {-age }) \end{gathered}$ | Surface Brightness, (mag/arc sec ${ }^{2}$ |
| 1.00 | 0.000 | 25 | $\begin{aligned} & -0.24 \\ & +0.31 \end{aligned}$ |
| 1.46 | 0.167 | 18 | -0.18 +0.22 |
| 1.78 | 0.250 | 10 | -0.10 +0.11 |
| 2.15 | 0.333 | 8 | -0.08 +0.09 |
| 2.61 | 0.417 | 12 * | $\begin{aligned} & -0.12 \\ & +0.14 \end{aligned}$ |
| 3.16 | 0.500 | 20 | $\begin{aligned} & -0.19 \\ & +0.24 \end{aligned}$ |
| 4.65 | 0.667 | 18 | $\begin{aligned} & -0.18 \\ & +0.22 \end{aligned}$ |
| 5.62 | 0.750 | 23 | $\begin{aligned} & -0.23 \\ & +0.28 \end{aligned}$ |
| 6.81 | 0.833 | 20 | $\begin{array}{r} -0.19 \\ +0.24 \end{array}$ |
| 10.00 | 1.000 | 52 | $\begin{array}{r} -0.45 \\ +0.79 \end{array}$ |
| 17.78 | 1. 250 | 33 | $\begin{aligned} & -0.31 \\ & +0.43 \end{aligned}$ |
| 20.00 | 1.307 | 48 | $\begin{aligned} & -0.43 \\ & +0.71 \end{aligned}$ |

minimum RMS deviation occurs in the range 1.5 to 7 arc seconds; this is the region of matching in the stacking process. This is reasonable, as one would expect the RMS deviation to be a minimum in this intervai.

The values for the RMS deviations are somewhat larger for points interior radially and exterior radially from the matching region. The increase in the RMS deviation interior to the matching region; while due to all the above problems would also include differences in seeing and guiding of the plates, slit smearing and centering when the plates were traced on the microdensitometer. For points exterior to the matching region, the increase in the RMS deviation is attributable to the fact that one is a very low light levels, and one is tracing the galaxy out to where it is becoming fainter than the sky.

- There is another source of error which will be included in the RMS deviations, and this will be the fact that the actual digitization process (using the Textronix 4662 plotter) will be repeatable only to a certain accuracy. It was found that the digitization process is repeatable to $\pm 0.15$ millimetres, which corresponds to $\pm 0.05$ arc-seconds on a tracing of magnification 200. While this error is not as great as those introduced due to the microdensitometry or even more so the characteristic curve, it will add its contribution to the RMS'deviation.

There are two remaining reasons for stacking the profiles of the galaxy. First, it allows one to cover a larger range of surface brightness within the system, by using short exposures to delineate the central regions and the longer (> 5 minutes) to delineate the outer regions to the halo. Second, stacking allows one to increase the signal-to-noise ratio, which is tied into the discussion above.

### 3.6 Calibration of the Profiles in Magnitudes

The relative interaction used in the construction of the intensity profiles were converted into standard surface brightnesses via two methods in this study.

- The first method uses the Newton-Ralphson iteration method to find a value for a constant. which is added to the relative magnitudes of - points on the profile to give a standard surface brightness in magnitudes per square arc-second. A more complete discussion of this method is found in English (1979).

The second method of calibrating the isophotal contours involves summing the relative-magnitudes of each of the isophotal contours (see Figure 2), to find a constant, which when added to the faintest contour will convert the relative magnitudes into standard surface brightness. Analytically this expression is of the form

N
$-2.5 \sum_{i=1} \log _{10}\left(I_{i}+R\right) a_{i}+M p e=\mu_{p e}$
where - $I_{i}$ is the relative intensity of the - $a_{i}$ is the area of the $i$ th contour

- R is the constant to be found
- Mpe is the apparent. photo-electric mag of each component of the A1775 double in the $V$. bandpass using a diaphram of $9.2 \mathrm{arc}-\mathrm{sec}$.

Two semi-axes of the Al775 system were calibrated using. both of the methods outlined above. It was found that the two methods gave the same central surface brightnesses to within. 0.08 mags/ $\boldsymbol{a}^{\top}$. Table VII lists the central surface brightnesses of the eight semi-axes, along with the averages each of the components.

Measured Values of Central Surface Brightnesses of the A1775 System . (in mags/ $\square^{\top}$ )

SE Component

| Semi-axis | NW | NE | SW | SE |
| :---: | :---: | :---: | :---: | :---: |
| Iteration Method | 19.27 | 19.24 | 19.15 | 19.31 |
| Analytic Method | - | - | - | 19.40 |
| NW Component |  |  |  |  |
| Semi-axis | NW | NE | SW | SE |
| Iteration Method | 19.60 | $19.46{ }^{\circ}$ | 19.50 | 19.56 |
| Analytic Method | - | - | 19.44 |  |

Averaged Surface Brightnesses of the Components
SE Component
19.24
NW Component
19.53


## SECTION IV: KING MODELS AND MODEL FITTING

### 4.1 King Models

The fitting functions used in this study are King's (1966) isothermal spheres.

King's models were developed in an attempt to form more realistic fitting function, and are a step away from the purely emperical function of devaucouleurs (1959) and Hubble (1930). The King hodels are based upon the dynamics of the stars within the system, and assume a verocity distribution which is everywhere within the system isothermal and Gaussian. To alleviate the problem of an infinite mass inherent in an isothermal . sphere, the Gaussian is trucated at a specific point corresponding to the escape velocity of the system.

The models were originally developed for use with globular star clusters; but have been found to describe elliptical galaxies reasonably well (King 1966, 1978).

The models have three parameters, two of which describe the core of the system. These two parameters are fo, the central surface brightness and $r_{\text {core }}$, the 'core-radius', at which the surface brightness has fallen to approximately one-half its central velue. If one defines a limiting radius $r_{\text {tidal }}$, then one can define the third parameter, of the form

$$
c=\log _{10}\left(\frac{r_{\text {tidal }}}{r_{\text {core }}}\right)
$$

A series of numerical calculations defining the profiles of the King models have been made available by Dr. King (1980). All the models have similar cores, but behave differently in the-outer regions.

Models with $c<1.5$ are cut off sharply and of elliptical galaxies, only tidally limited objects such as the dwarf elliptical systems of the

Local Group are well described by them (Hodge 1971).
It is these models which also fit the globular clusters. In the range $1.5<c<2.0$, the models are similar to the $r^{\frac{1}{4} / 4}$ deVaucouleurs relation. For $c>2.0$, the central concentration becomes so large that a distinct outer halo is formed that is brighter than the deVaucouleurs relation. For c >> 2.0, the surface brightness. of the models approach the $r^{-1}$ relation of the isothermal relation.

The King model used in this study has $c=2: 25$, the so-called 'standard' King models used by other investigators of elliptical galaxies (King 1966; Kormendy 1977, Oressler 1979).

The King model is used in this study in preference to either, deVaucouteurs (1959) or Hubble's (1930) because each of these relations implies an infinite central space density, whereas the King models do not.

However, there are difficulties with King's models. The models are based upon the idealized Gaussian distribution, which while applicable to relaxed systems such asglobular clusters, will not be applicable to galaxies. King's models were developed with tidally limited objects in mind, and elliptical galaxies are not limited, at least in the sense of the tidal interaction defined by King (1962; 1966).

There are other factors which differentiate elliptical galaxies from globular clusters, and these will be recapitulated briefly.

The profile of the halo of an elliptical. galaxy in the framework of the King model is related to the high-energy tail of the velocity distribution in the centre of the galaxy. There is no reason to bel ieve this true for elliptical galaxies. The models are based upon an isotropic 'velocity distribution. Again, there is no reason to believe this is true in the halo of an elliptical galaxy. Also, the model's are self-gravitating,
whereas there is a growing body of evidence that the outer portions of elliptical galaxies are dominated by an unseen mass which is distributed differently than the visible material. If this is so, then the halo density distribution (and hence the velocity distribution) depends on both the gravitational potential and velocity distribution of this unseen material.

It has been recently pointed out by Illingsworth (1977) that the velocity dispersions in the cores of elliptical galaxies are not isotropic. If this is found to be true, then the dynamical model for King's model's has no applicability to elliptical galaxies.

Finally, Schweizer (1979/81) recently pointed out that, except in galaxies in the Local Group, the central portions of galaxies are not being resolved. This would imply that King's models overestimate the core radii of galaxies (except in the Local Group) by an order of magnitude. While the dynamical construction of King's models are not affected by this fact; it does call into question any conclusions about the dynamic's in the cores of elliptical galaxies.

Thus, while the dynamical significance of the 'standard' King model is in some doubt with respect to elliptical galaxies, they are used in preference to the relations of deVaucouleurs and Hubble because they do盆 not fit the centres of elliptical galaxies well. A brief description of the dynamical calculations behind King's models is found in Appendix 3.

### 4.2 Model Fitting

The model fitting process consists of scaling the 'standard' King model to each of the intensity profiles for each of the eight semi-axes ) of the Al775 system. The scaling process is in both the radial and
intensity co-ordinates and the fitting process was carried out in two steps.

The first step involves plotting the eight averaged intensity profiles in the same scale as that of the 'standard' King model (the sćale used was $\frac{1}{4}$-inch for 0.10 in $\log _{10} r$ ). A sliding fit was made in the radial co-ordinate between the unscaled King model and the intensity profile in question. This was used to obtain the apparent core radius (in view of Schweizer's work, it would be inappropriate to denote this fitted value as the core radius - implying the intrinsic value). At the same time limits were found on this apparent core radius. This was found by finding where the sliding fit was only tolerable in both -directions radially.

- The points defining the given intensity profile, along with its the apparent core radius, and with the unscaled standard King model (in the Form of points) were used as input to a computer programe which performed s the second step in the model fitting process:

Three other parameters are also needed by this computer programme. These are arbitrary maximum and minimum trial values for the central intensity, and the number of steps desired between these two values.

The second step in the model fitting process was performed by a computer programmed called REDUCE (see Appendix 4 for a listing of the code). The points defining a given intensity profile and the unscaled King model ( $c=2.25$ ) are entered into the programe logarithmically and are converted into linear co-ordinates: The radial co-ordinate of the King model is then scaled by using the value of the apparent core radius, found in the above procedure, and the King model is truncated at the outermost point of the intensity profile. The King model is then scaled
for the central intensity. The central intensity of the King model is first set equal to the arbitrary minimum yalue of the intensity used as input to the programe. . This intensity is then increased by a given amount (determined by the difference between the maximum and minimum intensity values and the number of steps) and the central intensity is stepped through a series of values, from the minimum to the maximum input yalues for the intensity. Thus a family of king-models is formed, each with the same apparent core radius; but with different central intensities for each of the eight semi-axes of the Al775 system.

An interpolation between each of the King model's and the points defining the observed intensity profile is theit performed., The points defining a given scaled King model are numerically fịtted by a cubic spline in a smooth curve. At each radial value of the -observed intensity profile; a value of intensity on the King model is found by cubic spline interpolation. The observed and interpolated intensities are then compared at each radial, posption, and the differences (in the sense observed minus interpolated) calculated. A root mean square (RMS) deviation is" calculated from these differences, and the total RMS deviation is divided by the central intensity of the King model in question. The ratio formed by this process indicates the goodness of fit of the scaled model to the observed profile. The interpolation process was carried out for each of the scaled King models in each family for all eight semi-axes. The smallest. value of the ratio of RMS deviation over central intensity was considered to be the best fitting: model with a given apparent core radius. -The entire interpolation process and the construction of families of - King models outlined above was carried out for each of the eight semi-axes with different values of apparent core nadus.

Each of the core radii found in the firs't step of the model fitting process was found to have an uncertainty of about $\pm 0.20$ arc-seconds.

Each of the semi-axes of the Al775 system was fitted using a total of 5 values of appap core radii from the maximum to minimum values of the uncertainty The steps in apparent core radius were ~ 0.10 arcsectnds. Thereffe for each of the eight semi-axes, one was left with. five 'best-fitting!' 'King models. A plot of the RMS deviation central intensity ratio versus core radius was made for each of the semi-axes, and a. smooth curve was fitted by eye through the points. The apparent core radius and central intensity ( $r_{c}$, app and $I_{0}$ ) corresponding to the minimum of the smooth curve were used as the parameters for the overgall - best fitting King model. This was done for each of the eight semi-axes. Table VIII lists the best fitting apparent core radii and central intensities for each of the semi-axes. Figures 3 to 10 are calibrated intensity profiles for each of the semi-axes. Also on each of the figures are the best fitting. King models.

Naturally, the goodness of the fit of the models is determined by seeing effects and instrumental smearing of the observed data. By inspection of Figures 3 to 10 , one sees that the best fitting portions of the King models is found in the same radial interval as the region used for matching profiles in the stacking process (see Section 3.5). This is natural as the uncertainties in the data are minimized in this region. Inspection of the stacking runs (see Appendix 2) indicates that the central regions of a profile are fainter in intensity for longer exposure plates than shorter exposure ones. This is in part caused by seeing effects and slit smearing in the microdensitometer and is partly: the cause of the RMS deviation calculated in Section III. However,
stacking the plates was done partly to eliminate this very problem as fitting models to unaveraged profiles would lead one to conclude that the central intensity would be fainter for progressively longer exposures. By fitting standard King models to averaged profiles, one is minimizing the seeing and instrumental uncertainties; and it is felt that the best fitting King models found in this study are not substantially affected by the above uncertainties.

Finally, it should be noted that in the modelling fitting process, all the points on the observed intensity profiles were used, with the exception of points which defined identified field objects intersecting . the intensity profile. These points were omitted and replaced by points representing an eye interpolation of the galaxy's profile. An example of this process is seen in. Figure 3: The reason for using all the points on a profile [even in light of Schweizer's (1979) work, which indicates the problems involved especially in the central regions] is simple; there were not enough data points to allow one the option of omitting. points, either near the centre or in the outskirts. One was obliged ${ }_{\text {fo }}$ use all: the data points one could get.

The apparent core radii found in the above analysis were 2.74 and 1.68 arc-seconds for the NW and SE components respectively. Using a value of $H_{0}=75 \mathrm{~km} / \mathrm{sec} / \mathrm{Mpc}$, the linear sizes of these core radii are 3.9 and 2.4 kpc respectively. These values are quite a bit larger than the value of $r_{c}$, app for M87, a 'typical' elliptiçal galaxy, of ~ 0.8 kpc , but , half as large as $r_{c \text {, app }}$ for the A2029 cD galaxy of $\sim 7.5 \mathrm{kpc}$. Both of these values are calculated assuming $H_{0}=75 \mathrm{~km} / \mathrm{sec}-/ \mathrm{Mpc}^{-1}$ and are quoted by Dressier (1979). ${ }^{\circ}$

Figure 3. SE component, NE semi-axis versus $\log _{10}$ radius (in arc seconds). The open circles are data points and the full line is the best fitting standard King model.

The error bars are from the RMS deviations found in Table VI.


Figure 4. Same as Figure 3 except for SW semi-axis of SE component.


8 *

$v$


Figure 5. Same as Figure 3 except for NW semi-axis of SE component. Note this semi-axis is between the components and is terminated half way betweèn them.

S


Figure 6. Same as Figure 3 except for SE semi-axis of SE component.
-


Figure 7. NW component, NW semi-äxis; plot of surface brightness versus $\log _{10}$ radius (in arc seconds). The . open circles are data points and the full curve is the best fitting standard King model. The error bars are from Table VI. The triangles are interpotated points to avoid the field object.


$$
\text { Figure 8. Same as Figure } 7 \text { except for SW semi-axis of NW component. }
$$



Figure 9. Same as Figure 7 except for NE semi-axis NW component.


Figure 10. Same as Figure 7 except for SE semi-axis' of Nil comppnent. This semi-axis is midway between the components and is truncated at half way between them.


## SECTION V: COMPOSITE MODELS

### 5.1. Introduction

There are two composite models constructed in this study, and they are constructed using the best fitting King, models described in Section IV above. . The composite models consist of 2 components and are made up of two parts: (1) an isophotal contour map and; (2) an intensity profile of the system major axis.

### 5.2 The Composite Models; The CC Model

For ease of notation, the composite models will be denoted by CC and CE.

The CC composite model is composed of two circularly symmetric scaled ' King models, while the CE model is composed of one circularly symmetric King model and one elliptical King model. The elliptical King model was constructed because it was found by inspecting the isophotes of the Al775 double galaxy that the SE component was elliptical in form (see Figures 2 and 11).

The composite models were constructed in the following manner; standard King modets:(i.e. $c=2.25$ ) were scaled using the averaged values of the apparent core radius and central intensity found in Table VIII for the SE and NW components. The scaling procedure was carried out by means of a computer programme called MODEL (see Appendix 5). The computer programe uses as input the averaged values for the apparent core radius and central intensity for a given component of the A177.5.system, and also the points which define the unscaled standard King models. Also used as input is a cut-off radius. The cut off radius is an arbitrary radial limit to the component, being modelled. A cut off radius was used

Figure 11. Smoothed isophotes (taken from Figure 2a) of the Al775 double system. The dotted circles are the aperature diaphrams used in the photo-electric photometry of the system.


## TABLE VIII

Apparent core radii, modelled central intensities and calibrated surface.brightnesses for each semi-axis of the Al775 system

*The uncertainties are the standard deviations.
because a scaled King model with $c=2.25$ and $r_{c}$, app $=2$ arc-seconds can be extended out to $356 \mathrm{arc}-\mathrm{seconds}$ from the central regions. This angular extent is simply not seen in the present data, which is at most some 50 arc-seconds in extent. Therefore, the cut off radius was imposed upon the scaled King models in order to allow one to model the regions for which data exists. The cut off radii for the components were chosen to be 45 arc-seconds. Larger cyt off radii were tried, to determine whether or not the brightness distributions were seriously affected by the cut off. No such problems were encountered.

In constructing the composite models, the model isophotes were constructed firis.' The centres of the NW and SE model components were separated by an amount, which when scaled, gives the true angular separation of the components in the plane of the sky. This separation was measured to be $20.71 \pm 0.05$ arc-seconds, and was found by direct measurement of the isophotes in Figure 2.

The next step in the model construction is to use the output of the computer programme MODEL to obtain contours of equal intensity for each of the appropriately scaled components. Radial values are read off the scaled King models for each of the components in 2 arc-second intervals, from the centre to the cut off radius: As these radii are all associated with specific intensities (which is due to the circular symmetry of the models), one may draw in the contours of constant intensity simply by using a compass. This is not true for the CE model, as the SE.component. is modelled as elliptical. The CE model will be discussed below in more detail. Finally one displaces the centres of the models by the scaled separation. Where the contours of intensity intersect, they are identified and the net intensities (i.e. the sums of the individual intensities) are
calculated. The net intensities are then 'binned' into intervals of . 0.10 in the logarithim of the relative intensity. Any points within a given 'bin' were joined together, to form the contours of the modets. The 'bin' intervals correspond to 0.25 magnitudes per square arc-second in terms of surface brightness. The contours were drawn in by hand using French curves.

### 5.3 The CE Model

The same procedure was carried out for the. CE model, except that the SE component was modelled as elliptical in form. As mentioned above, eye inspection fef the isophotes of the Al775 system (Figures 2 or lí) show that the SE component is elliptical in form. The eccentricity of the elliptical contours was found by measuring major/minor axis pairs of the 10 inner-most smoothed isophotes (see Figure 11). The eccentricity was found for each majorfminor axis pair, and from these calculated eccentricities, an average value was found. The value of the average eccentricity was found to be

$$
\langle e\rangle=0.28 \pm 0.02
$$

The same process was carried out for the NW component, and it was found to be essentially circular.

In constructing the elliptical intensity contours of the SE component, the profile of the intensity, along the system major axis was constructed. first. By eye inspection; the major axis of the SE component was found 'to be in an essentially north-south orientation, as opposed to the system major axis, which had a posítion angle of $128^{\circ} \pm 0.5^{\circ}$ (by direct measurement). Then, given the known difference-in orientation between the system and component major axes; one could calculate radial values on the SE component's major and minor axes by the relations

$$
\begin{equation*}
a_{i}=\frac{r_{i}(1+e \cos D)}{\left(1-e^{2}\right)} \tag{5.1}
\end{equation*}
$$

and

$$
\begin{equation*}
b_{i}=\frac{r_{i}(1+e \cos D)}{\left(1-e^{2}\right)^{\frac{1}{2}}} \tag{5.2}
\end{equation*}
$$

where $D$ is the angle between the system and component major axes, and $r_{i}$ is a radial position along the system major axis. With the use of the above equations it was then possible to construct an elliptical model of the SE component which fitted the observed intensity profile along the syṣtem major axis, and hence it was possible to draw contours of intensity with an ellipsograph. The model contours were constructed in the same fashion as those for the CC model outlined above.. The model system major axes were constructed from the isophotal contour maps of both the CC and CE models: These model system major axes are plotted along with the data points defining the observed system major axis. The parameters for both the CC and CE models are found in Table IX. The CC model is found in Figures, 12 and 13, and the CE model is found in Figures 14 and 15.

### 5.4 Other Models

The CC and CE models were constructed by fitting each of the components separately, and then super-imposing them to form the composite models: By inspecting Figures 13 and 75 , one notes that the outskirts of the profiles along the system major axis are over-estimated by the composite models by - 0.5 mags/arc- $\sec ^{2}$. Therefore, modifications were made to the composite models to see if the problem in the outskirts could be rectified.

The approach taken was to try to fit the overat brightness profile

Figure 12. Model isophotes for the CC model of the Al 775 system. The scale is $2 \mathrm{~cm}=12^{\pi}$. The contour interval is $0.25 \mathrm{mag} / \mathrm{a}^{\pi}$.



3

－Fìgứre ${ }^{\text {s．}} 138^{\circ}$ Model system major axis for the CC model．The dashed
line is the model and the filled circles are the observed points
$f^{\prime \prime}$ of the profile．




Figure 14. Model isophotes of the CE model. The SE component is modelled as elliptical. Same contour interval as Figure -12 and the scale is $3 \mathrm{~cm}=12^{\pi}$.


Figure 15. Model system major axis for the CE model. The symbols have the same meaning as in Figure 13.


without worrying about whether or not the individual companents were well modelled. . To do so, the apparent core radii of each component was changed. The central intensities (and therefore the surface bright'nesses) were not changed, although it is certain that the values found under-estimate the true surface brightnesses at the centres, due to seeing and instrumentation effects. The reason for not changing the central surface brightnesses, is that the intermediate regions of the profiles (i.e. $r=2 \mathrm{arc}-\mathrm{sec}$ to $\mathrm{r}^{-8} 8 \mathrm{arc}-\mathrm{sec}$ ) are fairly well modelled, and in these regions the seeing and smearing effects of the instruments should be small, implying that the distribution of brightness seen.is the true distribution of light.

The values of the apparent core radii were reduced to one-half and $3 / 4$ of the values found in Table IX, for each component. With these modifications, model system major axes were constructed. It was found that by reducing the apparent core radii of each component by $1 / 2$, the brightness profile produced badly under-estimated the observed profile everywhere, including the outskirts (see Figure 16). By reducing the apparent core rad by a factor of $1 / 4$, the outskirts of the profile were fairly well modelled, but the inner portions of profile were, again, underestimated (see Figure 17).

To sum up then, in decreasing the apparent core radii; the fit to the observed brightness profile was not at all improved. One simply ended up exchanging an over-estimation in brightness in the outskirts for an under-estimation in the inner portions of the profile. As one is only redistributing the bad fit of the models and not elliminating,it, it was decided to retain the CC and CE models as originally constructed.

TABLE IX

## Components of the Composite Models

| Model | Components | $\begin{aligned} & \text { Appárent Core } \\ & \text { Radius } \\ & r_{c, a p p . ~}(\operatorname{arc}-\mathrm{sec}) \end{aligned}$ | Central Intensity (as in PROFIL) | Surface Brightness $\mu_{v}(\operatorname{mag} / \square \pi)$ |
| :---: | :---: | :---: | :---: | :---: |
| CC | $\begin{aligned} & \text { SE } \\ & \text { NW } \end{aligned}$ | $\begin{aligned} & 1.68 \\ & 2.75 \end{aligned}$ |  | $\begin{aligned} & 19.24 \\ & 19.53 \end{aligned}$ |
| CE | $\begin{aligned} & \mathrm{SE} \\ & \mathrm{NW} \end{aligned}$ | $\begin{aligned} & 1.68 \\ & 2.74 \end{aligned}$ | $\begin{array}{r} 1189 \\ 927 \end{array}$ | $\begin{aligned} & 19.24 \\ & 19.53 \end{aligned}$ |

8

Figure 16. The model system major axis of the Al775. system, with the values of the apparent core radii of the two components reduced by a factor of one-hal.f.

The model is the dotted line, while the observed data are the filled circles.



Figure 17. The model system major-axis with the values of the apparent core radii reduced by a factor of one-quarter. Symbols have the same meaning as in Figure 16.



SECTION VI: CONCLUSIONS AND DISCUSSION

It is possible to draw a few conclusions about the Al775 double galaxy upon the basis of the composite models constructed in Section $V$ above.

Consider Figure 11, the smoothed isophotes of the Al775 double. One notes that the most distinctive feature of these isophotes is the fact that the two centres of light are surrounded by a halo with a very distinct 'box' shape. This box-shaped halo is qualitatively well repro- duced by the isophotes of the CE model. (Figure 14).

By modelling the SE component of the Al775 system as elliptical, the fit to the observed isophotes in both the central regions of the SE componert and in the halo, is significantly improved over the isophotes of the CC model. On the face of it, this is what would be expected in the case of the superposition of two unrelated elliptical galaxies, but it may be possible to form such a distinctively box-like halo through af gravitational interaction as well.

If one now considers the brightness profile of the Al775 system along the system major axis, one notices immediately that in both the CC and CE models (Figures 13 and 15 respectively), the outskirts of the profile are over-estimated by about 3 times (or roughly 1 magnitude). As outlined in Section 5,4 above, experiments were carried out to see if it was possible to eliminate or at least minimize this over-estimation by adjusting the apparent core radiíof each of the components. The results of these experiments were either to shift the portion of the brightness profile where the composite models were bad fits or to make the overall fit to-the observed profile worse.

A couple of points can be made in Itght of the results. of the. models constructed. These points can best be outlined under the two major possibilities concerning the nature of the Al775 double system. The two possibilities are that: 1) the Al775 system is the result of a superposition of unrelated and non-interacting galaxies or; 2) the system is interacting (and is either bound or unbound).

Consider the case if the Al775 system is a superposition of two unrelated galaxies. From the results of the models anduthe results of trying to modify these models, as outlined in Section 5.4 above, one. can conclude that both of the galaxies which are superimposed must have brightness profiles which-are different than those of most other Targe isolated elliptical systems. This conclusion is unattractive for two reasons. First, if both of the components are members of the Al775 cluster, then the photometry is suç as to indicate that each is about as luminous as a typical bright elliptical galaxy. One would therefore expect the standard $(c \doteq 2.25)$ King model to be appropriate. The second reason is that this sets up a special case for the Al775 cluster. However, it should be pointed out that throughout this study, it has been tacitly assumed that the standard King model is appropriate. While this seems to be the case in light of the present photometry, some investigators (e.g. 'King 1978) have found that some elliptical galaxies in rich clusters of galaxies are better fitted by King models with $c \simeq 2.00$ (i.e. with steeper brightness profiles). This possibility does not seem too likely in view of the fact that one was able to get quite good fits to the data by fitting each component individually, using the standard King model. It is for these reasons, that the superposition model for the Al775 system is considered unlikely.

Consider next the possibility that the Al775 double is the result 2 of some sort of interaction. In this case, one might reasonably expect. the procedure of fitting the standard King models to each of the components individually and then superimposing them, to break down somewhat. This is because it is either not posstible or even valid to individully fit the components in a dynamical system. It might be more realistic to consider the overall profile of the system and try to fit it without any regard to how good the fit is to the individual components.

By way of a qualitative example, consider a King model with $\log _{10}$ $r_{t} / f_{c} \equiv c=2.00$. This model has a steeper brightness gradient, but in the centrat regions of the model', there is not too much difference from the standard $(c=2.25)$ Ring model $\ldots$. It, is only in the outskirts ©. (as is to be expected) where the differences of the profiles manifest
 the present data cüts off (at ${ }_{p}=24.5$ mags/arc-sec ${ }^{2}$ or' $r=32$ arcsec ), the difference between the fow King models is about a factom of 3 in intensity, with the $c=2.25$ model being-brighter. One might reasonably expect a comporite model formed from two appropriately scaled King models with $\mathrm{C}=2: 00$ to bealmoft the same as the CC and CE mets constructed using $c=2 \$ 5$, in the inner portions, but to be fainter in the outskirts. More over, not only might the fit be Better to the observed brkantness profile, but one would al so have a plausible explanation for the existanceof the steeper gradients in the central regions of the profile... The steeper gradients, would be attributable to tidal interaction affects, which would cause the surface obrightness to fall off more rapidly than in an isolated system. This sort of tidal steepening of the brightness profíte has been seen, in

NGC 4486B and NGC 5846A (e.g. Faber 1973).
Another interesting point which can be brought out is to consider the Al775 system not only to be interacting but to be bound. If this were the case, then one would expect the centres of light to be surrounded and embedded in a large diffuse halo. The halo could be formed by one of two mechanisms. If a binary galaxy is formed out of a proto-galactic nebula, then some of the original material may not have collapsed into the central regions. Assuming little overall net rotation (Illingsworth 1977), athìs material could be quitex extention and its distribution isotropic. If on the other hand, the A1775 system was fokmed by a tidal medger; thay the halo may be stripped material which, while lost to the individual galaxies, is.retained by both in common. In any event, one would expect a large diffuse compon halo about the Al775 system such as is seen in $\because$ the $A 2029$ and $A 2670$ CD gataxies

Unfortünately, Dressler's (1979) study of the A2029 cD galaxy only goes down to $\mu_{v} \simeq 24$ mags/arc- $\sec ^{2}$, which is to about the same surface brightness as the present study. Comparing the A1775 and A2029 systems does not lead to any useful results, as the A2029 sjistem looks just like the Ai775 system except.it has been radially scaled larger. There is no evidence in Dressler's sdata for A2029, that the halo has been detected. The study of the A2670 cD by Oemler (1973) is far more promising in that his data extends down to $\mu_{v}=30 \mathrm{mag} / \mathrm{arc}^{2}-\mathrm{sec}^{2}$, and there is - definite evidence for a large diffuse hato.

Unfortunately, the evidence for the halo first appears at $\mu_{v} \simeq 26$ mags/arc-sec ${ }^{2}$, which is about a magnitude-and-a-half fainter than the cut off of the surface-brightness data in this present data. Again no useful conctusions can be drawn by comparing the A1775 and A2670 systems due to
the lack of surface brightness data on the Al775 system. One cannot say anything about the existance of a large faint halo which may surround the Al775 system.

To sum up then, the A1775 double seems to be some sort of interacting system. It is unlikely to be a superposition of two unrelated ellipticals as these objects would not be fitted by a standard king $\because$ model, and this would require these ellipticals to have surface brightness profiles different than other isolated elliptical galaxies. Due to the lack of surface brightness/data, no conclusionscican be drawn about the nature of the interaction that the Al775 system is undergoing, nor can information be gained about the interaction by comparing the Al 775 data with other large galaxies.

Further study of the 11775 double system, and systems Jike it; are desirable as these systems may be intemediate between isolated ellipticals in clusters in rich clusters, and the large cannibalistic CD galaxies seen in the centres of rich clusters. Such a study will need to be carried out on large instruments (e.g. the KPNO 4 metre) in order to obtain high resolution and to reach fainter surface brightness levels. Thus, one may be able to come to morerpositive conciusions concerning the nature of these types of systems.

## NOTE TO APPENDICIES

All the programmes used in this study are listed excepting the programmes GALAXY and PRQFIL. More information on these two programmes may be obtained from their respective aúthors. Mr. T. J. Deveau and Dr. G. A. We]ch.

All the programmes are written in FORTRAN IV, except GALAXY which is written in POP BASIC. All of the FORTRAN programmes were run in the BATCH mode on the Dalhousie University Cyber 170. A. sample BATCH command programe is found in Appendix 2. The programme GALAXY was run on a time-sharing basis on the St. Mary's Unjversity PDP $11 / 7 \mathrm{Ca}$. All of the out-puts of the BATCH-run programmes were directed to the line printer at St. Mary's University.

The subsystem GALAXY is a computer routine written for the St.: Mary's University PDP $11 / 70$ computer and uses a-Tekronix 4662 plotter. The subsystem was written by Mr. T. J. Deveau and is used to digitize microdensitometer tracings, forming data files which may be processed further.

The digitizing process is as follows, the microdensitometer tracing is placed on the Tekronix ${ }^{\text {phot er, which is set in data trans- }}$ mission mode, and the system GALAXY is called. Three points are selected to form a co-ordinate axis on the tracing, and these points are digitized. Starting at the extreme left end of the tracing, points along the tracing are digitized at 1 mm intervals. The 3 points defining the coordinate axis are redigitized to check for any movement of the tracing during the digitization process. Upon successfully digitizing a tracing, one enters a name for the data file which has been created and stored on the PDP 11/70. These data files may be transferred to the Dalhousie University Cyber 170 by calling the programme CDC. QUE -and the file name of the data one wishes transmitted.

Further information concerning this subsystem may be obtained from either the present writer or Mr. Deveau:


APPENDIX 2: THE PROGRAMME PROFIL

The programme PROFIL converts the densities of the digitized microdensitometer tracings into relative intensity-radius profiles. The programme can plot the intensity profiles individually, or averaged together.

The input data cards, formats and type of data being entered into ,the programme are listed below. More information concerning profil can be found from English (1979). Included in this appendix are the file used to submit the programme and any data files to the Dalhousie University Cyber 170 computer and. a sample run of PROFIL in the averaging mode. The run is of 5 plates on the system major axis and is the NW semi-axis of the NH component.

## Input Data Cards (In order of appearance)



APPENDIX 2 (cont.)

| NAME | FORMAT | \# CARDS | COMMENTS |
| :---: | :---: | :---: | :---: |
| SKYCAL | F5. 1 | 1. | -density of sky (in mm) above clear plate level |
| ISKY | 2014 | $\leq 28$ | -digitized sky tracing |
| IGAL | 214 | 1 | -location of galaxy nucleus on sky tracing |
| IOBJ | 2014 | $\leq 28$ | -digitized galaxy tracing |
| ICENTR | 214 | 1 | -location of nucleus on galaxy tracing |
| NPLOT - | 12 | 1 | $\begin{aligned} -00 & =\text { no plots } \\ 01 & =\text { plots on a plotter } \\ 10 & =\text { plots on a line printer } \end{aligned}$ |
| DEL | F4. 2 | 1 | -included if NPLDT $\neq 00$. It governs the incremient for selecting the points to be plotted. |

$=$

## Sample Run of PROFIL

NW component, NW semi -axis.
All five plates of this study have been stacked (averaged) to form the $10 t h$ profile found in this run.
$4-2$


AOL.XY, TOTO, CH11 10000.
USER;SMASOO4 GIALAXY.
PRINTAT(SMU)
GET(BINBRT1)
GET(K.IAXIB)
BINBRT 1 (M, IAXIS)
BEPLACESOUIPUITHUTFILE/NE)
NOTIFY, TRACING RAN OK.
SKIP,EOJ.
EXIT,
NOTIFY, TRACTNG ROMBED
ENDIF,EOJ.
DAYFILE(DAYFILE)
REPL.ACE(HATFILE)
JEOF
1

0





















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## APPENDIX 3: INTRODUCTION

The following appendix deals very briefly with the dynamical arguments used to construct the King models. Only the essential ideas are presented, along with some of the more important equations. The actual description of the construction of a King isothermal model is to be found in King's original paper of 1966.


APPENDIX 3: MATHEMATICAL BACKGROUND FOR STAR CLUSTERS

After a long enough time, an isolated cluster of stars will be in complete statistical equilibrium. Indeed, even the richest star Alusters will be in statistical equilibrium in their centres. Chandrasekhar (1960) and Freeman (1975) have shown that the mean freepath of a star in a cluster is many times the radius of the cluster. Therefore, spatial mixing is more important than relaxation through stellar encounters. Chandrasekhar (1960) has also shown that the structure of a star cluster is closely represented by a solution of the encounterless Liouville equation, with the stellar encounters producing a slow evolution from one solution to another.
$\therefore$ The general solution to the steady-state encounterless Liouville equation was given by deans (1915). The distribution function in phase space must be expressible as a function of the isolating integrals of the equations of motion of a star. For the case of spherical symmetry in position space the only known isolating integrals for a general potential function $V(r)$ are the energy and angular momentum per unit mass,

$$
\begin{align*}
& E=1 / 2 v^{2}+V(r)  \tag{1}\\
& h=r v_{t}
\end{align*}
$$

In the above, $v$ is the magnitudes of the velocity and $v_{t}$, its tangential component. Even after specifying these integrals, Jeans' theorem permits a very large range of cluster models. One needs only to glance at real cluster and galaxies to see that there are strong


similarities between one object of a given class and others. The real clusters and galaxies are best described by a particular set of distribution functions. If one assumes that at the relaxation time at the centre of a galaxy is a small fraction of its age, it is natural for stellar encounters to provide the regularizing mechanism in galaxies. One approach to the problem is to ask that the stellar encounters determine a velocity distribution, which then in turn determines the spatial characteristics' of the model. This approach has been used by a number of investigators (Chandrasekhar 1960; Spitzer and Härm 1958; King 1966), each of whom chose somewhat different velocity distributions.

The remainder of this discussion follows the work of King.
To start with, consider the ideal, but unattainable Gaussian velocity distribution. This distribution has been shown by Chandrasekhar (1960) to lead to a density distribution that corresponds to an isothermal gas sphere. The model has a total mass that is infinite; when $r$ is large, the mass contained within that radius increases in direct proportion to $r$. In fact, the Gaussian distribution could have been rejected initially, as a cluster or galaxy cannot retain stars whose velocity exceeds a finite escape velocity. What is needed is a velocity distribution that will be produced by ste?lar encounters, yet drops to zero at a finite limiting velocity. The mechanism for solving this problem was provided by Chandrasekhar (1943), who introduced the Fokker-Planck equation into stellar dynamics to calculate the effect of encounters on a velocity distribution. The Fokker-Planck equation is a differential equation involving the probability function governing the occurrence of a velocity at a given time. The probability function must satisfy the diffusion equation:


$$
\frac{\partial W}{\partial t}=q \nabla^{2} w
$$

where $q$ is the diffusion co-efficient and $w$ is the probability function.
To describe the veiocity distribution in acluster, Chandrasekhar found a steady-state solution of the Fokker-planck equation with a finite cut off velocity. He used this solution only to determine the rate of escape of stars, and not the velocity distribution expicicitly.

Spitzer and Härm (ig58) tabulated the steady-state velocity distribution and attempted to use this distribution to derive a cluster model but were unsuccessful. No matter what they chose for the central value of the potential, the density of their model went to zero at some finite value of the radius, contradicting their identification of the cut off velocity with the escape velocity of the cluster.

In the approach used by King (1966), the model is constructed by choosing a velocity distributionat the centre of the model of the form.

$$
\begin{equation*}
f(0, v)=k\left[\exp \left(-j^{2} v^{2}\right)-\exp \left(j^{2} v_{e}^{2}\right)\right] \tag{4}
\end{equation*}
$$

where $v_{e}$ is the escape velocity and $j$ is given by

$$
\begin{equation*}
j^{2}=2 / 3\langle v\rangle^{2}, . \tag{5}
\end{equation*}
$$

- 

where <v> is the mean velocity of a Maxwell-Boltzmann distribution.
The energy integral for a star is

$$
\begin{equation*}
E=1 / 2 v^{2}+V(r) \tag{5}
\end{equation*}
$$

If one lets $V(r)$ equal zero at the surface of the cluster, a star with zero energy then is barely able to reach the surface. The escape velocity is thus given at any point by
-

$$
\begin{equation*}
\because v_{e}^{2}=-2 v(r) \tag{6}
\end{equation*}
$$

Above zero energy, $f(v)$ is taken to be zero since stars of positive energy have escaped from the cluster. Tidal distortion of the shape of the cluster has been neglected in the discussion. It affects only the outermost regions and is taken into account by applying perturbation theory at a-later stage.

In terms of $E$, the distribution function at the centre of the cluster is

$$
\begin{equation*}
f(0, v)=k\left(\exp \left(-2 j^{2} \cdot v(0)\right)\left[\exp \left(-2 E j^{2}\right)-1\right] .\right. \tag{7}
\end{equation*}
$$

But according to Jeans' theorem, the distribution function must be the same at all points; therefore at any point the distribution is of the form.

$$
\begin{equation*}
f(r, v)=k\left(\operatorname { e x p } \left(-2 j^{2}(v(r)-v(0))\left[\exp \left[-j^{2} v^{2}\right]-\exp \left(-j^{2} v^{2}\right)\right]\right.\right. \tag{8}
\end{equation*}
$$

Equation (8) is of the same form as equation (4). That is, the velocity distribution at every point within the cluster is the appropriate steadystate solution to the Fokker-Planck equation for the velocity cut off that applies at that point. Stellar encounters are automatically taken into account everywhere in the cluster, not only at the centre.

The density at any point is found by integrating the velocity distribution $f(r, v)$ with respect to velocity. As a simplification, the following substitutions are made. .

$$
\begin{align*}
& w=-2 v(r) j^{2}  \tag{9}\\
& \text { and } \\
& n=j^{2} v^{2} \tag{10}
\end{align*}
$$

However, in performing this substitution, one no longer has an equation for the density in terms of potential (and hence radius); but
as a function of the variabies $w$ and $n$. The $w(r)$ relation is solved by using Poisson's equation
$2 \quad \frac{\partial^{2} w(r)}{\partial r^{2}}+\frac{2}{r} \frac{\partial V(r)}{\partial r}=4 \pi G \rho$
In terms of a dimensionless radius $R\left(\equiv \frac{r}{r_{\text {core }}}\right)$, equation (10) maybe rewritten as

$$
\begin{equation*}
\frac{\partial^{2} W}{\partial R^{2}}+\frac{2}{R} \frac{\partial W}{\partial R}=-8 \pi G \rho j^{2} r_{\text {core }}^{2} \tag{11}
\end{equation*}
$$

Upon substituting for a power series for the function of $w(R)$, one will obtain values of $r$ which will be close to $r_{\text {core }}$ if. the central value of the equation (11) is set to -9 . Therefore, if one sets the constant term on the righthand side of (11) to -9 , one only needs the relative values of the density and not the density itself;

$$
\begin{equation*}
\text { i.e. } \frac{\partial^{2} W}{\partial R^{2}}+\frac{2}{R} \frac{\partial W}{\partial R}=-9 \mathrm{p} / \rho_{0} \tag{12}
\end{equation*}
$$

The calculations to obtain the relation between intensity and radius are performed from equation (12) on the basis that the density is, related to the distribution of iight within the cluster. These caTculations are.... found in King (1966). The interested reader may consult that paper for further details.

The resulting models are modifications of an isothermal gas sphere. Near the centre their densities are very close to isothermal, but projected it falls below the isothermal curve and drops to zero. The projected densities are shown in Figure A-1 (taken from Figure 1 of King 1966), where the surface density is called $f$. The quantities $f$ and $r$ are unitless ratios of the central values $f_{0}$ and $r_{\text {core }}$, respectively. The curves are labelled with the values of the parameter.c.

Figure A-1. The family of King's isothermal spheres, plotted in terms of brightness relative to the central brightness and radius in terms of the 'core radius' logarithmically.



APPENDIX 4: programme reduce

The programme REDUCE uses as input a dat file containing the points defining a standard King model ( $c=2.25$ ), and points defining the average intensity-radius profile produced.by PROFIL. It then. scales the King model in both radius and intensity and compares the observed data points to the fitting King model. The best fitting King model is printed out, a listing of the programme is given below, as is a iist of the input data cards.

Input Data Cards (In Order)

| NAME | FORMAT | \# CARDS | COMMENTS |
| :---: | :---: | :---: | :---: |
| NPKR, NPKB, NPOR, NPOB | 4(2X, 13) | 1 | - -number of points in the standard King model and the observed profiles |
| KINGR | 1058.3. | <20 | -the radial values of the King model |
| KINGB | 10FB. 3 | $<20$ | -the intensity values of the King model |
| OBSR | 10F8. 3 | $<20$ | -the radial value of the data points |
| OBSB ${ }^{\text {- }}$ | 10F8.3 | <20 | -the iptensity values of the data points |
| RCORE | $F 5.2$ | 1 | -the value of the core radius of the King. <br> - model. |
| BVALU, NSTEPS, aXIS, VALUE, PLATE | F10.3, 3X, 13, 3X, A5, 3X, F5.0, 3X, A6 |  | -the min. value of intensity to start scaling process; the number of steps for the scaling. The semi-axis being fitted. The maximum value of intensity and the name of the galaxy. |


FOFFinT ( $10=8.3$ )
IF (055R(KM). 50.0) 60 T0 5001
cominut

6010690
-
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GRIIE $(6,1034)$

WRITE(G,1035) (CTUKP (J), J=1,LAST3)
1035

C.
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$006000 \mathrm{~J}=1,15$
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FORKAT(10FB.3)
IFOESSR(LL).ED.0) 50 TO 6001
6000

101

STEPS=FLOAT(NSTEPS)
IELTA=(VALUE-WUALU/STEPS
DELTA (VALUE-
DO $50 . \mathrm{KK}=1$ +NSTEFS
CALL MAX (KBRT, MAXBFT, IEND)
DO 15 II $=1$ IEND
BNCRKIKK:II)=(EVALU/KAXERT)LSEKTUII
15


$\mathrm{CHI}(\mathrm{KK})=\mathrm{CHIOHE}$
RATIO(KK)=CHI (KK)/RUALI
bualuzqualutbleta
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XKTN=RATID:1)
DO $51 \mathrm{KK}=1$, NSTEPS
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B. CORTINJE

C
C . NOU TO URITE EVERYTHING OUT.



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84 FORMAT(/),15X,' RADILUS(ARC-SEC, ' 'BX;' INIENSIIY '


## AXI5, FLATE





82. FG:HAT $\left(Y, 16 X, F 10.4,14 X, F 10,4,18 X_{F} F 10.5 ; 12 \times, F 12.6\right)$



 MRITE 0,85 ) AXIS HRIE (6.99) RCIRE


- 2 -

2 Mr

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1111 COHITMUE
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$C$
$C$
$C$
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15 Cohtinue
EAG
$\stackrel{C}{c}$

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REAL LBRT
GIMENSION LEAT (NP), RRT(SM)
DO $20 \quad \mathrm{JJ}=1, \mathrm{KP}$
RRT(J) $=10 . * *(\operatorname{LRRT}(J J))$
REIURH
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${ }_{c}^{c}$

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$C$
$c$
$c$

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a cohinus

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END.
C

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$1 P 1=10+1$


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11 comtinue
IPV $=0$
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$U K=U(x)$





## APPENDIX 5: PROGRAMME MODEL

After all 4 -semi-axes of each nucleus have been fitted using the programme REDUCE (Appendix 3), one obtains average values for the core radius and the central intensity. The programme MODEL is designed to scale a King model from the average core radii and central intensities. These model nuclei are then used to construct the model system major ones and isoplates.

A listing of the programe and the input data cards is given below.

Input Data Cards (In Order)

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/list
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$C$
$C$
$C$
$C$
$C$
$C$
$C$
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C DATA POINTS OF THE KING KITEL AND NOEMALIZES JHEK TO
ع The degerver core rajius aid centahl intenstit.
E THE OUTPUT FDRKGT OF THIS FROGKAMEE IS THE SGKI AS THAT
PROERRK MODEL (IMPUT,


conkon Lastenafi
REAL KRAD, KRRT, MAXBRT
REAL KINGR, KINGB
READ (5,11) MPRKR, NPKB
11 FORHAT(2(2X,13))

CRDII AMB INTESSITIES FOR THE KING MDDEL.

$\begin{array}{ll}\mathrm{y}=1 \\ 80 & 3000 \\ \mathrm{~N}=1,20\end{array}$

1007
IF (KINGRT(AN), Ea.0) 60 TO 3001
$\mathrm{N}=\mathrm{N}+10$
3000
3001

10770 I=1,NPKR
IF(KINGR(I).ME.O) G0 to 770
$60 \mathrm{ASI} 10 \mathrm{I}-1$
77
g

CORTLIUE

WFITE ( 6,1033 ) (ADUKP( J$), J=1,1$ AST1)

'c

FCRRAT (10F8.3)

$k=k+10$
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DO $730 \mathrm{I}=1$, NPKB
IF(KINGE(i), ME O) 6010730
LAST2:I-1
G0 GO 7
-
ADOP(I)=ELMEB(I
COMTINUE
HPRTEE 61634



CALL BKGTHS(KINGE,KEKT,LASI2)

1107


- 22 READ (5,22) RCORE
DO 10 TI=1, LA5I

IF (RABKS (II) ILE: RMAK) 60 IO 10
IENTEII
is EONTMUE
,


1 - FORMAT (FE,O)
CALL HAX (KBRT, MAXBRTIIENB)
DO $15 \cdot 11=1,1 E \mathrm{NB}$

15
CORTINEE $81=1$, IEND

BKLOE(I)=ALOE1OTBNORM(I))
B CONTINUE
NOU TO YRITE EVERYTHITS OUT.


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op
胞 720 I 1, LAST2
AMMP(I)=KLAEG(I)
CONTINUE
URITE $(6,1634)$
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1, :1X, 'KiNG kODE AKE;')

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15
CONTINUE,
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8
$I=1, I E N D$
RKLOG(I)=ALOG10(RADKS(I))
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$c^{c}{ }_{c}^{8}$
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DIMENSION LBRTINP), BRTINP)
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20
contimue
RETURM
END
$\stackrel{c}{c}$
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c this roltile finis the haxigil value of the bata set involvel and

DIMENSIOA X(NP)
$X \mathrm{XAA}=0.0$
DO $111 \mathrm{KI}=1$, NP
IF(XMAX,LT,XKKI)) GOTO 110
60 T0 111
110
XHAXK $\mathrm{x}(\mathrm{Ki})$
CNTIMUE
(


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[^0]:    Note: *value calculated with $\mathrm{H}_{0}=75 \mathrm{~km} / \mathrm{sec} / \mathrm{Mpc}$
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