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The [α/Fe] ratios of very metal-poor stars within the integrated galactic initial mass function theory

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ABSTRACT

The aim of this paper is to quantify the amplitude of the predicted plateau in [α/Fe] ratios associated with the most metal-poor stars of a galaxy. We assume that the initial mass function (IMF) in galaxies is steeper if the star formation rate (SFR) is low – as per the integrated galactic initial mass function (IGIMF) theory. A variant of the theory, in which the IGIMF depends upon the metallicity of the parent galaxy, is also considered. The IGIMF theory predicts low [α/Fe] plateaus in dwarf galaxies, characterized by small SFRs. The [α/Fe] plateau is up to 0.7 dex lower than the corresponding plateau of the Milky Way. For a universal IMF one should expect instead that the [α/Fe] plateau is the same for all the galaxies, irrespective of their masses or SFRs. Assuming a strong dependence of the IMF on the metallicity of the parent galaxy, dwarf galaxies can show values of the [α/Fe] plateau similar to those of the Milky Way, and almost independent of the SFR. The [Mg/Fe] ratios of the most metal-poor stars in dwarf galaxies satellites of the Milky Way can be reproduced either if we consider metallicity-dependent IMFs or if the early SFRs of these galaxies were larger than we presently think. Present and future observations of dwarf galaxies can help disentangle between these different IGIMF formulations.

Key words: stars: abundances – stars: luminosity function, mass function – supernovae: general – galaxies: dwarf – galaxies: evolution – galaxies: star clusters: general.

1 INTRODUCTION

The relative distribution of stellar masses in a given generation (the so-called initial mass function or IMF) is a fundamental entity linking the microphysics of molecular cloud fragmentation with the global characteristics of galactic stellar populations. The IMF regulates the proportion of high- to low-mass stars, and hence, the study of diagnostics such as abundance ratios can constrain its main features, i.e. its normalization, upper and lower ends and shape. The integrated galactic initial mass function (IGIMF) theory (Kroupa & Weidner 2003; Weidner & Kroupa 2005) is aimed at providing a self-consistent description of how the IMF varies as a function of another fundamental characteristic in galaxy evolution models, namely the star formation (SF) history. The IGIMF is based on the fundamental assumptions that (i) most stars form in small stellar groups, i.e. in embedded clusters. Within each cluster, the stars are distributed according to a universal two-slope power-law IMF: $\alpha_1 = 1.3$ below 0.5 $M_{\odot}$ and $\alpha_2 = 2.35$ above 0.5 $M_{\odot}$. The maximum stellar mass in an individual cluster is a function of the cluster mass; (ii) the stellar clusters are distributed according to a single-slope power law and (iii) the maximum possible mass of a star cluster increases with the galactic star formation rate (SFR). The main implication is that low-mass galaxies, characterized by low SF values, present a steeper (top-light) IMF than the Milky Way (MW).

In two previous papers, the IGIMF theory has been applied to study the abundance ratios in different classes of galaxies. Recchi, Calura & Kroupa (2009, hereafter R09) investigated the integrated abundances in local early-type galaxies, showing with a chemical evolution model that the IGIMF theory, coupled with their downsizing character (i.e. the fact that the SF time-scale inversely correlates with the mass of the system), well accounts for the observed correlation between integrated [α/Fe] ratios and mass, or velocity dispersion σ, of the galaxy. In particular, these models predict a change in the [α/Fe] versus σ relation not achievable with a constant IMF. Calura et al. (2010, hereafter C10) applied the IGIMF to a model for

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the solar neighbourhood, showing that a large set of observables can be accounted for and suggesting how the present-day mass function can be used locally in order to disentangle between different IMF scenarios.

In all these studies, the determination of the IMF parameters required the assumption of an SF history, which is unknown a priori and which renders the problem degenerate. In fact, it is well known from chemical evolution studies that a higher SF efficiency mimics the effects of an IMF skewed towards massive stars (see e.g. Matteucci 1994).

A different diagnostic able to provide crucial information on the IMF is the [α/Fe] plateau observed at low metallicity in the Galactic halo (Clegg, Tomkin & Lambert 1981; Barbuy 1988; Edvardsson et al. 1993). The importance of the low-metallicity plateau is that it depends solely on the IMF (Tsujimoto et al. 1997), in the sense that the plateau's value is set entirely by the IMF slope (and massive star yields), irrespective of the uncertain role played by Type Ia supernovae (SNeIa).

The situation is different within the IGIMF theory, in that the plateau depends on the SF history. If it is possible to establish the existence of a [α/Fe] plateau of metal-poor stars in different galaxies, its value will depend on the IGIMF slope and on the upper mass limit, hence it will be a function of the SF history of that system. On the other hand, with a constant, SF-independent IMF, all the galaxies will have the same [α/Fe] plateau, irrespective of the SF history.

To make the previous reasoning more quantitative, let us assume that all the stars with masses larger than 8 $M_\odot$ explode as SNeII. It is possible to establish an average [α/Fe]$_h$ of the interstellar medium in the halo as a consequence of SNII pollution (hence of the halo stars) through the formula

$$\left(\frac{\alpha}{\mathrm{Fe}}\right)_h = \left(\frac{\alpha}{\mathrm{Fe}}\right)_\odot \times 10^{[\alpha/\mathrm{Fe}]_h} = \frac{\int_8^{m_{\mathrm{up}}} m_\alpha^{ej} \varphi(m)\,\mathrm{d}m}{\int_8^{m_{\mathrm{up}}} m_{\mathrm{Fe}}^{ej} \varphi(m)\,\mathrm{d}m}, \qquad (1)$$

where $\varphi(m)$ is the adopted IMF, m_{up} is its upper limit and m_α^{ej}, m_{Fe}^{ej} are the theoretical yields from SNeII (see Tsujimoto et al. 1997; Gibson 1998). Once a compilation of SNII yields is established (for instance, the widely used Woosley & Weaver 1995 compilation), the [α/Fe] plateau is determined solely by the IMF, in particular by its slope at the high-mass range and upper limit. Tsujimoto et al. (1997) showed in this way that the [O/Fe] in Galactic metal-poor halo stars is consistent with an IMF with slope 2.3–2.6 above 1 $M_\odot$ (i.e. similar or slightly larger than the Salpeter index) and an upper mass limit $m_{\mathrm{up}} = 50 \pm 10$ $M_\odot$.

Although Gibson (1998) pointed out all the limits and weaknesses of this approach, nevertheless a point should be made. If an [α/Fe] plateau of metal-poor stars in different galaxies is established, its value can only reflect the IMF slope(s) and upper mass limit. The yields of massive stars, although still very uncertain, cannot change from galaxy to galaxy.[1] If the IMF is the same in all galaxies, the most metal-poor stars must have on average the same [α/Fe], irrespective of the galaxy they belong to. If instead a variation of the [α/Fe] plateau among different galaxies is established, this is a clear indication that the IMF is not universal.

This provides a potentially interesting test for the IGIMF theory because the latter predicts that dwarf galaxies, characterized by average low SFRs, should have steeper IMFs (and lower values of m_{up}) than the MW (Weidner & Kroupa 2005). At variance with the previously published works of R09 and C10, this test for the IGIMF is independent of the weakly constrained role of SNeIa and low- and intermediate-mass stellar yields on the abundance patterns.

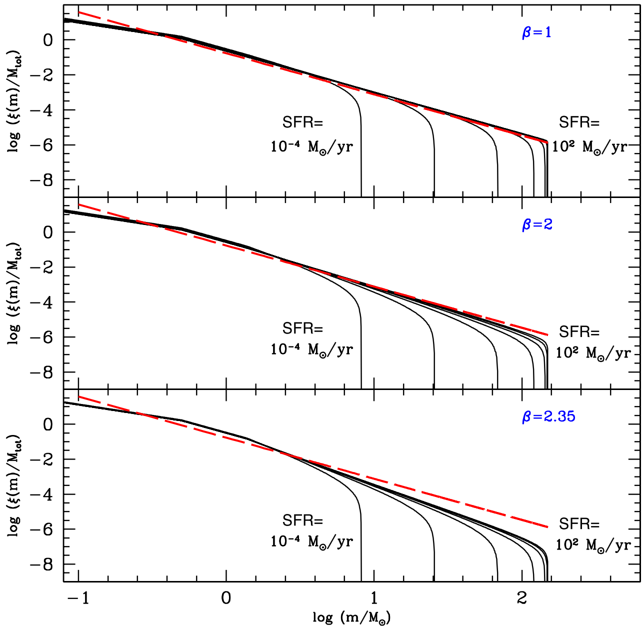

Figure 1. IGIMFs (in logarithm) for different cluster mass functions distributions. Upper panel: $\beta = 1$; central panel: $\beta = 2$; lower panel: $\beta = 2.35$. In each panel, we have considered seven possible values of SFRs, ranging from 10^{-4} $M_\odot$ yr^{-1} (lowermost solid lines) to 100 $M_\odot$ yr^{-1} (uppermost solid lines), equally spaced in logarithm. The thick dashed line is the standard Salpeter IMF. The IGIMFs and the Salpeter IMF are all normalized through the equation $\int m\xi(m)\,\mathrm{d}m = M_{\mathrm{tot}} = 1$ $M_\odot$.

It is worth pointing out that the IMF variations we are dealing with here are concentrated in the upper range of stellar masses. The IGIMF theory predicts little changes in the IMF for stars with low and intermediate masses, whereas the change of the IMF slope for high-mass stars and of the upper mass cut off can be extremely significant (see e.g. fig. 1 of R09 or Fig. 1 in this paper). The change of the IMF in this mass range can directly and significantly affect the [α/Fe] plateaus. Other forms of IMF variations among galaxies have been inferred/deduced recently. On the one hand, it seems that also the low-mass end of the IMF changes according to the galaxy mass (Conroy & van Dokkum 2012; Ferreras et al. 2013). On the other hand, also the metallicity can have a significant role in the SF process, regulating the transition between Population III (PopIII) and PopII stars, whose IMFs are supposed to be very different (see e.g. Bromm et al. 2001; Schneider et al. 2002; see also Section 6 for further discussions on the dependence of the IMF on metallicity).

The paper is organized as follows. In Section 2, the main assumptions behind the IGIMF theory will be shortly summarized; in Section 3.1, we summarize the present knowledge regarding [α/Fe] plateaus in dwarf galaxies. Given the wealth of available data, much of this section will be devoted to the abundance patterns of individual stars in the most luminous dwarf galaxy satellites (DGSs) of the MW, namely the ones for which we know spectroscopic data of single stars with high precision. In Section 3.2, we will also summarize our present knowledge regarding the SFRs and SF histories of these galaxies, given the key role that SF histories play in the IGIMF

[1] Actually, one might be able to envision some scenario where yields could differ from galaxy to galaxy, for instance the fraction of high rotators might differ as a function of environment. However, this hypothesis is very speculative at this stage.

theory. In Section 4, the main assumptions and the calculations required to predict the [α/Fe] plateaus of our model galaxies will be outlined. In Section 5, the results of our investigation will be presented. We will concentrate first (Section 5.1) on the study of the Galactic MW plateau, in order to identify from the literature the most accurate value of the [α/Fe] plateau and to calculate the best set of nucleosynthetic yields able to reproduce it with our simplified approach. In Section 5.2, we will present results concerning the variations in the [α/Fe] plateau as a function of the SFR that we predict with our model. In Section 6.1, results obtained by using new, metallicity-dependent IGIMF formulations (Marks et al. 2012, hereafter M12) are presented. In Section 6.2, we compare our results with available observations. Finally, in Section 7, some conclusions are drawn.

2 THE INTEGRATED GALACTIC INITIAL MASS FUNCTION

In this section, we briefly summarize the main assumptions behind the IGIMF theory, already shortly anticipated in Section 1. A more complete description is provided in Weidner & Kroupa (2005) and R09.

The main assumption of the IGIMF theory is that all the stars in a galaxy form in star clusters. These star clusters need not be gravitationally bound entities, but should be seen as space–time correlated stellar populations, deeply embedded in gas. These embedded clusters have, prior to the removal of their remaining gas, radii of about 0.15 pc, in agreement with molecular cloud cores and the filamentary structures of molecular clouds (André et al. 2010; Hennemann et al. 2012; Marks & Kroupa 2012; Schneider et al. 2012).[2] Within each embedded cluster, the stellar IMF has the canonical form $\xi(m) = km^{-\alpha}$, with $\alpha_1 = 1.3$ for $0.08\ \mathrm{M}_\odot \le m < 0.5\ \mathrm{M}_\odot$ and $\alpha_2 = 2.35$ (i.e. the Salpeter slope) for $0.5\ \mathrm{M}_\odot \le m < m_{\max}$. The upper mass $m_{\max}$ is a function of the mass of the embedded cluster $M_{\rm ecl}$; this is simply due to the fact that small clusters do not have enough mass to produce very massive stars.

On the other hand, the mass distribution function of the correlated SF entities (embedded clusters) is assumed to be a single-slope power law, $\xi_{\rm ecl} \propto M_{\rm ecl}^{-\beta}$, with a slope β close to 2 (Zhang & Fall 1999; Lada & Lada 2003). In this work, as in our previous IGIMF studies, we have considered values of β: 1.00, 2.00 and 2.35. According to the quoted papers and the results of R09 and C10 the reference value for β will be 2, but it is of interest to evaluate the dependence of our results on the choice of β. The combination of the stellar IMF with the mass distribution of embedded clusters yields the IGIMF, i.e. the IMF integrated over the whole population of embedded clusters forming in a galaxy as a function of the SFR $\psi(t)$:

$$\xi_{\rm IGIMF}(m;\psi(t)) = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}(\psi(t))} \xi(m \le m_{\max})\xi_{\rm ecl}(M_{\rm ecl})\,\mathrm{d}M_{\rm ecl}, \qquad (2)$$

where $M_{\rm ecl,min}$ and $M_{\rm ecl,max}(\psi(t))$ are the minimum and maximum possible masses of the embedded clusters in a population of clusters, respectively, and $m_{\max} = m_{\max}(M_{\rm ecl})$. For $M_{\rm ecl,min}$, we assume the value of 5 $\mathrm{M}_\odot$, i.e. the mass of a Taurus–Auriga aggregate, which is arguably the smallest star-forming 'cluster' known. On the other hand, the upper mass of the embedded cluster population depends on the SFR; this fact renders the whole IGIMF dependent on ψ. The existence of a correlation between $M_{\rm ecl,max}$ and SFR has been observationally established (Larsen & Richtler 2000; Weidner, Kroupa & Larsen 2004) and originates from the sampling of clusters from the embedded cluster mass function given the amount of gas mass being turned into stars per unit time (Weidner et al. 2004). In accordance with Weidner et al. (2004), we assume this function to be

$$\log M_{\rm ecl,max} = A + B \log \psi, \qquad (3)$$

with $A = 4.83$ and $B = 0.75$.

In Fig. 1, we show the IGIMF as a function of the SFR for the three values of β considered in this work, compared to a single-slope, Salpeter IMF. The IGIMFs are characterized by a nearly uniform decline, nearly approximated by a power law, and a sharp cutoff at mass values close to $m_{\max}$. The steepest distribution of embedded cluster (in our case the model with $\beta = 2.35$) produces also the steepest IGIMF; this occurs since this distribution is biased towards embedded clusters of low mass, therefore, the probability of finding high-mass stars in this cluster population is lower. Moreover, the dependence of the IGIMF on the SFR is strong for SFR $\le 1\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$ whereas it is weak for SFR $\ge 1\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$ (see R09). This is due to the fact that for SFRs larger than $\sim$1 $\mathrm{M}_\odot\ \mathrm{yr}^{-1}$, the maximum possible value of the embedded cluster mass is very high; therefore, it is always possible to sample massive stars in a galaxy up to a mass very close to the empirical limit (which is assumed to be $m_{\max,*} = 150\ \mathrm{M}_\odot$; see Weidner & Kroupa 2005).

It is worth noticing that here the IMF within each embedded cluster is assumed to always be invariant. That is, neither the possibility of having a top-heavy IMF in starburst clusters (which leads to a top-heavy IGIMF for SFR $> 10\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$; see Weidner, Kroupa & Pflamm-Altenburg 2011; Kroupa et al. 2013), nor the alleged dependence of the IMF in clusters on the metallicity (M12; Kroupa et al. 2013) are incorporated here. The first hypothesis will not be considered in detail since this paper is mostly concerned with dwarf galaxies, whose SFRs cannot reach high values. We will instead study in Section 6.1 the possibility of having a dependence of the IMF on the metal content of the parent galaxy. We remark already, however, that M12 mostly based their conclusions on the study of the mass distribution functions in globular clusters (GCs). It is not clear at this stage if this model can be safely extended to entire galaxies.

A summary of the main IGIMF parameters and their reference values used in this study is provided in Table 1, whereas the main chemical evolution parameters assumed in the study of the MW (Section 5.1) are reported in Table 2.

3 THE SATELLITES OF THE MILKY WAY AS A BENCHMARK OF THE IGIMF THEORY

The best benchmark to test possible variations in the [α/Fe] plateau among different galaxies and link it to the variations of the IMF is the Local Group, with particular emphasis on the MW DGSs. Most of these dwarf galaxies (in particular the *classical* satellites; Kroupa et al. 2010) are very well known and studied, both photometrically and spectroscopically.

[2] Some authors (e.g. Bressert et al. 2010) claim that a significant fraction of massive stars might have formed far from dense environments. It is however worth noticing that a log-normal surface density distribution (taken by Bressert et al. as an evidence of significant SF far from dense environments) can be achieved also by means of stars forming in bound clusters (see e.g. Gieles, Moeckel & Clarke 2012).

Table 1. IGIMF parameters and their reference values used in this study.

Quantity	Adopted values		Ref.	Section
Constant stellar IMF (IMF within a star cluster)	$\xi(m) = k\,m^{-\alpha}$	$\alpha_1 = 1.30$; $0.08\ \mathrm{M_\odot} \le m < 0.5\ \mathrm{M_\odot}$ $\alpha_2 = 2.35$; $0.5\ \mathrm{M_\odot} \le m < m_{\rm max}$	Weidner & Kroupa (2005)	2
Empirical stellar mass limit	$m_{\rm max,*} = 150\,\mathrm{M_\odot}$	$(m_{\rm max} < m_{\rm max,*})$	Weidner & Kroupa (2005)	2
Cluster mass function	$\xi_{\rm ecl} \propto M_{\rm ecl}^{-\beta}$	$\beta = 2$	Zhang & Fall (1999)	2
Minimum cluster mass	$M_{\rm ecl,min} = 5\ \mathrm{M_\odot}$		Weidner & Kroupa (2005)	2
Maximum cluster mass	$M_{\rm ecl,max} = A + B \log \psi$	$A = 4.83$, $B = 0.75$	Weidner & Kroupa (2005)	2

Table 2. MW chemical evolution parameters and reference values used in this study.

Quantity	Adopted values	Ref.	Section
$\langle \mathrm{SFR} \rangle_{1\,\mathrm{Gyr}}$ ($\mathrm{M_\odot\ yr^{-1}}$)	0.5	C10	5.1
[O/Fe]	0.47 ± 0.15	Cayrel et al. (2004)	5.1
[Si/Fe]	0.37 ± 0.15	Cayrel et al. (2004)	5.1
[Mg/Fe]	0.27 ± 0.13	Cayrel et al. (2004)	5.1
$\hat{y}_{\rm Fe}$	0	Fig. 2	5.1
$m_{\rm thr}$	8 $\mathrm{M_\odot}$	–	5.1
$y_{\rm Fe,150}/y_{\rm Fe,40}$	1	–	5.1

3.1 The [α/Fe] plateau in DGSs

A wealth of data concerning the chemical composition of individual stars in DGSs is now available (see for instance Kirby et al. 2011, and references therein).

Useful data concerning [α/Fe] ratios in DGSs come from the Dwarf Abundances and Radial Velocities Team (DART) experiment (Tolstoy et al. 2004), which is dedicated to the measure of abundances and velocities for several hundred individual stars in a sample of four nearby (luminous) dwarf galaxies: Sculptor, Fornax, Sextans and Carina. Among these galaxies, perhaps the best example of a [α/Fe] halo plateau is given by Fornax (Letarte et al. 2010). Looking at the [Mg/Fe] ratios in Fornax, there does not seem to be a change in [α/Fe] for stars more metal-poor than [Fe/H] $= -1$ and the average value is about 0.1 dex. Only one star in their sample has [α/Fe] $\simeq 0.4$; the other [α/Fe] ratios are all close to solar (see also de Boer et al. 2012b). In addition, GCs in Fornax have an average [Mg/Fe] $= 0.025$ (Larsen, Brodie & Strader 2012). In Sextans, too, most of the observed stars have [Mg/Fe] ratios close to the solar value (Aoki et al. 2009).

Also Sagittarius seems to lack (or to have very few) metal-poor stars with the same [Mg/Fe] of metal-poor halo stars of the MW, although in this case the establishment of a plateau is much more uncertain (see Tolstoy, Hill & Tosi 2009). Moreover, recently McWilliam, Wallerstein & Mottini (2013) analysed three very α-deficient stars in Sagittarius and concluded that their abundance ratios can only be compatible with an IMF very deficient in massive stars. They thus argued that the abundance ratios in Sagittarius can be well explained by the IGIMF theory. Carina shows large dispersions of [Mg/Fe] ratios, perhaps indicative of inhomogeneous mixing of supernovae ejecta (Venn et al. 2012). A more recent, larger sample of Carina stars (Lemasle et al. 2012) seems also to indicate that the [Mg/Fe] of metal-poor stars is not far from the MW plateau. However, if we consider only the stars with [Fe/H] < -2, the average [Mg/Fe] ratio of the Lemasle sample is close to zero (see Section 5.2). Moreover, looking at the [Ca/Fe] ratios of all stars older than 10 Gyr, the average [α/Fe] is less than that observed in the Galaxy. Sculptor instead seems to have a distribution of [Mg/Fe] ratios in metal-poor stars similar to the Galactic one (de Boer et al. 2012a). Data about the [Mg/Fe] ratios in the most metal-poor stars of Draco, Carina, Sextans and Sculptor are further discussed and compared with model results in Section 5.2.

Medium-resolution spectra of about 3000 stars in a larger sample of DGSs, including ultrafaint ones are also available (Kirby et al. 2010; see also Kirby et al. 2011). This study challenges some of the conclusions of the DART experiment in the sense that the average [Mg/Fe] of metal-poor ([Fe/H] < -1.8) stars in DGSs seems to be even slightly higher than that of the MW. Moreover, they do not detect any [α/Fe] plateau for [Fe/H] > -2.5, with maybe the exception of Sculptor, and conclude that SNeIa, responsible for the production of the bulk of iron, substantially contributed to the chemical evolution even at these low metallicities. Also the analysis of Frebel et al. (2010) of the ultrafaint dwarf galaxies UMa II and Coma Berenices, indicates either the presence of a plateau with relatively high [α/Fe] or a constant decrease of [α/Fe] with [Fe/H], i.e. no plateau (see also Vargas et al. 2013). The ultrafaint dwarf galaxy Leo IV has been analysed, too (Simon et al. 2010), and, again, no compelling signs of a [Mg/Fe] plateau significantly lower than the Galactic one have been found. To finish, Norris et al. (2010) analyse a star in Bootes I with [Fe/H] $= -3.7$ and high [α/Fe].

As a summary of this section, only a fraction of the DGSs show signs of an enrichment of α-elements in metal-poor stars significantly lower than the one observed in the MW, whereas some other DGSs seem to show either [α/Fe] plateaus similar to the ones found in the MW, or no signs of plateau at all. Of course, deeper and more detailed observations might change this picture. It is also worth pointing out that non-local thermodynamic equilibrium effects can significantly change the measurements of the abundances of individual stars, as shown by Fabrizio et al. (2012) in the case of Carina. In spite of the incredible progresses made in the last years, the study of the abundances of individual stars in dwarf galaxies is still in its infancy.

3.2 The star formation histories of DGSs

In many cases, the whole SF history of DGSs has been recovered by means of comparison of synthetic colour–magnitude diagrams (CMDs) with observed ones (see Tolstoy et al. 2009, and references therein). Particularly relevant in this sense is the Local Cosmology from Isolated Dwarfs (LCID) project (Monelli & Lcid Team 2007), aimed at obtaining CMDs reaching the oldest main-sequence turnoff stars for a sample of nearby isolated dwarf galaxies, using the Advanced Camera for Surveys (ACS) camera on board of the *Hubble Space Telescope*. The LCID team has already obtained detailed SFRs for the dwarf galaxies LGS-3, Tucana, Cetus and Phoenix (see Monelli et al. 2010a,b; Hidalgo et al. 2011).

It would be extremely interesting to compare the SFRs in the *early* phases of the evolution of these galaxies, when the contribution of SNeIa to the chemical evolution is supposed to be negligible, and to compare it with the [α/Fe] plateau (if any) in the most metal-poor stars. These two quantities should correlate. However, tidal interactions and other environmental effects can be very strong in DGSs and this could lead to the loss of a very significant fraction of the baryonic mass initially present in the galaxy. In the extreme cases, the mass of a galaxy can be reduced by two or three orders of magnitude (Kroupa 1997; Klessen & Kroupa 1998; Mayer et al. 2006). Therefore, the evaluation of the SF history of DGSs made for instance by the LCID team by means of the analysis of the present-day CMD cannot correctly take into account the large fraction of long-living stars that the galaxy has lost orbiting around the MW. The present-day CMDs of DGSs contain only the (long-living) stars that survived $\sim$12 Gyr of galactic evolution; they do not contain all long-living stars ever formed in the galaxy. Of course, if some of these stars got stripped off the galaxy the CMDs will not contain them, and the resulting SFR determinations will necessarily be lower limits.

In some cases, there is direct observational evidence of tidal disruption of DGSs. The archetypal example is Sagittarius (Ibata, Gilmore & Irwin 1994; Majewski et al. 2003), but also Segue 2 is a bare remnant of a tidally stripped galaxy (Kirby et al. 2013). Other DGSs like Draco (Cioni & Habing 2005) and Ursa Minor (Palma et al. 2003) clearly show *S*-shaped morphologies, indicative of tidal interactions. It is also important to point out that modern techniques of reconstruction of the SF histories of galaxies still do not have adequate resolution for very old stars and an ancient SF peak, if present, would be smeared out and the deduced peak SFR could be underestimated by a factor of up to 10 (see de Boer et al. 2012b, their fig. 10). Therefore, only the *relative* SF histories deduced by these authors, i.e. the fraction of stars formed during different periods of the galactic evolution, should be considered reliable (see also the discussion about SF histories in DGSs in Section 6.2). Real SFRs in the early evolution of these galaxies might have been two orders of magnitude larger than commonly reported.

For later convenience, we summarize here some typical derived early SFRs in DGSs, bearing in mind the caveats expressed above. Results of the LCID project suggest that the SFRs of dwarf galaxies, even the ones for which a short episode of SF is deduced (for instance Cetus; Monelli et al. 2010a), are mild, often below 10^{-4} M$_\odot$ yr^{-1}, although the SFR of Tucana might have been comparable to the SFR deduced in present-day blue compact dwarfs. Early SF histories with similar SFRs are suggested also for Sagittarius (Karachentsev, Aparicio & Makarova 1999) and Fornax (Coleman & de Jong 2008), although the early SFR in Fornax might have been as high as 10^{-3} M$_\odot$ yr^{-1} (de Boer et al. 2012b). On the theoretical side, the models of Salvadori, Ferrara & Schneider (2008) indicate (very short) early bursts of SF in DGSs with SFRs of at least 0.1–0.15 M$_\odot$ yr^{-1} (but see Fenner et al. 2006; Lanfranchi, Matteucci & Cescutti 2006; Calura & Menci 2009; Brook et al. 2012).

4 THE GALAXY MODEL

We want to establish the correlation between the IMF of a galaxy and its [α/Fe] plateau (or at least the average [α/Fe] of the most metal-poor stars) using an approach similar to the one used by Tsujimoto et al. (1997) (namely, by means of equation 1), but in the framework of the IGIMF theory.

In order to do that, we make the following assumptions.

(i) All the stars above a threshold mass $m_{\rm thr}$ explode instantaneously as SNeII.

(ii) The stars with masses below $m_{\rm thr}$ do not contribute to the chemical enrichment. This assumption is always justified for α-elements whereas it is not justified, in general, for iron. In fact, as already mentioned, a large fraction of iron comes from SNeIa, whose progenitors have non-negligible lifetimes. However, by definition [α/Fe] plateaus are regions in the [Fe/H]–[α/Fe] plane where the contribution of SNeIa can be neglected (a non-negligible contribution from SNeIa would decrease the [α/Fe] as a function of [Fe/H]). Hence, provided that we can really identify [α/Fe] plateaus in galaxies (see Section 3.1), our assumption is fully reasonable.

(iii) The chemical products of SNeII are instantaneously and homogeneously mixed with the surrounding interstellar medium. Although this assumption is not fully justified theoretically, it is the only possible assumption in the present framework. This hypothesis can be relaxed only by means of detailed chemodynamical simulations (in progress). Results of Spitoni et al. (2009) show that the inclusion of a delayed mixing does not dramatically alter the [α/Fe] tracks.

(iv) The initial metallicity of the galaxy is set to be $Z = 10^{-4}$ Z$_\odot$. This assumption is further discussed in Section 5.1.

The [α/Fe] plateaus in galaxies as defined in equation (1), but with the universal IMF $\varphi(m)$ replaced by the (ψ-dependent) IGIMF $\xi_{\rm IGIMF}$ is defined as

$$[\alpha/{\rm Fe}]_{\rm pl} = \log\left[\frac{\int_{m_{\rm thr}}^{m_{\rm up}} m_\alpha^{ej}\xi_{\rm IGIMF}(m,\,\psi)\,{\rm d}m}{\int_{m_{\rm thr}}^{m_{\rm up}} m_{\rm Fe}^{ej}\xi_{\rm IGIMF}(m,\,\psi)\,{\rm d}m}\right] - \log\left(\frac{\alpha}{\rm Fe}\right)_\odot, \quad (4)$$

where $m_{\rm up}$ in this case is the upper mass in the largest star cluster of the galaxy (i.e. $m_{\rm up} = m_{\rm max}[M_{\rm ecl,\,max}(\psi)]$) and [$\alpha$/Fe]$_{\rm pl}$ indicates the [α/Fe] plateau we seek and, as we can see, it depends on the SFR ψ.

5 RESULTS

5.1 The optimal set of yields to reproduce the Milky Way [α/Fe] plateau

We wish to apply equation (4) and calculate [α/Fe]$_{\rm pl}$ in model galaxies characterized by different values of the SFR ψ. In order to do that, we first need to check if such an approach allows us to reproduce the MW [α/Fe] plateau. We thus apply equation (4) for a value of the SFR characterizing the early phases (for instance the first Gyr) of the MW evolution. In particular, we take $\langle{\rm SFR}\rangle_{1\,{\rm Gyr}} \simeq 0.5$ M$_\odot$ yr^{-1}. This value comes from the calculations of C10 of the chemical evolution of the solar neighbourhood. It is important to remark that, in the work of C10, the results are calibrated on the empirical data available for the inner halo of the MW; at present, there is insufficient high-resolution data available for field stars of the outer halo where the impact of individual accretion events on the position/tightness of the plateau might be more important. However, Carollo et al. (2010) argue that the inner and outer haloes have mean [Fe/H] $= -1.6$ and -2.2, respectively. If this is the case, our emphasis of stars with [Fe/H] < -2.0 in the solar neighbourhood could be prone to contamination by the outer halo. It is important to point out that our results are quite insensitive to the choice of $\langle \mathrm{SFR}\rangle_{1\,\mathrm{Gyr}}$ (see Appendix A), hence we keep $\langle \mathrm{SFR}\rangle_{1\,\mathrm{Gyr}} = 0.5$ M$_\odot$ yr^{-1} as our reference value. This value together with all other values adopted as standard values for our MW plateau calculation are reported in Table 2. We wish to find out for which set of yields equation (4) correctly reproduces the abundance ratios observed in the most metal-poor MW stars. This set of yields can be safely applied to other model galaxies, since we do not expect galaxy-to-galaxy yields variations (see also Section 1).

The reference data set we will use for very metal-poor stars in the MW halo comes from Cayrel et al. (2004). This data set is sufficiently rich, uniform and possesses little intrinsic scatter. It seems to be the best reference to obtain a value for the observed [α/Fe] MW plateau. The four most commonly (and best) measured α-elements in galaxies are O, Mg, Si and Ca. Calcium is perhaps the best measured element among these four because it has more lines that can be measured than the other α-elements, especially in DGSs. The dispersion of the [Mg/Fe] ratios observed by Cayrel et al. (2004) about the lines of the best fit is also smaller than the dispersion of other α-elements. However, Ca yields are very uncertain (much more than O, Mg and Si yields) because they are strongly affected by explosive burning and 'fallback' (Woosley & Weaver 1995, hereafter WW95; Shigeyama & Tsujimoto 1998). This makes the comparison between models and observations very problematic and uncertain. In this sense, Mg is a much more robust α-element because its production is dominated by hydrostatic burning processes and it has been often used as 'reference element' (Shigeyama & Tsujimoto 1998; Cayrel et al. 2004). Moreover, SNeIa contribute substantially to the production of Ca. According to Iwamoto et al. (1999, see their table 3), a typical (IMF averaged) SNII produces 5.8×10^{-3} M$_\odot$ of Ca, whereas the (SNIa) W7 model predicts a yield of 1.2×10^{-2} M$_\odot$, i.e. SNeIa are responsible for the majority of the Ca production. On the other hand, the average Mg from SNeII is 8.8×10^{-2} M$_\odot$, whereas the Ia production of Mg is only 8.5×10^{-3} M$_\odot$ (and also Mg from intermediate-mass stars is negligible). Finally, the slope of the [Ca/Fe] versus [Fe/H] correlation identified by Cayrel et al. (2004) is not as flat as for the other α-elements (see table 9 of Cayrel et al. 2004). Given the large number of Mg abundance measurements in individual stars of DGSs (see also Section 5.2), we will use Mg as a reference element throughout this paper, although we will consider also O, Si and, occasionally, Ca. In what follows, we will assume [Si/Fe] $= 0.37 \pm 0.15$, [O/Fe] $= 0.47 \pm 0.15$ and [Mg/Fe] $= 0.27 \pm 0.13$ as typical values of the [α/Fe] MW plateau. Cayrel et al. (2004) report also the measurements of other α-elements. The inclusion of other, less accurately measured, α-elements (S, Ar or Ti) adds nothing to the conclusions of this paper. We will assume solar abundances from Asplund et al. (2009), namely 12+log(O/H) = 8.69, 12+log(Si/H) = 7.51, 12+log(Mg/H) = 7.60 and 12+log(Fe/H) = 7.50.

As shown by C10, chemical evolution MW models based on the IGIMF theory are able to reproduce, among other things, the [α/Fe] of the most metal-poor MW stars, in particular if the slope β of the embedded cluster mass function is equal to 2. Concerning these results, it is important to remark that they are based on the WW95 yields[3] who, however, calculated nucleosynthetic stellar products only for stars in the interval of initial masses [12,40] M$_\odot$ ([11,40] M$_\odot$ if the initial composition is solar). Below 12 M$_\odot$, yields are usually interpolated (and they are extrapolated above 40 M$_\odot$).

Such an interpolation is expected to have measurable effects on the computed abundances, since for a standard IMF, a non-negligible mass fraction of stars forms in the mass range between 8 and 12 M$_\odot$. In this section, we aim at testing the effects regarding the assumption of the yields in this mass range, and assess how this affects the computed abundance ratios.

Assuming that all stars with initial masses larger than 8 M$_\odot$ end their lifetimes as SNeII (i.e. assuming $m_{\mathrm{thr}} = 8$), by means of equation (4) we can explore the effect of the yields in the interval of initial masses 8, 12 M$_\odot$ on the [α/Fe] MW plateau. The IGIMFs are calculated as in R09, using SFR $= 0.5$ M$_\odot$ yr^{-1} and taking as reference value $\beta = 2$ (see Table 1). Our reference set of nucleosynthetic yields is the one of WW95 with Z $= 10^{-4}$ Z$_\odot$, case B. Gibson (1998) showed that the oxygen yields do not significantly depend on metallicity, at least for initial metallicities larger than 0.

Of course what determines the final [α/Fe] is the ratio between α-element yields and Fe yields in this interval, not their absolute value. Therefore, we fix the Si, Mg and O yields at 10 M$_\odot$ as the average between the 12 M$_\odot$ yields and the 8 M$_\odot$ yields calculated by van den Hoek & Groenewegen (1997) and leave the 10 M$_\odot$ Fe yield $y_{\mathrm{Fe},\,10}$ as a free parameter, which is allowed to vary between the 8 M$_\odot$ yield ($y_{\mathrm{Fe,low}}$) predicted by van den Hoek & Groenewegen (1997) and the 12 M$_\odot$ yield ($y_{\mathrm{Fe,up}}$) of WW95. The yields in the intervals]8,10[and]10,12[M$_\odot$ are linearly interpolated between the tabulated values. We define the 'normalized' iron yield

$$\hat{y}_{\mathrm{Fe}} = \frac{y_{\mathrm{Fe},10} - y_{\mathrm{Fe,low}}}{y_{\mathrm{Fe,up}} - y_{\mathrm{Fe,low}}}. \tag{5}$$

The quantity $\hat{y}_{\mathrm{Fe}}$ varies between 0 and 1. The [O/Fe], [Mg/Fe] and [Si/Fe] predicted by means of equation (1) as a function of $\hat{y}_{\mathrm{Fe}}$ are shown in Fig. 2 (left-hand panels). The IGIMF parameters assumed to calculate these [α/Fe] ratios are the reference values reported in Table 1. In particular, $\beta = 2$ is assumed. These values are in agreement with the results of Calura et al. (see their fig. 3, upper panel, for [O/Fe], figs 6, 9 and 14, lower panels, for [Si/Fe])[4] and are within the observed ranges, irrespective of $\hat{y}_{\mathrm{Fe}}$, with the exception of [Mg/Fe] for $\hat{y}_{\mathrm{Fe}} > 0.6$.[5]

From Fig. 2, we can observe that the dependence of the [α/Fe] ratio on $\hat{y}_{\mathrm{Fe}}$ is moderate, with variations of $\sim$0.08 dex at most. This dependence, together with the changes many other parameters produce in the [α/Fe] plateaus, is summarized in Table 3. From Fig. 2, left-hand panels, we can also observe that the best match between observations and model results is obtained for $\hat{y}_{\mathrm{Fe}}$ between 0 and 0.5. We are mostly interested in the [Mg/Fe] abundance ratios, thus, we choose $\hat{y}_{\mathrm{Fe}} = 0$ as our reference value (see Table 2), since

[3] Actually, they are based on the François et al. (2004) yields, but the difference between François's yields and WW95 ones is negligible for the chemical elements we are analysing.

[4] It is worth pointing out that C10 adopted the Asplund et al. (2004) solar abundances; however, these values differ very little from our assumptions, so the comparison with Calura et al.'s results is still meaningful.

[5] Notice that here we have assumed the photospheric solar Mg abundance of Asplund et al. (2009). Had we chosen the meteoric Asplund et al.'s abundance 12+log(Mg/H) = 7.53, the [Mg/Fe] would be consistent with the observed MW plateau for the whole interval in $\hat{y}_{\mathrm{Fe}}$.

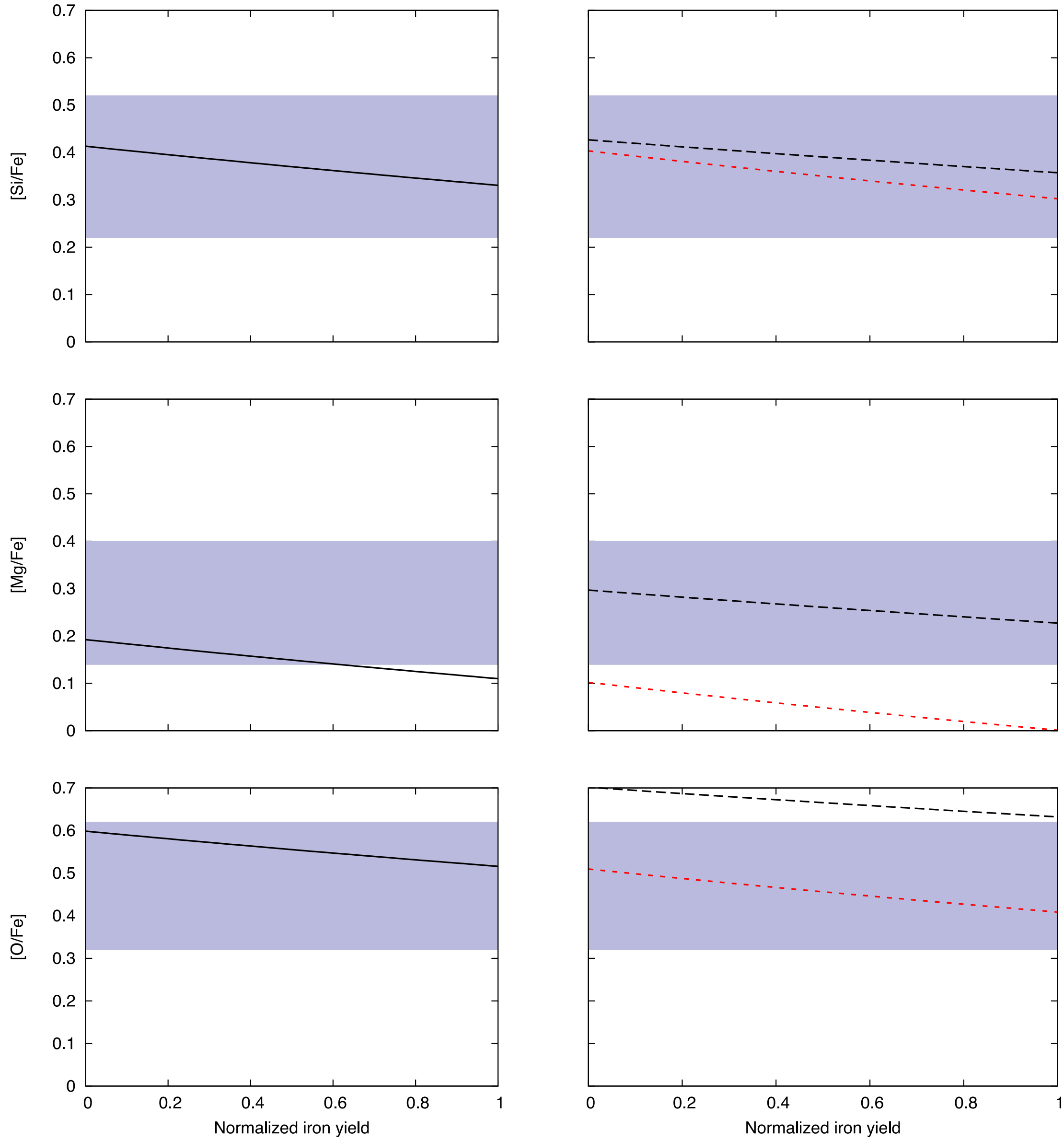

Figure 2. Left-hand panels: [O/Fe] (lower panel), [Mg/Fe] (middle panel) and [Si/Fe] (upper panel) obtained applying equation (4) with $m_{\rm thr} = 8\ {\rm M}_{\odot}$, with a variable Fe yield at 10 $\rm M_{\odot}$, expressed through the normalized value $\hat{y}_{\rm Fe}$ (see equation 5). IGIMF parameters are the ones reported in Table 1. Shaded areas in each panel are the average MW [α/Fe] plateau values as obtained from Cayrel et al. (2004) $\pm 1\sigma$. Right-hand panels: same as left-hand panels but with different values of β: $\beta = 1.00$ (long dashed lines) and $\beta = 2.35$ (short dashed lines).

Table 3. Modified parameters (as compared to the reference values of Tables 1 and 2) and their effect on the predicted [α/Fe] ratios.

Quantity	Modified value/ set of values	[α/Fe] variation (dex)	Section/Appendix
$\hat{y}_{\rm Fe}$	[0,1]	0.08	5.1, 5.2
$m_{\rm thr}$	10 $\rm M_{\odot}$	0.04	A
β	1; 2.35	0.2	5.1
$y_{\rm Fe,\,150}/y_{\rm Fe,\,40}$	[1,10^4]	0.005	A
$\langle{\rm SFR}\rangle_{\rm 1\,Gyr}$	100 $\rm M_{\odot}\ yr^{-1}$	0.015	A
$M_{\rm ecl,min}$	20 $\rm M_{\odot}\ yr^{-1}$	<0.01	A
$m_{\rm max,\,*}$	300 $\rm M_{\odot}\ yr^{-1}$	0.03–0.04	A
α_2 (Mild Z dependence)	2.3 + 0.0572 [Fe/H]	0.2	6
α_1, α_2 (Hard Z dependence)	$\alpha_1 = 1.3$+0.5 [Fe/H] $\alpha_2 = {\rm Min}(2.63+0.66{\rm [Fe/H]},2.3)$	0.7–0.8 dex	6

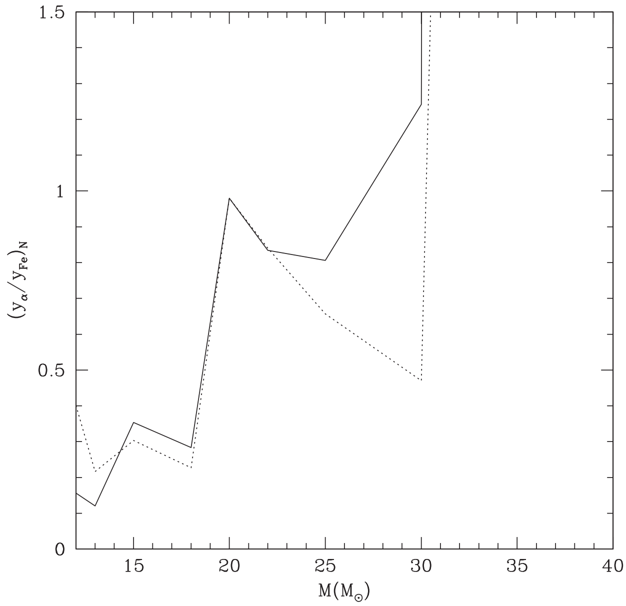

Figure 3. $y_{\rm O}/y_{\rm Fe}$ (solid line) and $y_{\rm Si}/y_{\rm Fe}$ (dotted line) tabulated by WW95 for $Z = 10^{-4}$ Z$_\odot$, as a function of the initial stellar mass. The yield ratios have been normalized to the corresponding yield ratio for a 20 M$_\odot$ star.

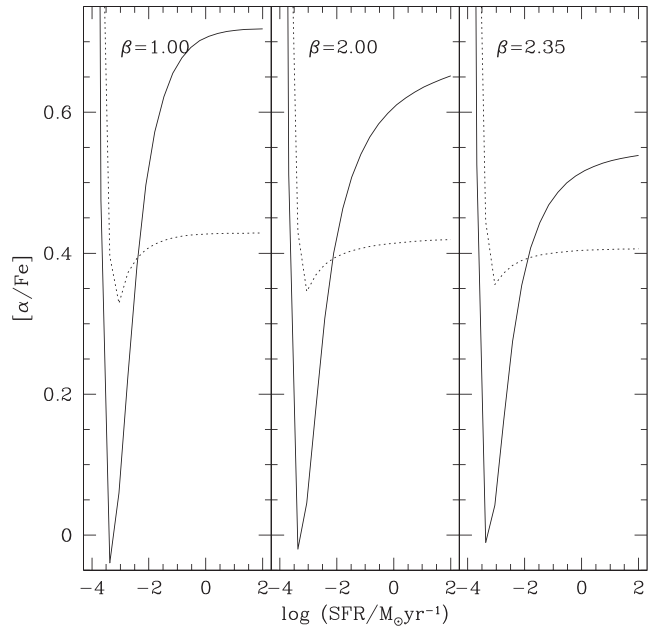

Figure 4. [O/Fe] (solid lines) and [Si/Fe] (dotted lines) ratios as a function of the galaxy SFR, for different values of β (indicated on the top of each panel). We assume in this plot the reference value $\hat{y}_{\rm Fe} = 0$.

it allows us to reproduce at best the [Mg/Fe] plateau as observed in local stars.

Fig. 2 (right-hand panels) shows the dependence of the [α/Fe] MW plateaus on β. It is clear that β affects quite significantly the results (see also R09 and C10). The fact that [O/Fe] with $\beta = 1$ and [Mg/Fe] with $\beta = 2.35$ do not fit the observations corroborates our choice of $\beta = 2$ as reference value. We can also observe that the [Si/Fe] results are less dependent on β than the [O/Fe] and [Mg/Fe] ratios, in agreement with the results of C10. This is due to the fact that the pattern of the Fe yields as a function of initial stellar mass is more similar to the Si yields pattern as compared to the oxygen and magnesium one. In other words, the mass dependence of the ratio $y_{\rm Si}/y_{\rm Fe}$ is flatter than the function that describes $y_{\rm O}/y_{\rm Fe}$ (or $y_{\rm Mg}/y_{\rm Fe}$) as a function of mass. This is shown in Fig. 3, where the yield ratios have been divided by the ratios of stellar yields at 20 M$_\odot$. From this figure, it is also worth noticing the large Si yield at 12 M$_\odot$, which has consequences for our results.

It is important to explain in more detail the reason of our choice of an initial metallicity of $Z = 10^{-4}$ Z$_\odot$ (see also Section 4). The WW95 set of yields at 10^{-2} Z$_\odot$ cannot be considered because the MW plateau extends to metallicities much below this value. The most metal-poor stars in the Cayrel et al. (2004) sample have in fact [Fe/H] slightly larger than −4. This forces us to consider only initial metallicities equal to or smaller than 10^{-4} Z$_\odot$. We have considered the predicted WW95 yields at $Z = 0$ (case B). Metal-free stars produce a limited amount of α-elements, resulting in very low [α/Fe] ratios. Only the models with $\beta = 1$ attain [α/Fe] larger than 0.2 dex and none of the considered models reach the assumed [α/Fe] MW plateau value. Even the more recent, detailed calculations of nucleosynthetic yields from metal-free massive stars by Heger & Woosley (2010) do not agree with the Cayrel plateaus for many elements (see their fig. 12). We thus discard this set of yields. Since the WW95 set of yields does not consider metallicities intermediate between $Z = 10^{-4}$ Z$_\odot$ and 0, it is clear that in the present study the only possible choice for the initial metallicity is $Z = 10^{-4}$ Z$_\odot$.

We have thus seen in this section that, besides the SFR (the independent variable in our study), our calculations of the [α/Fe] plateaus depend sensitively on the choice of β and, to a lesser extent, on the choice of $\hat{y}_{\rm Fe}$, i.e. on the choice of the yields in the interval 8, 12 M$_\odot$. Many other parameters have been investigated in the Appendix and they turn out to have a negligible effect on our results (see Table 3).

5.2 The [α/Fe] ratios of the most metal-poor stars in dwarf galaxies

We can now relax the hypothesis of a fixed SFR and explore the predicted values of the [α/Fe] plateau in dwarf galaxies, characterized by mild SFRs. In order to illustrate our results, we plot in Fig. 4 the calculated [O/Fe] and [Si/Fe] plateaus as a function of SFR in the case in which $\hat{y}_{\rm Fe} = 0$ and $m_{\rm thr} = 8$ M$_\odot$ (i.e. our reference values). Although our reference value of β is 2, we show in this plot also the results obtained with different values of this parameter.

As one can see from this figure, [O/Fe] shows very large variations as a function of SFR below SFR $\simeq 0.1$ M$_\odot$ yr^{-1}. Again, the variation of [Si/Fe] is more limited (at least for SFRs larger than 10^{-3} M$_\odot$ yr^{-1}) and it is significant only below SFR $\simeq 10^{-1}$ M$_\odot$ yr^{-1}. The main reason of this different behaviour is still the different pattern of $y_{\rm O}/y_{\rm Fe}$ and $y_{\rm Si}/y_{\rm Fe}$ shown in Fig. 3. From Fig. 4, we can notice that the [α/Fe] of the models with the weakest SFR are even higher than the MW plateau [α/Fe]. This is due to the fact that, below 12 M$_\odot$, the Fe production is negligible. It is only due to secondary production which, being the considered metallicity of the stellar population 10^{-4} Z$_\odot$, is very tiny. Also the O and Si production is very low, but still much larger than the Fe production. However, due to the very limited quantity of Fe and α-elements synthesized by these model galaxies, these results cannot be safely compared to the observations.

The [O/Fe] has a pronounced minimum at log SFR $\simeq -3.7$. The difference between this minimum and the [O/Fe] MW plateau is of the order of $\simeq$0.5 dex. Overall, if weneglect the initial steep decline of [O/Fe], there are variations of the order of up to 0.7 dex of the [O/Fe] ratios as a function of the SFR. If we repeat this calculation with $\hat{y}_{\rm Fe} = 0.5$ (which is still compatible with the MW [α/Fe] plateau) the resulting [O/Fe] ratios reach minima of the order of ~-0.2 dex, almost 0.7 dex below the MW plateau value. Overall, the differences with the model with $\hat{y}_{\rm Fe} = 0$ are of the order of $\sim$0.1 dex, at least for SFR > 10^{-3} M$_\odot$ yr^{-1}.

A general conclusion we can draw from Fig. 4 is that, irrespective of the SFR, the predicted [Si/Fe] plateau of dwarf galaxies is always $\geq$0 and never departs significantly from the observed MW plateau. On the other hand, the predicted [O/Fe] plateau can differ by up to 0.5 dex compared to the MW halo value. Concerning oxygen abundances, one might argue that they are known to be relatively low in a large fraction of the red giants within Galactic GCs (e.g. Carretta et al. 2009), and that this could be true also in dwarf galaxies. However, this point should not obviate any of our analysis, since O-poor stars in GC are likely second-generation stars born out of gas containing the ejecta of older, first-generation stars, as indicated also by the well-established O/Na anticorrelation in GCs (Gratton, Sneden & Carretta 2004; Gratton, Carretta & Bragaglia 2012). Moreover, as suggested by Carretta et al. (2010), such an anticorrelation is not observed in the body of Saggitarius field stars. The same may be true for other dwarf galaxies. At the present stage, in the light of the results of Carretta et al. (2010), the [O/Fe] values adopted here for the stars of local dwarfs galaxies can be considered as representative of the bulk of their stellar population.

Although illustrative, the [O/Fe] and [Si/Fe] ratios cannot be easily compared with available observations. The determination of [O/Fe] ratios in DGSs is difficult and it has been performed only by a handful of authors. There are more data available about [Si/Fe] but, as can be seen from Fig. 4, we predict moderate variations of [Si/Fe] as a function of the SFR, hence it is difficult from this abundance ratio to constrain the IMF or the SFR. A larger amount of data is available for the [Mg/Fe] abundance ratios (see also Section 3.1). In Table 4, we collect available data of [Mg/Fe] ratios in stars of the DGSs Draco, Carina, Sextans and Sculptor, having [Fe/H] < -2. Average [Mg/Fe] ratios for each galaxy have been calculated by considering data coming from different papers (reported in the table, as well). Since the average [Mg/Fe] in Cayrel et al. (2004) is $\simeq$0.27, we can notice from Table 4 that average [Mg/Fe] ratios in the four considered dwarf galaxies lie below this MW plateau value, although the differences are of the order of $\sim$0.15 dex at most.

We choose to concentrate on the range of iron abundances with [Fe/H] < -2 in order to be consistent with the range of [Fe/H] ratios of the Cayrel et al. (2004) sample for MW halo stars. However, it is important to remind the reader that some metal-poor stars with relatively low [Mg/Fe] ratios have [Fe/H] only slightly below -2. If we consider as a threshold metallicity [Fe/H] = -2.3 instead of -2, the average [Mg/Fe] ratios in Draco and Sculptor remain unaltered, the average [Mg/Fe] in Sextans increases by only $\sim$0.02 dex, whereas the average [Mg/Fe] ratio in Carina changes dramatically from 0.144 to 0.328. However, looking at Starkenburg et al. (2013) one could also argue that extremely metal-poor stars (i.e. stars with [Fe/H] < -3) have also [Mg/Fe] ratios below the average and that the [Mg/Fe] ratio has a peak at [Fe/H] $\simeq -3$.

We calculate the variation of the [Mg/Fe] abundance ratio as a function of the SFR exactly as we did for the [O/Fe] and [Si/Fe] abundance ratios. We concentrate in what follows on the reference values of $\hat{y}_{\rm Fe} = 0$ and $\beta = 2$. We compare the results of our models with the data presented in Table 4 for the dwarf galaxies Draco, Sextans, Carina and Sculptor in Fig. 5. Given the uncertainties in the determinations of the SFR in the earliest phases of galaxy evolution described in Section 3.2, we draw the measurements of the observed [Mg/Fe] ratios as horizontal shaded areas.

At least in principle, this plot could be used to constrain the early SFR in the four considered dwarf galaxies, in the framework of the IGIMF theory. From this plot, it seems that the early SFR of these galaxies is either very low, with SFRs below 10^{-3} M$_\odot$ yr^{-1}, or moderately intense, with SFRs larger than 10^{-2} M$_\odot$ yr^{-1}, depending on the galaxy. The former possibility should be ruled out because of the large variations of the [Mg/Fe] as a function of the SFR, which would make unlikely the observations of similar (within a factor of 0.3 dex) [Mg/Fe] ratios in different galaxies. The latter is a viable possibility and we have summarized the range of allowed SFRs for each galaxy in Table 4. However, one should not forget the uncertainties implied by this kind of comparison, among which: (i) not always abundance ratios are determined with the same methodologies by different groups and different methodologies can lead to different results. (ii) Even the solar abundance of Mg is still uncertain. (iii) At variance with the MW halo, the statistics of very metal-poor stars in DGSs is still quite poor. (iv) There are uncertainties related to the parameters and to the formalism of the IGIMF theory. Some of these uncertainties have been discussed in this paper and summarized in Table 3.

6 DISCUSSION AND RESULTS WITH A METALLICITY DEPENDENT IGIMF

6.1 Results with a metallicity-dependent IGIMF

As already mentioned, it has been recently suggested that metallicity can play a role in shaping the IMF in galaxies. In particular,

Table 4. Average [Mg/Fe] ratios with relative errors for the most metal-poor stars (stars with [Fe/H] below -2) in the Draco, Carina, Sextans and Sculptor DGSs.

Galaxy	Average [Mg/Fe]	σ	References	Allowed log SFRs (in M$_\odot$ yr^{-1})
Draco	0.228	0.051	Shetrone, Côté & Sargent (2001)	log SFR>-0.52
			Cohen & Huang (2009)[a]	
Carina	0.144	0.099	Koch et al. (2008)	$-1.82 <$ log SFR < 1.70
			Venn et al. (2012)	
			Lemasle et al. (2012)	
Sextans	0.166	0.029	Tafelmeyer et al. (2010)	$-1.10 <$ log SFR < -0.15
			Shetrone et al. (2001)	
			Aoki et al. (2009)	
Sculptor	0.162	0.055	Starkenburg et al. (2013)	$-1.40 <$ log SFR < 0.45
			Tafelmeyer et al. (2010)	

[a]Notice that Cohen & Huang (2009) had to increase the [Mg/Fe] determined by Cayrel et al. (2004) by 0.15 dex to make it compatible with their measurements.

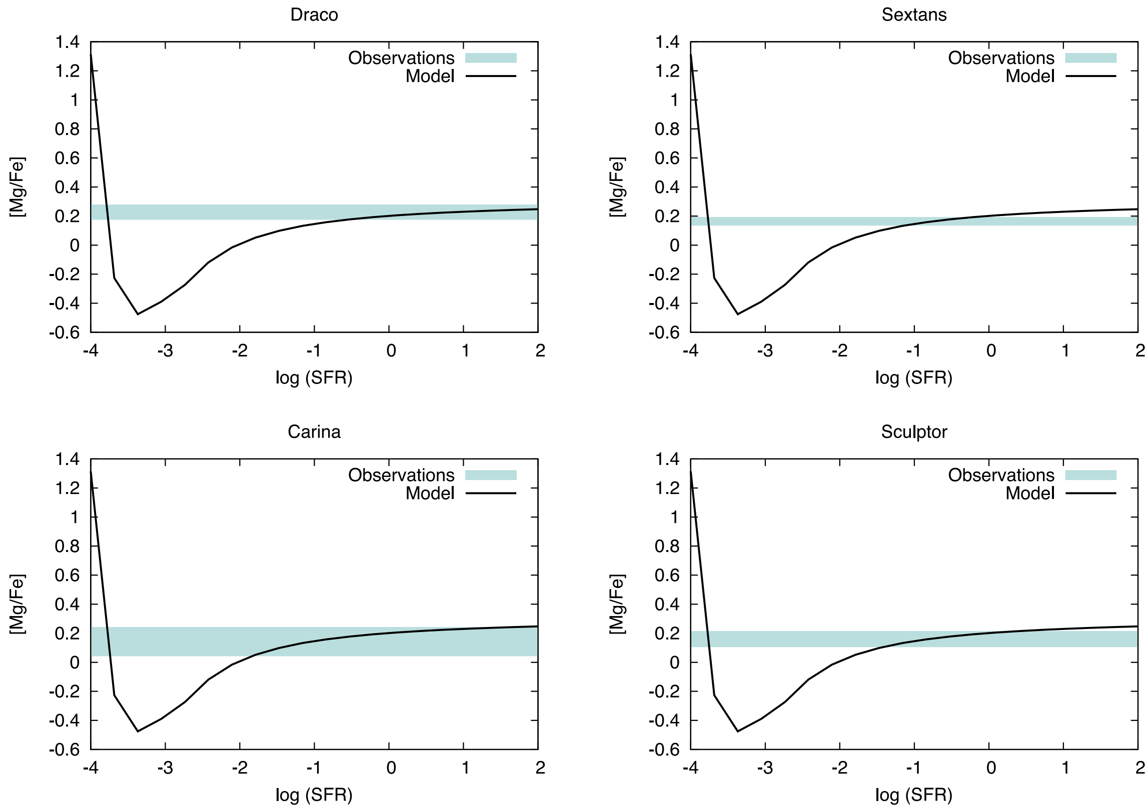

Figure 5. Comparison of computed [Mg/Fe] plateau as a function of the SFR (solid lines) with observed averages of the most metal-poor stars in four well-studied DGSs: Draco, Carina, Sextans and Sculptor (shaded areas, see Table 4).

galaxies with low metallicities should have top heavier IMFs than high-metallicity galaxies with the same SFRs (see M12; Kroupa et al. 2013, and references therein). It is important to stress that the results of M12 are mostly based on studies of the mass distribution in GCs, therefore, it is not clear whether they can be directly applied to galaxies. Nevertheless, we believe that it is interesting to study the [α/Fe] ratios in the framework of this new IGIMF formulation also because it is the first time to our knowledge that the chemical composition and evolution of a galaxy having a metallicity-dependent IGIMF has been studied.

M12 consider indeed the combined effect of cluster densities and metallicities. Since we have no information on the densities of our model galaxies, we will consider only the results of M12 concerning the metallicity dependence of the IMF. If we assume that all the star clusters in galaxies, irrespective of their masses, always have the same density, we end up with a mild dependence of the IGIMF on the metallicity (*mild model*). The metallicity of our model dwarf galaxies at different masses (and SFRs) is calculated according to equation 2 of Mannucci et al. (2010; the so-called *fundamental metallicity relation*). The correlation between SFR and mass of a model galaxy is obtained exactly as in R09. The obtained (O/H) is converted into [Fe/H] by means of the solar abundances of Asplund et al. (2009).[6] The relation we obtain in the end, adopting equation 15 of M12 with a constant value of the cluster density $\rho_{\rm cl}$ is

$$\alpha_2 = 2.3 + 0.0572[\mathrm{Fe/H}], \tag{6}$$

where α_2 is the IMF slope for high-mass stars already introduced in Section 2 and in Table 1. This formula ensures that a galaxy like the MW has a present time high-mass IMF slope of 2.3, in agreement with the assumptions of M12, and very similar to the high-mass IMF slope of 2.35 adopted by R09. It is important to stress that we modify the IMF *within each star cluster* according to this formula. Once a distribution function of star clusters has been assumed, the integrated IMF is then calculated by integrating the IMF of the various star clusters, according to the standard recipes (see Section 2 and R09).

Considering for simplicity only the reference case $\beta = 2.00$, the comparison between the standard and the metallicity dependent IGIMF for two representative metallicities is presented in Fig. 6. As one can see from this figure, at [Fe/H] $= -1$ the metallicity-dependent IGIMF is very similar to the reference, Z-independent one (the one obtained in R09). At [Fe/H] $= -3$ the difference is more significant but still not very large. The effect of this metallicity-dependent IMF is to make IGIMFs in low-mass (hence metal-poor) galaxies flatter, thus partially counter-balancing the effect of the SFR on the IGIMF. However, since the metallicity effect is mild in this formulation, dwarf galaxies are still characterized by steeper IGIMFs than large ones.

A more pronounced metallicity dependence (*hard model*) is obtained by assuming that also the IMF slope α_1 below 0.5 $\mathrm{M}_\odot$ in each star cluster depends on [Fe/H]. In agreement with equation 12

[6] This calculation of [Fe/H] is quite approximate and a more detailed calculation of a metallicity-dependent IGIMF in a galaxy of a given mass results from an iterative procedure. Our main aim here is to have a reasonable dependence of the metallicity with the SFR (namely, to assign a reasonable metallicity [Fe/H] to each model galaxy) and, through it, to compare calculations with and without metallicity-dependent IMFs. Thus, we prefer to use a simple estimate of [Fe/H] and hence of the IGIMF.

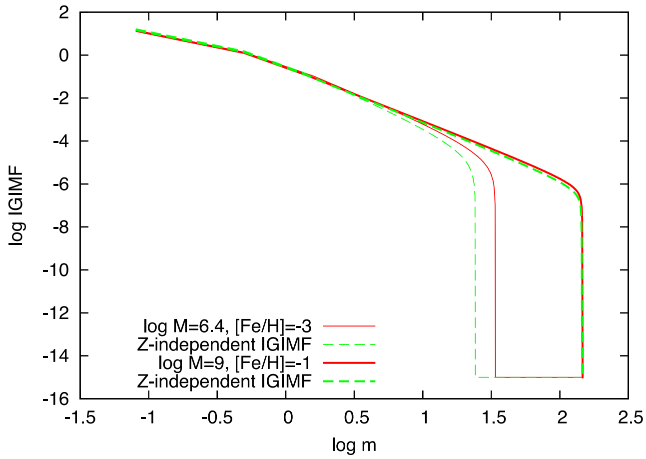

Figure 6. The comparison of the logarithm of the IGIMF as a function of the stellar mass (m) obtained by means of the mild model (equation 6; solid lines) for two representative values of the metallicity ([Fe/H] = −3: light line; [Fe/H] = −1: heavy line). The two dashed lines represent calculations made for two metallicity-independent IGIMFs (with $\beta = 2.00$) at two different galaxy masses: log $M = 6.4$ (light line); log $M = 9$ (heavy line).

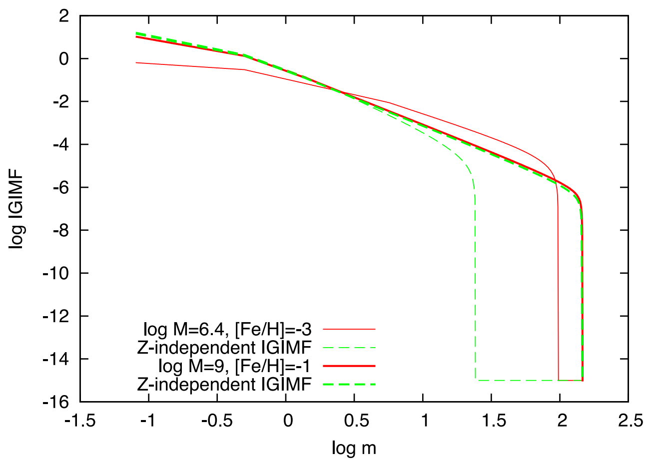

Figure 7. As Fig. 6 but for the hard dependence of the IGIMF on metallicity (equations 7 and 8).

of M12 we assume

$$\alpha_1 = 1.3 + 0.5[\mathrm{Fe/H}]. \tag{7}$$

Moreover, following equation 11 and table 3 of M12 we assume

$$\alpha_2 = \mathrm{Min}(2.63 + 0.66[\mathrm{Fe/H}], 2.3). \tag{8}$$

In this way, α_2 grows linearly with [Fe/H] until [Fe/H] = −0.5 and then it remains constant at $\alpha_2 = 2.3$ for [Fe/H] > −0.5. The comparison between the standard and the metallicity dependent IGIMF in the hard model for two representative metallicities is presented in Fig. 7. As one can see from this figure, at [Fe/H] = −1 the metallicity-dependent IGIMF is still quite similar to the reference one but at [Fe/H] = −3 the difference is extremely large. According to this model, low-mass galaxies, in spite of their low SFRs, have *flatter* IGIMFs than high-mass ones. It is worth noticing that Zhang et al. (2007) found indeed a dependence of the IMF slope on the metallicity of the parent galaxy. Galaxies at higher metallicities were found to have steeper IMFs, with the slope index ranging from ∼1.00, for the lowest metallicity, to ∼3.30 for the highest metallicity. Such direct determinations of the IMF slopes in galaxies are extremely useful to understand how the IMF is related to the galactic metal content and to assess the validity of the metallicity-dependent IGIMF put forward by M12.

We should pause here to consider more carefully the assumptions implied by our semi-analytical calculations. The fundamental metallicity relation of Mannucci et al. (2010) does not consider the detailed chemical evolution of a galaxy. It just reveals a correlation between the SFR and the metallicity of a star-forming galaxy at the moment at which we observe it. A galaxy with a large SFR appears to have also a large metallicity. As we have seen, such a large metallicity tends to produce steep IMF slopes. A large SFR on the other hand tends to make the IMF flatter. According to the hard model, the metallicity effect prevails. Consequently, at the time of observation, the IMF in high-SFR, high-metallicity galaxies is steeper than the corresponding IMF in smaller galaxies.

Of course, also the halo of the MW has experienced phases of its evolution characterized by very low metallicities, during which the IMF was flat. To keep track of this time evolution of the IMF, we need a theoretical approach like the one employed in C10. However, our semi-analytical approach still can give us important indications on the average IMF during the period of the halo formation. According to one of the most fundamental results of galactic chemical evolution, the larger the SFR, the higher the [Fe/H] reached before SNeIa contribute significantly to the chemical enrichment (Matteucci 2001). During the formation of the halo, the SFR was relatively high. That allowed the [Fe/H] to reach a value of −1, without significant contribution from SNeIa. In the framework of the metallicity-dependent IGIMF, we thus assume that the stars in the MW halo stem from stellar populations having different metallicities, up to [Fe/H] = −1. In the most metal-poor stellar populations the IMF is flat but it steepens as the metallicity increases. Plateau stars are stars for which the contribution from SNeIa is still negligible. Since the MW plateau stars have metallicities up to [Fe/H] = −1, we expect IMFs corresponding to metallicities up to that value to be relevant for the stars we consider in this work. This is also the reason why we plot in Figs 6 and 7 the IMFs of model galaxies with [Fe/H] = −1. On the other hand, in dwarf galaxies, characterized by lower SFRs, the iron enrichment by SNeII is less pronounced and SNeIa start contributing to the chemical evolution when [Fe/H] is still quite low. In other words, the plateau extends up to lower metallicities. For these galaxies, the IMF corresponding to [Fe/H] = −1 plotted in Figs 6 and 7 does not play any role in the chemical evolution of the most metal-poor, plateau stars of their respective galaxies. According to the hard model, we thus expect plateau stars in dwarf galaxies to be made of stellar populations with low metallicities, characterized by flat IMFs as shown in Fig. 7.

We can repeat the calculations of Section 5.2 and evaluate the dependence of the [α/Fe] plateaus on the galaxy mass assuming the mild and the hard IGIMF dependences on [Fe/H]. We consider only the reference values $\beta = 2.00$ and $\hat{y}_{\rm Fe} = 0$. Fig. 8 shows the [Mg/Fe] plateau versus SFR plot for the mild (solid line) and the hard (long dashed line) models. The [Mg/Fe] calculated with the metallicity-independent IMF employed in Section 5.2 is also shown for comparison (short dashed line). The mild model shows a trend similar to the one of the reference model, although the minimum [Mg/Fe] is up to ≃0.2 dex above the one shown by the reference (metallicity-independent) model. This difference reduces considerably for larger values of SFR. On the other hand, the hard model shows quite a different pattern: the variations of [Mg/Fe] as a function of SFR are of modest magnitude and [Mg/Fe] remains almost constant at ∼0.3 dex. This means that low-mass galaxies could

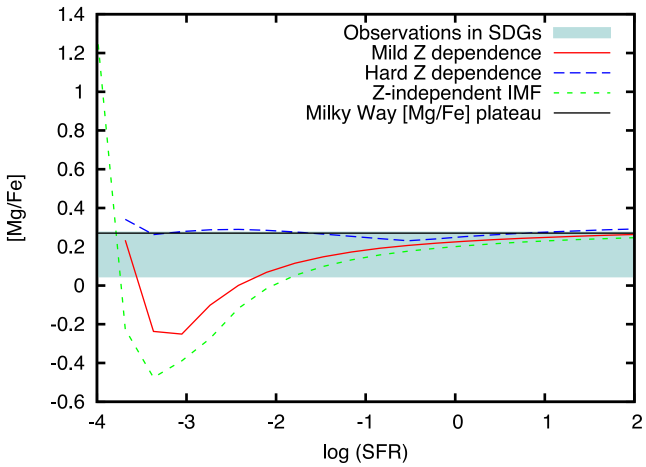

Figure 8. The variation of the [Mg/Fe] plateau as a function of the SFR for the mild (solid line) and hard (long dashed line) dependences of the IGIMF on the metallicity. Also the [Mg/Fe] predicted by the Z-independent IMF is shown (short dashed line). This is the same curve shown in Fig. 5. The average MW [Mg/Fe] plateau as derived by Cayrel et al. (2004) is shown as a horizontal solid line (see Table 2). The range of variation of [Mg/Fe] in the considered DGSs is reported as a shaded area (see Table 4).

indeed have [α/Fe] plateau values similar to the MW ones. In Fig. 8, we also plotted (shaded area) the data about the [Mg/Fe] ratios of the most metal-poor stars in Draco, Carina, Sculptor and Sextans, putting together all the calculated averages (with sigma deviations) for the four DGSs (see Table 4). The curve relative to the hard model is always close to the upper edge of this area, indicating that the hard model can be compatible with the MW [α/Fe] plateau and predicts [Mg/Fe] ratios that are only slightly larger than those of the most metal-poor stars in DGSs. On the other hand, the mild model is always closer to the shaded area than the Z-independent IGIMF model. It appears to be compatible with observations provided that the SFR is larger than $\sim 6 \times 10^{-3}$ M$_\odot$ yr^{-1} or smaller than $\sim 4 \times 10^{-4}$ M$_\odot$ yr^{-1}.

It is interesting to understand why the [α/Fe] plateau remains almost constant as SFR increases in the hard model. On the one hand, galaxies with low SFR have, according to the fundamental metallicity relation, low metallicities, hence flat IGIMFs (see Fig. 7). On the other hand, galaxies characterized by low SFRs attain a quite low value of $M_{\mathrm{ecl,\,max}}$ (see equation 2) and, consequently, a more limited amount of massive stars. These two competing effects (nearly) cancel out in this formulation.

6.2 On the comparison between our model predictions and the observations

As we have already anticipated in Section 3 and seen in Section 5.2, in spite of the enormous wealth of data on spectroscopy of metal-poor stars in dwarf galaxies and on SF histories of DGSs, detailed comparisons between our models and the observations are still difficult. Our results should be seen thus as *predictions* and detailed comparison with observations will be possible only when more data will be available and the literature results will be more homogeneous. However, we can already comment on the results of our study in the light of the available observations/knowledge of the DGSs. We recall briefly from Section 3.2 that typical early SFRs in DGSs are deduced to be of the order of 10^{-4} M$_\odot$ yr^{-1} but, for some galaxies, they could have been one or two orders of magnitude larger. Some theoretical studies even suggest them to be of the order of 10^{-1} M$_\odot$ yr^{-1} or more (Salvadori et al. 2008). As we have seen in Section 5.2, at this level of SF the predicted [α/Fe] plateaus are very close to the ones observed in the MW. Such an SFR would make our computation of [Mg/Fe] fully compatible with the observed plateaus in Sextans, Carina and Draco (see again Fig. 5). On the other hand, large differences are predicted if the SFR is significantly below 10^{-2} M$_\odot$ yr^{-1}.

We have reported in Table 4 the ranges of acceptable SFRs for each galaxy, according to the comparison between (Z-independent) IGIMF model results and observations shown in Fig. 5. We can see that Carina, Sextans and Sculptor can have had early SFRs as low as 0.1 M$_\odot$ yr^{-1} or less, whereas Draco is the only considered DGS requiring larger SFRs (at least 0.3 M$_\odot$ yr^{-1}). Also Fornax, with a [Mg/Fe] plateau of ∼0.1 dex (see Section 3.1), is compatible with an early SFR of ∼0.04 M$_\odot$ yr^{-1}. These SFRs are in general larger than the ones found in the literature for DGSs and summarized above. We remark again that, according to Fig. 5, the [Mg/Fe] ratios of the most metal-poor stars in DGSs could be compatible also with a very low SFR (less than 10^{-3} M$_\odot$ yr^{-1}). However, we believe that this solution is unlikely because, in this interval of SFRs, the average [Mg/Fe] ratios vary strongly. The average [Mg/Fe] ratios in the four considered DGSs are however quite similar (Table 4) and this fact seems to be hard to reconcile with the derived strong dependence of [Mg/Fe] with SFR. The SFRs suggested by the models (Table 4) and the ones available from the literature seem thus to be different. The reasons for this mismatch could be the following.

(i) As we have already argued, the determinations of the SFR in DGSs based on synthetic CMDs do not correctly take into account the stars lost during the DGS orbit around the MW and, thus, underestimate the intrinsic early SFR.

(ii) The IMFs in galaxies are indeed metallicity dependent. As we have shown, both mild and hard models predict [Mg/Fe] plateaus that are closer to the observations than what predicted by the Z-independent IGIMF model (see Fig. 8).

(iii) At variance with our assumptions, the IMF in galaxies is indeed universal and, therefore, the [α/Fe] plateaus of galaxies must always be the same (see also Kirby et al. 2011).

Based on the presently known observational facts, none of these three possibilities can be definitely ruled out. As described in Section 5.2, available observations for Draco, Carina, Sextans and Sculptor suggest that the most metal-poor stars in these DGSs have apparently slightly lower average [Mg/Fe] ratios than the most metal-poor MW stars. This is an indication that the IMF can indeed be non-universal. However, once again one should not forget that other high-quality data in other DGSs (Frebel et al. 2010; Norris et al. 2010; Simon et al. 2010; Vargas et al. 2013) indicate the presence of very metal-poor stars with [α/Fe] ratios equal or even larger that the Cayrel plateau. Clearly, these data also demand a theoretical explanation. We can at the present not even exclude the possibility that [α/Fe] ratios of the most metal-poor stars in some (or most of) DGSs of the MW are larger than the MW plateau value. In this case, our results could be compatible with the observations only if we consider the 'hard' Z-dependent IMF model (see again Fig. 8).

Again, our results should be seen as predictions and, once data concerning SFRs and [α/Fe] plateaus in dwarf galaxies will be more reliable, we will be able to establish more precisely if and to what extent metallicity affects the slopes of the IMF in galaxies.

Another hypothesis could be considered here. Some authors (see Kroupa et al. 2010; Kroupa 2012, and references therein) find that DGSs cannot be of cosmological origin. At least a fraction of them can be originated by the tidal disruption of a much larger galaxy (see also Lynden-Bell 1976). A key probe of that is the alignment of most of the DGSs along a disc (the *disc of satellites*), almost perpendicular to the MW disc. Another spectacular example of disc of satellites has been recently discovered around Andromeda (Ibata et al. 2013). If this is the case, of course the abundance patterns of the metal-poor stars in these galaxies reflect the early conditions of this large, disrupted galaxy. The [α/Fe] plateaus in the fragments that emerge after the disruption of the parent galaxy should be determined by the SFR in the early evolution of this primordial galaxy. They should therefore be quite similar to the MW plateau, without big differences from galaxy to galaxy. Of course, better measurements of the [α/Fe] ratios in DGSs and larger samples can shed light on this scenario, too.

7 CONCLUSIONS

In this work, we have studied the dependence of the [α/Fe] ratios in the most metal-poor stars of a galaxy (the [α/Fe] plateaus) on the galaxy's initial mass distribution of stars. The working hypothesis we have assumed in this study is that the most metal-poor stars in a galaxy are made out of gas that has been solely polluted by SNeII. Ejecta from intermediate-mass stars and from SNeIa are released on longer time-scales and cannot contribute to the chemical enrichment at this early stage. In this way, we can establish a connection between the high-mass slope of the IMF and the [α/Fe] plateau. If the high-mass IMF slope is universal, then the [α/Fe] plateau cannot change from galaxy to galaxy. If instead different galaxies are characterized by different IMF slopes, then a correlation between [α/Fe] plateau and IMF could be theoretically established.

In particular, the IGIMF theory (Kroupa & Weidner 2003) predicts a steep IMF in dwarf galaxies, characterized by mild SFRs. One could thus test the IGIMF theory by calculating the [α/Fe] plateaus as a function of the average SFR in a galaxy and compare them with available observations in the best observed sample of dwarf galaxies, namely the satellites of the MW. However, as summarized in Section 3.1, neither the early SF histories nor the [α/Fe] plateau values can be firmly established for most of the MW DGSs. Our results should be seen as predictions of the IGIMF models, to be compared with observations only once the determinations of these quantities will be more reliable.

Moreover, recent works (M12; Kroupa et al. 2013) suggest that the IMF in star clusters (hence also the integrated galactic IMF) could depend on the star cluster's metallicity. We have derived from M12 a mild and a hard dependence of the IGIMF on the average galaxy metallicity and we have calculated the [α/Fe] plateaus accordingly.

Our main results can be summarized as follows.

(i) Dwarf galaxies characterized by low SFRs, i.e. SFRs smaller than 10^{-2} M$_\odot$ yr^{-1}, can show very low [O/Fe] or [Mg/Fe] plateaus, up to 0.7 dex lower than the MW [O,Mg/Fe] plateau. This is at variance with observations, indicating [Mg/Fe] plateaus in dwarf galaxies only moderately lower than the MW [Mg/Fe] value. On the other hand, at SFRs smaller than $\sim 10^{-3.5}$ M$_\odot$ yr^{-1}, the [O,Mg/Fe] plateau can be much larger and consistent with observations.

(ii) If the SFR of the dwarf galaxy is larger than 10^{-2} M$_\odot$ yr^{-1}, then the variations of the [α/Fe] plateaus with SFR are of the order of 0.2 dex at most.

(iii) The [α/Fe] plateaus depend on the (uncertain) stellar yields between 8 and 12 M$_\odot$.

(iv) The slope β of the mass distribution function of star clusters has a significant impact, too, on the chemical abundance pattern.

(v) Our study of the observed [Mg/Fe] plateau in local dwarf galaxies suggests that they should be characterized by early SFRs either of the order of 10^{-4} M$_\odot$ yr^{-1} or larger than 10^{-2} M$_\odot$ yr^{-1}. However, in the former hypothesis, given the steep [Mg/Fe]–SFR relation predicted within the IGIMF, it seems difficult to conceive similar [Mg/Fe] ratios in different galaxies as observed today. It is also worth stressing that current determinations of the early SFRs in dwarf galaxies tend to be lower than 10^{-2} M$_\odot$ yr^{-1}. We have argued though that these SFR estimates are necessarily lower limits because they cannot take into account stars stripped off the main body of the dwarf galaxy by tidal interactions.

(vi) For the first time, we have considered the effects of a metallicity dependent IMF on the chemical abundance ratios. In case of a weak dependence of the IMF on [Fe/H], our results do not change substantially from the ones of the Z-independent IMF. The minimum of [Mg/Fe] as a function of the SFR lies $\simeq$0.2 dex above the corresponding minimum for the standard, non-metallicity-dependent IGIMF model. For this model, there is agreement between model predictions and observations provided that the early SF in the dwarf galaxies is larger than $\sim 6 \times 10^{-3}$ M$_\odot$ yr^{-1} or smaller than $\sim 4 \times 10^{-4}$ M$_\odot$ yr^{-1}.

(vii) If instead the IMF strongly depends on the metallicity (hard model), galaxy models characterized by low SFRs will have flatter IGIMFs than galaxy models with high SFRs. Unexpectedly, almost no dependence of the [α/Fe] plateau on the SFR is predicted in this case. The predicted [Mg/Fe] ratio is very close to the one observed in the most metal-poor stars of the MW and only slightly larger than that observed in Local Group dwarf galaxies.

The enormous progress made recently in the determination of the SF histories and of the [α/Fe] ratios of the most metal-poor stars in local dwarf galaxies offers a valuable benchmark for chemical evolution studies. The next few years will see an increase of the samples of stars with observable abundances in low-mass, low-metallicity galaxies. With improved statistics and more homogeneous data, our predictions for the variation of the [α/Fe] plateaus as a function of the SFR, will represent valuable tools to constrain the most basic chemical evolution parameters of dwarf galaxies, such as the IMF and the SF history.

To conclude, given the uncertainties related to the early SFRs in dwarf galaxies, we believe there are two ways to better assess the reliability or our results: (i): a better and more precise determination of the [α/Fe] plateaus in satellite dwarf galaxies, to check whether these plateaus, at least for some galaxies, could really differ from the MW halo values and (ii) a more direct determination of the IMF in low-metallicity dwarf galaxies (see e.g. Zhang et al. 2007) which could be directly used as an input for our models for calculating abundance ratios of metal-poor stars in DGSs.

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REFERENCES

André P. et al., 2010, A&A, 518, L102
Aoki W. et al., 2009, A&A, 502, 569
Asplund M., Grevesse N., Sauval A. J., Allende Prieto C., Kiselman D., 2004, A&A, 417, 751
Asplund M., Grevesse N., Sauval A. J., Scott P., 2009, ARA&A, 47, 481
Barbuy B., 1988, A&A, 191, 121
Bressert E. et al., 2010, MNRAS, 409, L54
Bromm V., Ferrara A., Coppi P. S., Larson R. B., 2001, MNRAS, 328, 969
Brook C. B., Stinson G., Gibson B. K., Roškar R., Wadsley J., Quinn T., 2012, MNRAS, 419, 771
Calura F., Menci N., 2009, MNRAS, 400, 1347
Calura F., Recchi S., Matteucci F., Kroupa P., 2010, MNRAS, 406, 1985 (C10)
Carollo D. et al., 2010, ApJ, 712, 692
Carretta E., Bragaglia A., Gratton R., Lucatello S., 2009, A&A, 505, 139
Carretta E. et al., 2010, A&A, 520, A95
Cayrel R. et al., 2004, A&A, 416, 1117
Cioni M.-R. L., Habing H. J., 2005, A&A, 442, 165
Clegg R. E. S., Tomkin J., Lambert D. L., 1981, ApJ, 250, 262
Cohen J. G., Huang W., 2009, ApJ, 701, 1053
Coleman M. G., de Jong J. T. A., 2008, ApJ, 685, 933
Conroy C., van Dokkum P., 2012, ApJ, 760, 71
Crowther P. A., Schnurr O., Hirschi R., Yusof N., Parker R. J., Goodwin S. P., Kassim H. A., 2010, MNRAS, 408, 731
de Boer T. J. L. et al., 2012a, A&A, 539, A103
de Boer T. J. L. et al., 2012b, A&A, 544, A73
Edvardsson B., Andersen J., Gustafsson B., Lambert D. L., Nissen P. E., Tomkin J., 1993, A&A, 275, 101
Fabrizio M. et al., 2012, PASP, 124, 519
Fenner Y., Gibson B. K., Gallino R., Lugaro M., 2006, ApJ, 646, 184
Ferreras I., La Barbera F., de la Rosa I. G., Vazdekis A., de Carvalho R. R., Falcón-Barroso J., Ricciardelli E., 2013, MNRAS, 429, L15
François P., Matteucci F., Cayrel R., Spite M., Spite F., Chiappini C., 2004, A&A, 421, 613
Frebel A., Simon J. D., Geha M., Willman B., 2010, ApJ, 708, 560
Gibson B. K., 1998, ApJ, 501, 675
Gibson B., MacDonald A. J., Sánchez-Blázquez P., Carigi L., 2008, The Metal-Rich Universe. Cambridge Univ. Press, Cambridge
Gieles M., Moeckel N., Clarke C. J., 2012, MNRAS, 426, L11
Gratton R., Sneden C., Carretta E., 2004, ARA&A, 42, 385
Gratton R. G., Carretta E., Bragaglia A., 2012, A&AR, 20, 50
Heger A., Woosley S. E., 2010, ApJ, 724, 341
Hennemann M. et al., 2012, A&A, 543, L3
Hidalgo S. L. et al., 2011, ApJ, 730, 14
Ibata R. A., Gilmore G., Irwin M. J., 1994, Nat, 370, 194
Ibata R. A. et al., 2013, Nat, 493, 62
Iwamoto K., Brachwitz F., Nomoto K., Kishimoto N., Umeda H., Hix W. R., Thielemann F.-K., 1999, ApJS, 125, 439
Karachentsev I., Aparicio A., Makarova L., 1999, A&A, 352, 363
Kirby E. N. et al., 2010, ApJS, 191, 352
Kirby E. N., Cohen J. G., Smith G. H., Majewski S. R., Sohn S. T., Guhathakurta P., 2011, ApJ, 727, 79
Kirby E. N., Boylan-Kolchin M., Cohen J. G., Geha M., Bullock J. S., Kaplinghat M., 2013, ApJ, 770, 16
Klessen R. S., Kroupa P., 1998, ApJ, 498, 143
Koch A., Grebel E. K., Gilmore G. F., Wyse R. F. G., Kleyna J. T., Harbeck D. R., Wilkinson M. I., Wyn Evans N., 2008, AJ, 135, 1580
Kroupa P., 1997, New Astron., 2, 139
Kroupa P., 2012, Publ. Astron. Soc. Aust., 29, 395
Kroupa P., Weidner C., 2003, ApJ, 598, 1076
Kroupa P. et al., 2010, A&A, 523, A32
Kroupa P., Weidner C., Pflamm-Altenburg J., Thies I., Dabringhausen J., Marks M., Maschberger T., 2013, Planets, Stars and Stellar Systems. Volume 5: Galactic Structure and Stellar Populations. Springer, Berlin
Lada C. J., Lada E. A., 2003, ARA&A, 41, 57
Lanfranchi G. A., Matteucci F., Cescutti G., 2006, A&A, 453, 67
Larsen S. S., Richtler T., 2000, A&A, 354, 836
Larsen S. S., Brodie J. P., Strader J., 2012, A&A, 546, A53
Lemasle B., Hill V., Tolstoy E. et al., 2012, A&A, 538, A100
Letarte B. et al., 2010, A&A, 523, A17
Lynden-Bell D., 1976, MNRAS, 174, 695
Majewski S. R., Skrutskie M. F., Weinberg M. D., Ostheimer J. C., 2003, ApJ, 599, 1082
Mannucci F., Cresci G., Maiolino R., Marconi A., Gnerucci A., 2010, MNRAS, 408, 2115
Marks M., Kroupa P., 2012, A&A, 543, A8
Marks M., Kroupa P., Dabringhausen J., Pawlowski M. S., 2012, MNRAS, 422, 2246 (M12)
Matteucci F., 1994, A&A, 288, 57
Matteucci F., 2001, The Chemical Evolution of the Galaxy. Kluwer, Dordrecht
Mayer L., Mastropietro C., Wadsley J., Stadel J., Moore B., 2006, MNRAS, 369, 1021
McWilliam A., Wallerstein G., Mottini M., 2013, preprint (arXiv:1309.2974)
Monelli M., Lcid Team, 2007, Proc. IAU Symp. 241, Stellar Populations as Building Blocks of Galaxies. Cambridge Univ. Press, Cambridge, p. 369
Monelli M. et al., 2010a, ApJ, 720, 1225
Monelli M. et al., 2010b, ApJ, 722, 1864
Norris J. E., Wyse R. F. G., Gilmore G., Yong D., Frebel A., Wilkinson M. I., Belokurov V., Zucker D. B., 2010, ApJ, 723, 1632
Palma C., Majewski S. R., Siegel M. H., Patterson R. J., Ostheimer J. C., Link R., 2003, AJ, 125, 1352
Recchi S., Calura F., Kroupa P., 2009, A&A, 499, 711 (R09)
Salvadori S., Ferrara A., Schneider R., 2008, MNRAS, 386, 348
Schneider R., Ferrara A., Natarajan P., Omukai K., 2002, ApJ, 571, 30
Schneider N. et al., 2012, A&A, 540, L11
Shetrone M. D., Côté P., Sargent W. L. W., 2001, ApJ, 548, 592
Shigeyama T., Tsujimoto T., 1998, ApJ, 507, L135
Simon J. D., Frebel A., McWilliam A., Kirby E. N., Thompson I. B., 2010, ApJ, 716, 446
Spitoni E., Matteucci F., Recchi S., Cescutti G., Pipino A., 2009, A&A, 504, 87
Starkenburg E. et al., 2013, A&A, 549, A88
Tafelmeyer M. et al., 2010, A&A, 524, A58
Tolstoy E. et al., 2004, ApJ, 617, L119
Tolstoy E., Hill V., Tosi M., 2009, ARA&A, 47, 371
Tsujimoto T., Yoshii Y., Nomoto K., Matteucci F., Thielemann F.-K., Hashimoto M., 1997, ApJ, 483, 228
van den Hoek L. B., Groenewegen M. A. T., 1997, A&AS, 123, 305
Vargas L. C., Geha M., Kirby E. N., Simon J. D., 2013, ApJ, 767, 134
Venn K. A. et al., 2012, ApJ, 751, 102
Weidner C., Kroupa P., 2005, ApJ, 625, 754
Weidner C., Kroupa P., Larsen S. S., 2004, MNRAS, 350, 1503
Weidner C., Kroupa P., Pflamm-Altenburg J., 2011, MNRAS, 412, 979
Woosley S. E., Weaver T. A., 1995, ApJS, 101, 181 (WW95)
Zhang Q., Fall S. M., 1999, ApJ, 527, L81
Zhang W., Kong X., Li C., Zhou H.-Y., Cheng F.-Z., 2007, ApJ, 655, 85

APPENDIX A: WIDENING THE PARAMETER SPACE

In this section, we extend the analysis made in Sections 5.1 and 5.2 and explore further parameters that might change our results concerning [α/Fe] plateaus in galaxies in the framework of the IGIMF theory. A summary of the results included here is also presented in Table 3.

As reported in Table 2, the reference threshold mass for SNII explosion used in equation (4) is 8 $M_\odot$. If we increase it to 10 $M_\odot$, the [α/Fe] variations as a function of $\hat{y}_{Fe}$ become very weak ($\sim$0.04 dex). This was expected because, although the fraction of massive stars with masses between 10 and 12 $M_\odot$ is not negligible

($\sim$16 per cent for $\beta = 2.00$), the heavy element production in this mass range is quite limited. For instance, only $\sim$1.8 per cent of oxygen is produced by stars in the interval [10, 12] $M_\odot$ for $\beta = 2$ and $\hat{y}_{Fe} = 0.5$. Conversely, we can observe that, modifying the standard value of m_{thr} from 8 to 10 $M_\odot$, we also obtain very small changes (of the order of 0.04 dex) in the [α/Fe] ratios.

The value of the average MW SFR in the first Gyr of 0.5 $M_\odot$ yr^{-1}, adopted as a standard value, comes from the C10 simulations. This value is however both model and assumption dependent. Significantly different (usually larger) values can be found in the literature. For instance, Gibson et al. (2008) show different simulations of the evolution of MW-like galaxies and the average SFR in the early phases is much larger than 0.5 $M_\odot$ yr^{-1}, reaching SFRs up to $\simeq$ 20–30 $M_\odot$ yr^{-1}. We have repeated the calculations of the [α/Fe] ratios using an SFR of 100 $M_\odot$ yr^{-1}, the largest SFR considered by R09. The results differ very little from the ones presented in Fig. 2: the [O/Fe] ratios are at most 0.015 dex larger than the reference values. This confirms the general conclusion of R09 that only for SFRs below 0.1–0.01 $M_\odot$ yr^{-1} are the deviations of the IGIMF from a standard (not SFR-dependent) IMF significant.

Yields above 40 $M_\odot$ turn out to affect negligibly our results. Even if we increase the iron yields above 40 $M_\odot$ by a factor of 10^4, the [α/Fe] ratios change by less than 0.01 dex (see Table 3). We have also varied the parameters A and B characterizing the dependence of the maximum embedded cluster mass $M_{ecl,max}$ and the SFR in equation (2). A variation of A of the order of 0.5 and of B of the order of 0.1 change very little our results. In fact, [O/Fe] changes by $\sim$0.05 dex at most. We remind however that the correlation between $M_{ecl,max}$ and ψ is based on observations; therefore, it should not be subject to large variations.

We have varied also the assumed lower mass for an embedded cluster $M_{ecl,min} = 5\,M_\odot$ and the theoretical upper stellar mass within an embedded cluster $m_{max*} = 150\,M_\odot$ (see Table 1) that are used in the IGIMF formula (equation 4). Since the upper end of the IGIMF is affected by the most massive star cluster, not by the smallest ones, assuming $M_{ecl,min}$ larger than 5 $M_\odot$ produces negligible effects. For instance, if we take $M_{ecl,min} = 20\,M_\odot$ we obtain results differing by less than $\sim$0.01 dex compared to the reference models. Since the work of Crowther et al. (2010) indicates the existence of stars with masses larger than 150 $M_\odot$, it is worth studying what changes in our results if we assume $m_{max*} = 300\,M_\odot$. At low SFRs the results with $m_{max*} = 300\,M_\odot$ are practically indistinguishable from the ones we have presented so far. This is understandable because in this case the upper mass predicted by the IGIMF theory is much below 150 $M_\odot$. At large SFRs, some differences start emerging, but they never exceed $\sim$0.03–0.04 dex. Again, these results are summarized in Table 3.

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