A Comprehensive Study of the Market Price of Canadian Private and Public Firms

by

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A research project submitted in partial fulfillment of the requirements for the degree of Master of Finance

Saint Mary's University

Written for MFIN 6692 under the direction of
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Abstract

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Private companies represent a large percentage of the world's businesses. Interestingly, most corporate finance research focuses on public companies. The only reason for this is that the wealth of information on public companies is considerably superior to that of private companies.

The present study contributes to the body of knowledge about private companies by finding a model that is able to explain what is the fair price of a Canadian private firm.

To put the private firm pricing model in perspective, this Project also determines a model that calculates the price of Canadian public firms.

The models are easy to use by professionals of the corporate finance world, so that some selected variables are able to explain as much of the variation in prices as possible.

The models are also used to extract valuable information for practitioners, i.e. owners of private firms, private equity firms, speculative investors and owners of public companies who want to acquire the portfolio of a private firm.

Key words: Canada; Mergers & Acquisitions; M&A; private firms; valuation; pricing; price discount

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Chapter 1

Introduction

1.1. Background

Private companies represent a large percentage of the world's businesses. In Canada, the total number of registered employer businesses (businesses with at least one employee on payroll) as of December 2010, was 1,138,761[1]. However, as of April 2012, only 1,588 of them were listed on the Toronto Stock Exchange[2].

Interestingly, most corporate finance research focuses on public companies. The only reason for this is that the wealth of information on public companies is considerably vaster compared to that of private companies.

In the field of pricing of private firms, practitioners accustom to use an indirect method to calculate the price of a private company, by first calculating the price of the private firm as if it were public -through the same methods used for the valuation of public companies- and then adjusting the result by a *price discount factor*, which represents the relative illiquidity of the private, restricted stock.

There is a large number of research works that deal with the calculation of such discount factor for the US, method that provides an indirect way to calculate the price of a US private firm.

[1] Industry Canada, December 2010.
[2] Market Intelligence Group (MiG), April 2012.

The discount factor method shows a basic weakness: while the theoretical base is spotless, its results have to be tested against real, empirical data. And for this purpose we would need to find two companies similar in every respect (e.g. revenues, assets, earnings, book value), except that one of them is public and the other one is private, and then observe the prices that the market allocates to each of them. The difference between both prices would be used to draw the price discount which, in turn, would have to be compared against the result of the theoretical model. But in real life it is very difficult to find a substantial number of pairs of similar private-public firms, especially in smaller markets like Canada. And, even if such pairs can be found, private companies are not always for sale, limiting the number of possible observations of market prices. The consequence is that a price discount model is difficult to be tested empirically in Canada.

Moreover, the adjusted-R^2 of all price discount models developed for the US, i.e. the statistical measure of how well price discounts are predicted by the models, is on average 30%, a very low figure. And no model could obtain an adjusted-R^2 higher than 41%. One of the main reasons for these meagre results is that the data samples used are relatively small. Wruck (1989) uses data of 37 unregistered sales, Hertzel & Smith (1993) 106 private placements and Silber (1991) only 69 private placements. This Project uses a larger data set to further improve the results obtained by prior researchers.

1.2. Purpose and objectives of study

The purpose of the present Master's Research Project is to contribute to the body of knowledge about private companies by finding a model that is able to explain what is:

- the fair *price* (not the value, but the price that an acquirer is most likely going to pay)
- of a *Canadian private firm*
- with a *high statistical reliability* of the results.

The model has to be easy to use by professionals of the corporate finance world, so that a few selected variables are able to explain as much of the price of Canadian private companies as possible.

As the absolute price of a private company by itself does not give much information, this Research Paper will also determine a second model that calculates the price of Canadian *public* firms. With both the private firm and the public firm models, it is then possible to calculate *price discounts*, and compare them against the results of researchers of the price discount model for the US.

The models obtained for Canadian private and public companies will then be used to extract valuable information for practitioners.

1.3. Limitations to the study

As this paper deals with market prices of private and public firms, the feasibility of the project is given by the data available from commercial databases.

For public companies a wealth of information is accessible, specially all related to their financial situation. However, for private firms, the amount of information made public varies very widely; for some companies we may have all key financial information accessible, while for others we may only get e.g. their revenues.

The selection of the most complete database at an affordable cost is, thus, essential. For the present work we will use the database 'Financial Post Crosbie Mergers & Acquisitions in Canada', created by Infomart, which provides daily information on mergers, acquisitions and divestitures involving Canadian companies between 1978 and 2012. This is further discussed in Chapter 3.

1.4. Outline of this paper

This current chapter sets the stage for the discussion of the study.

Chapter 2 reviews the literature that sheds some light on the pricing of private companies, identifying what is still left to improve.

In Chapter 3 we will describe the data set used for the present work, and then we will use such data set to review the main features of Canadian private and public

firms, i.e. prices paid in transactions performed during the last 34 years, statistical distributions of revenues, assets, net book value, net income, debt/equity ratios and percentage of firm that uses to be acquired. Always discriminating between private and public firms.

Chapter 4 deals with the derivation of statistical models for pricing private and public companies, using the data set from Chapter 3. Conclusions are then drawn using the models to provide valuable information to sellers and acquirers of companies, e.g. owners of private companies, owners of public companies and investors.

In Chapter 5 we summarize the results of this study and discuss possible directions for future research.

Chapter 2

Literature Review

2.1. Price discount models to calculate the price of a private firm

All research works that deal with the calculation of the price of a private firm use the so-called price discount method. This method considers that a private firm is a restricted public firm, limited in its access to capital markets. And because of this restriction, its price is assumed to be less than the price of a similar public firm.

The first contemporary study of the price of private securities can be attributed to Solberg (1979). He collected cases judged by US courts and the US Internal Revenue Service in which a discount from market could be observed. The cases spanned the period 1942 - 1978, but the sample size was limited, as it only considered 18 decisions. The mean discount observed was 37.4%.

Later works by Wruck (1989), Silber (1991), Hertzel & Smith (1993), Longstaff (1995), Koeplin et al. (2000), Bajaj et al. (2001), Barclay et al. (2001), Das et al. (2002), and Capron & Shen (2007) contributed with the formalization of models that are able to explain the price of private firms in the US market.

As an example of the work performed by former researchers we will review in more detail four of the most influential papers, those written by Wruck, Silber, Hertzel & Smith and Longstaff.

Wruck (1989) studied the change in firm value when companies carry out private and public sales of securities. The sample size used was 37 transactions that occurred in the US between 1979 and 1985. One of her findings was that unregistered, private sales of stock performed at an average discount of 13.5%, with a minimum of -48.2% and a maximum of 95%. Unregistered shares are closer to private stock because they are less marketable.

Silber (1991) analyzed the impact of illiquidity on stock prices. His main contribution was the development of a regression model that calculates the estimated price discount applied to a private company when compared vs. a similar public company. This model can be considered the benchmark used still today. Silber used data of 69 private placements that happened in the US between 1981 and 1989, and found that restricted stocks were selling at an average price discount of 33.75%, with a minimum of -12.7% and a maximum of 84%. However, the adjusted-R^2 of the model, i.e. the statistical measure of how well price discounts are predicted by the model, is only 29%. Silber admitted "that there is substantial unexplained variability in the dependent variable" (p. 63).

Similarly to Silber, Hertzel and Smith (1993) tried to explain price discounts observed for private sales of stock. The sample used was 106 private placements

that happened in the US between 1980 and 1987. They also developed a least squares regression equation for private placements that is able to explain a mean discount of 20.14%. Even though Hertzel's model is statistically more accurate than Silber's (Hertzel's adjusted-R^2 of 41% vs. Silber's 29%), the mean discount is very different (Hertzel's 20.14% vs. Silber's 33.75%), and different from the mean discount found by most researchers, which is close to 30%.

Differently to the three prior studies, Longstaff (1995) developed a theoretical price discount model without taking a look at real sales of private stock. Instead, he derived an options model that uses as inputs the standard deviation of returns of the US market and the number of days that the stock is restricted to be marketed. For a volatility of returns of 0.25 to 0.35, and a marketability restriction period of 2 years, the expected price discount varies between 31.6% and 45%.

We can summarize the results of the five works above and all others reviewed with the help of the following table (Table 2.1):

Table 2.1

Summary of main works that deal with a price discount method to calculate the price of a private firm

Author	Year of publication	Years of data sample	Country analyzed	Size of private firm sample	Mean discount found	If regression model applied to discounts, what is R^2	Number of citations[3]	Other remarks
Solberg	1979	1942-78	US	18	37.4%	Not applicable	Unknown	Not a research work, but a listing of court decisions
Wruck	1989	1979-85	US	37	13.5%	Not given	662	The statistics refer to observations of private sales of unregistered securities
Silber	1991	1981-88	US	69	33.8%	29%	257	The work is based on private transactions of stock of public firms
Hertzel	1993	1980-87	US	106	20.14%	41.3% (adjusted-R^2)	448	The work is based on private transactions of stock of public firms
Longstaff	1995	Not an empirical study	US	Not applicable	31.6-45%	Not applicable	238	Not an empirical study, but a theoretical way to determine the discount due to lack of marketability
Koeplin	2000	1984-98	US & foreign	84	20-28%	Not given	98	Excludes financial firms and regulated utilities
Bajaj	2001	1990-95	US	51	28.13%	32.27% (adjusted-R^2)	70	The work is based on private transactions of stock of public firms

[3] As reported by Google Scholar on July 27, 2012.

Table 2.1 - Continued

Author	Year of publication	Years of data sample	Country analyzed	Size of private firm sample	Mean discount found	If regression model applied to discounts, what is R^2	Number of citations	Other remarks
Barclay	2001	1978-97	US	594	19%	15% (adjusted-R^2)	64	The work is based on private transactions of stock of public firms
Das	2002	1980-2000	US	52,322 private equity financing rounds in 23,208 firms	11% for late stage companies, 80% for early stage companies	Not given	73	The discount calculated includes not just illiquidity effects, but also remuneration for venture capitalists
Capron	2007	1988-92	US & foreign	92	38%	Not given	84	Only considers manufacturing industries

2.2. What is still subject to improvement?

In addition to their positive contributions, the research works identified show some common features that are subject to improvement:

- Most of the works are based on private transactions of stock of public firms, but they do not deal with pure private companies. This questions the applicability of the results to companies that are not traded.
- The adjusted-R^2 of all price discount models, i.e. the statistical measure of how well price discounts are predicted by the models, is on average 30%, a very low figure. Only Hertzel declared an adjusted-R^2 of 41%. This is probably the consequence of working with relatively small data sets.
- Data used are at least 12 years old.
- There is no work dealing with prices of Canadian firms. All of them are based on the US.

The present paper intends to improve the results of the former works by using a large database of private and public firms acquired in Canada, up to the year 2012, what will lead to reliable pricing models for both private and public firms.

Chapter 3

Methodology

3.1. Process to follow

To get to the goal determined in Chapter 1 we will follow the process described on Figure 3.1, below.

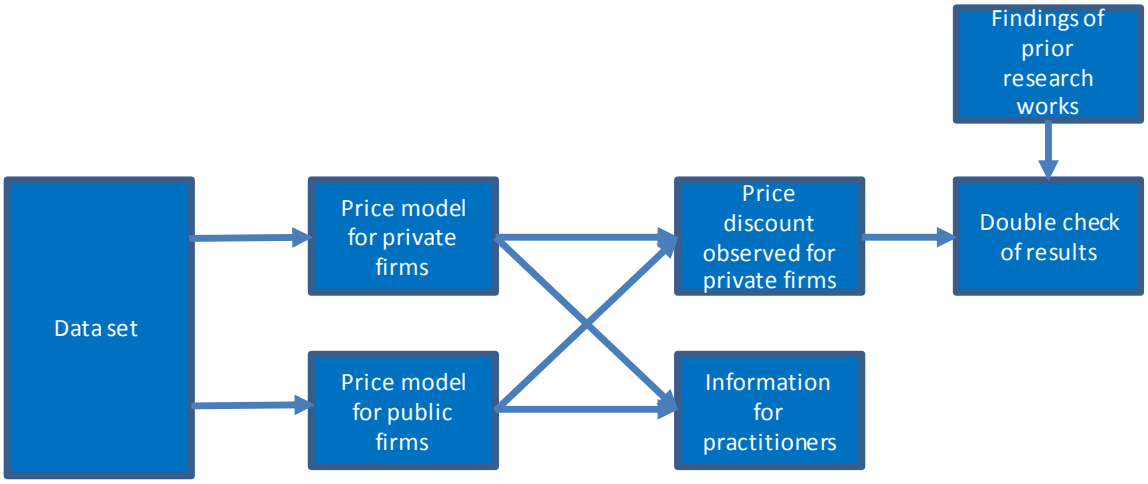

Figure 3.1

Process followed in the present Research Project

From the data available we will first develop regression equations that allow us to model the price of private and public firms in Canada.

To double check that these results are consistent with prior research works, we will then calculate the price discount for an average Canadian company and compare it against Silber's model.

The final contribution of this study is to provide applications of the pricing models developed, as this paper intends to provide very practical information for corporate finance practitioners.

3.2. Data source

For the present work we will use the database 'Financial Post Crosbie Mergers & Acquisitions in Canada', created by Infomart, which provides daily information on mergers, acquisitions and divestitures involving Canadian companies between 1978 and 2012.

Infomart is a Canadian media intelligence agency owned by Postmedia Network Canada Corp., with more than 25 years of delivering media monitoring and research solutions.

The database holds records of 19,664 transactions that occurred between 1978 and June 5, 2012. The content can be classified as follows:

- 5,336 transactions where the acquired firm is foreign. This group of transactions will be excluded from any further analysis, as they cannot be measured by the same standard as companies acquired in Canada.
- 4,197 divestitures of Canadian firms. The database does not differentiate between public and private vendors, so this group will not be considered for the analysis.

- 2,829 publicly owned Canadian targets: 2,189 of them include financial information (revenues, assets, net book value, net income or a combination of them). 640 of them do not.
- 7,302 privately-owned Canadian targets: 345 of them include financial information. 6,957 of them do not.

The usable sample size consists thus of a total of 2,534 transactions, 2,189 of them where the target is a public firm and 345 transactions where the target is a private firm.

This can be summarized with the help of the following chart:

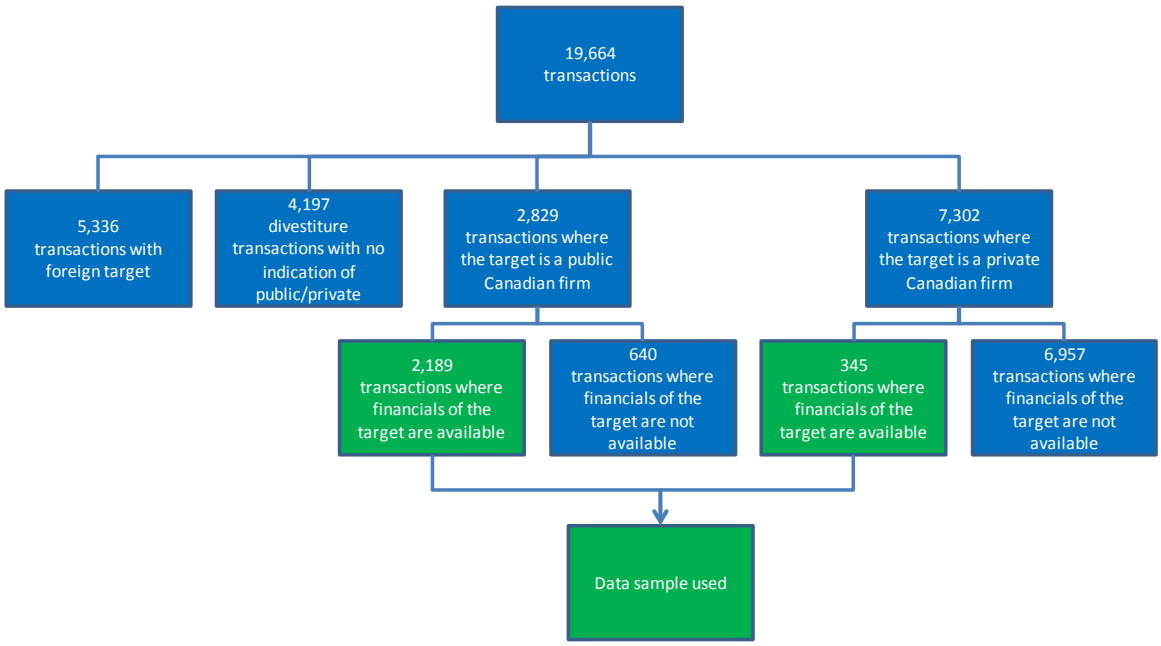

Figure 3.2

Database available and data set used

3.3. Available variables

For each of transactions registered on Infomart the following data are available:

Table 3.1

Informart database description

Infomart's variables and their descriptions	Units
Infomart's identifier (ID) of the transaction	
Announcement date of the transaction	
Revised date of the transaction	
Status of the transaction: complete / pending / terminated	
Classification of the transaction: acquisition, change of control, merger...	
Deal currency: USD, CAD, GBP...	
Deal value: amount presented in the original currency of the transaction	Deal currency
Canadian dollar (CAD) equivalent: deal value in CAD	CAD
Payment terms: cash, debt assumption, stocks...	
% bought: percentage of firm acquired	%
Deal description	
Industry category	
Deal type: privately owned, publicly traded, divestiture, foreign target	
Role of party: acquirer, target, vendor	
Name of acquired firm	
Location of acquired firm	
Standard Industrial Classification code (SIC) of the acquired firm	
Business description	
Financial advisors	
Year	
Revenue	CAD
Assets	CAD
Net book value (NBV)	CAD
Net income (NI)	CAD
Times revenue: deal value in CAD divided by revenue	
Times NBV: deal value in CAD divided by net book value	
Times NI: deal value in CAD divided by net income	
Offer/sh: for public companies, deal value in CAD divided by the number of shares traded	CAD
Market price: for public companies, share market price on the announcement date of the transaction	CAD
Premium to market: for public companies, relative price difference offer/sh vs. market price	%

It is necessary to take a closer look at the variable that records the price paid for the acquisition of a company. Infomart's variable "Canadian dollar equivalent" gives us the price paid for the percentage of the firm acquired. To find out what would be the price of the total company from the perspective of the acquirer, we need to take the Canadian dollar equivalent of the deal and divide it by the percentage of the company bought. In this way we can create a new variable "price of the company" defined as:

price of the company = Canadian dollar equivalent / percentage bought [1]

It could also be interesting to know what is the capital structure for Canadian private and public companies. This information is not directly given by Infomart, but we can calculate the ratio debt / equity (D/E) in this way:

D/E = (assets - net book value) / (net book value) [2]

As identified by Silber (1991), most of the relevant financial variables are easier to use in their logarithmic form that in their absolute values. This is due to the large dispersion of the financial figures. For our models we will also calculate the logarithmic transformations of the variables price, revenue, assets, net book value, net income and D/E.

Lastly, we can analyze each variable separately for private and public firms. For this purpose we will create a dummy variable, DPRIVATE, that will be assigned a value DPRIVATE=0 if the company is public and DPRIVATE=1 if it is private.

All these additional variables will be included in the database used for this Project, and added to the ones provided by Infomart. The final database that will be used when running regressions is thus the one given in Table 3.2.

Table 3.2

Final database used for deriving pricing models

Infomart's and additional variables used	Units	Variable name used in Stata
Price of the company: deal value in CAD divided by the percentage bought, as given by equation [1]	CAD	PRICE
Percentage of firm bought		BOUGHT
Revenue	CAD	REVENUE
Assets	CAD	ASSETS
Net book value	CAD	NBV
Net income	CAD	NI
Debt/equity ratio, as given by equation [2]		DE
ln (PRICE)		LNPRICE
ln (BOUGHT)		LNBOUGHT
ln (ASSETS)		LNASSETS
ln (NBV)		LNNBV
ln (NI)		LNNI
ln (DE)		LNDE
Dummy variable to differentiate private from public firms		DPRIVATE

3.4. Characterization of the data set

Once we have the list of the variables available it would be interesting to know what their typical values are.

For instance, let's find out what is the distribution of the price paid for the acquisition of a company, assuming that the acquirer buys 100% of it (in logarithmic form, variable LNPRICE):

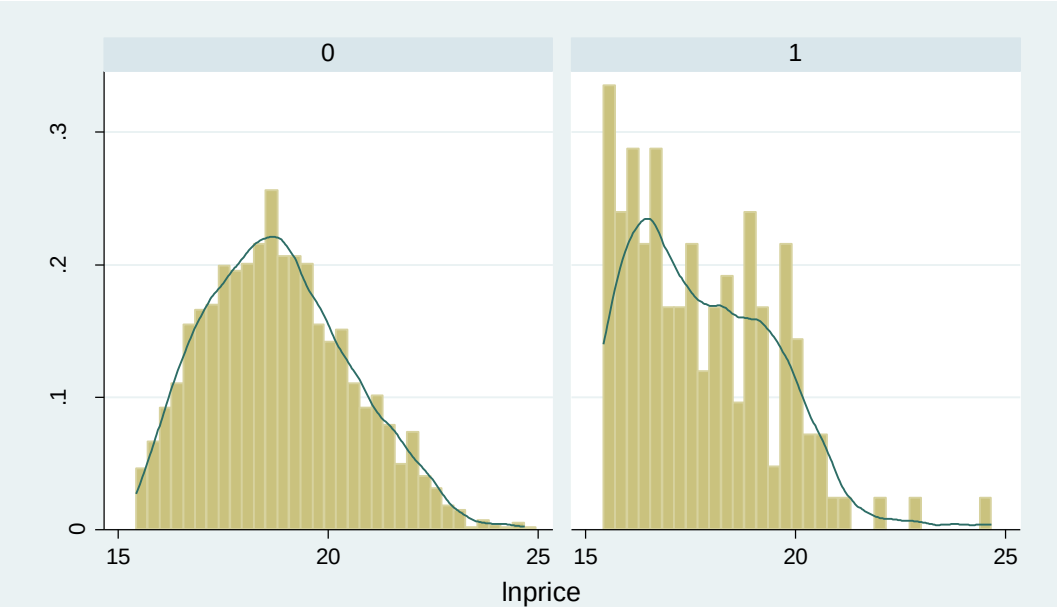

Figure 3.3

Histogram and kernel density of LNPRICE, for public (left, DPRIVATE=0) and private (right, DPRIVATE=1) firms

The mean price of a private firm is considerably less than the mean price of a public company. And the variability of prices of private companies is considerably higher.

Similarly, we can identify the distribution of the revenues of the companies acquired (in logarithmic form, variable LNREVENUE):

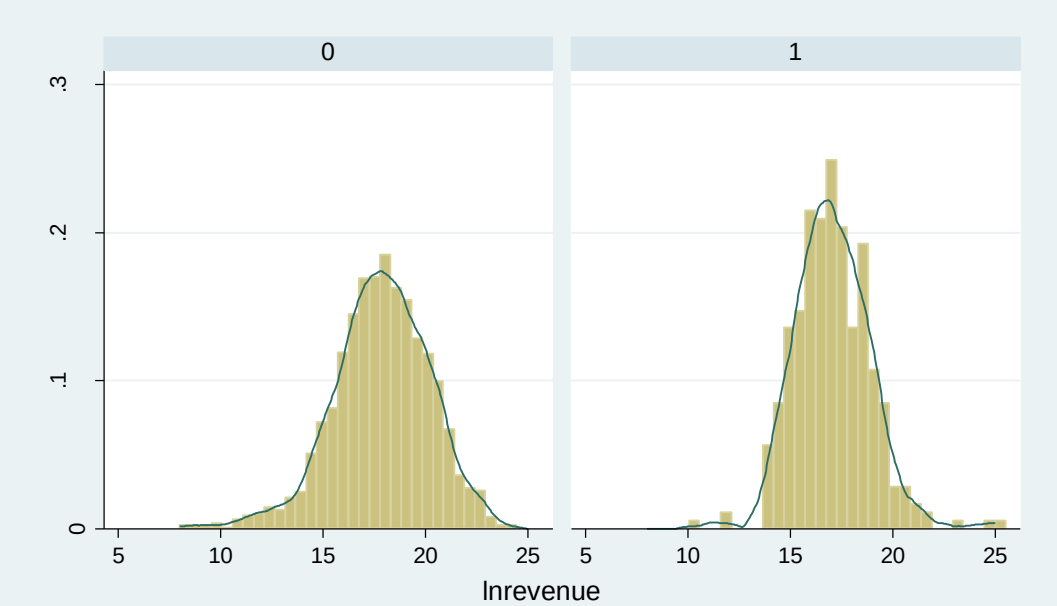

Figure 3.4

Histogram and kernel density of LNREVENUE, for public (left) and private (right) firms

Being the median similar in both cases, the variance of the revenue of private firms is considerably less than that of public companies.

For assets (in logarithmic form, variable LNASSETS) the median is similar for both public and private firms, but the variability of the value of assets is considerably higher for private companies:

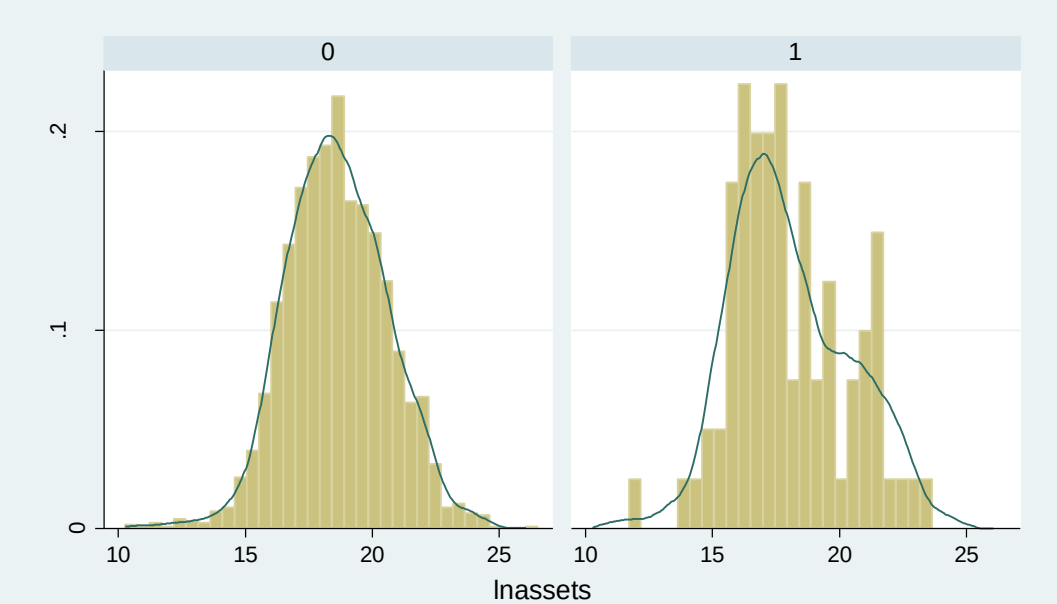

Figure 3.5

Histogram and kernel density of LNASSETS, for public (left) and private (right) firms

For net book value (in logarithmic form, variable LNNBV) the median is similar for both public and private firms, but the variability of the net book value is considerably higher for private companies:

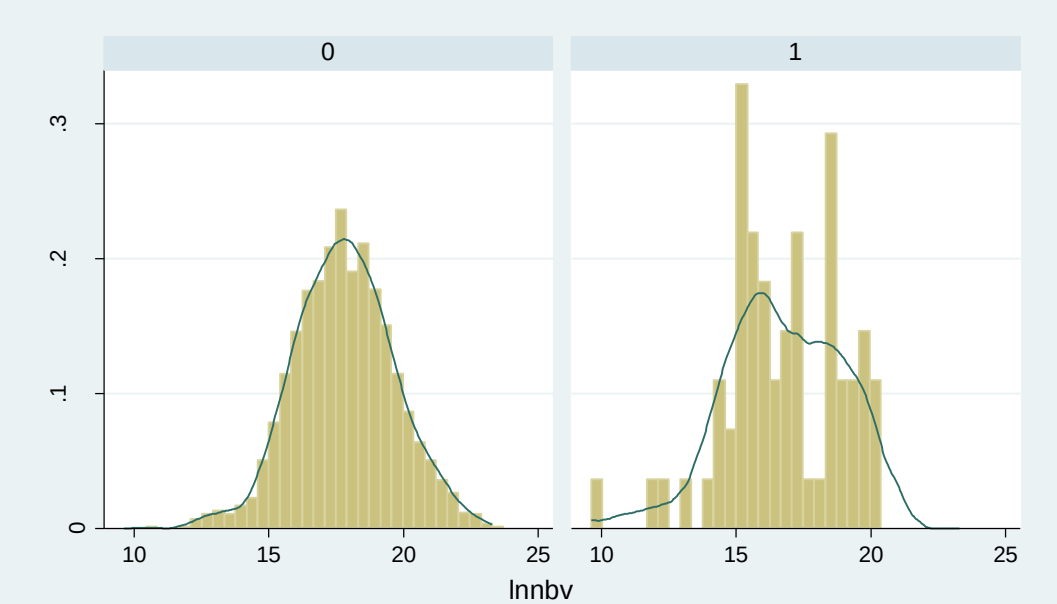

Figure 3.6

Histogram and kernel density of LNNBV, for public (left) and private (right) firms

For the net income (in logarithmic form, variable LNNI) the median is similar for both public and private firms, but the variability of the net income is considerably higher for private companies:

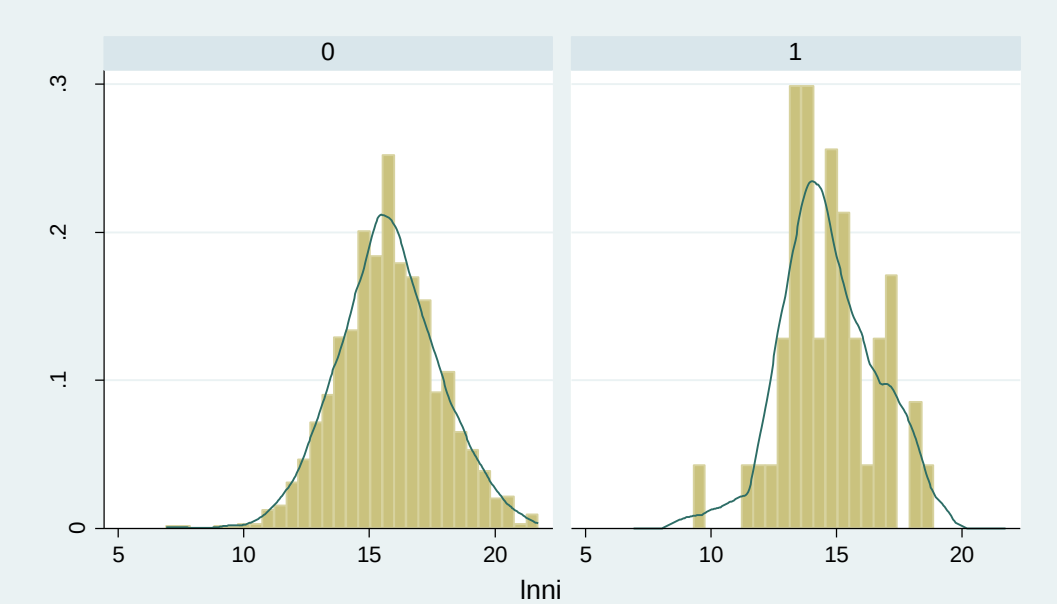

Figure 3.7

Histogram and kernel density of LNNI, for public (left) and private (right) firms

For the D/E ratio (in logarithmic form, variable LNDE) the median is similar for both public and private firms, but the variability of the ratio is considerably higher for private companies:

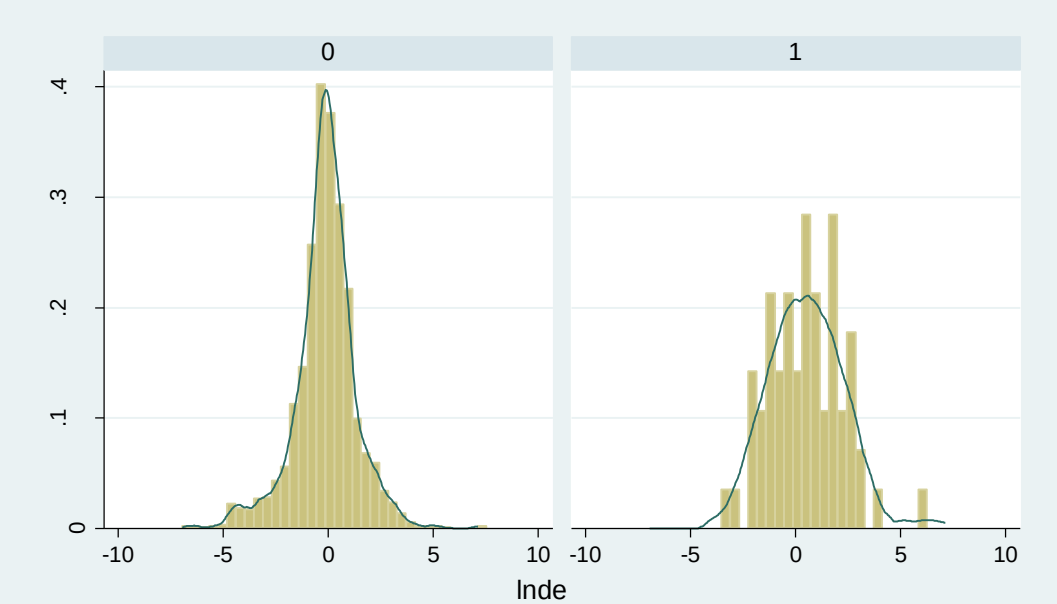

Figure 3.8

Histogram and kernel density of LNDE, for public (left) and private (right) firms

Regarding the percentage of the firm that is acquired (in logarithmic form, variable LNBOUGHT) there are differences between public and private firms, too. More private companies are purchased completely than public companies:

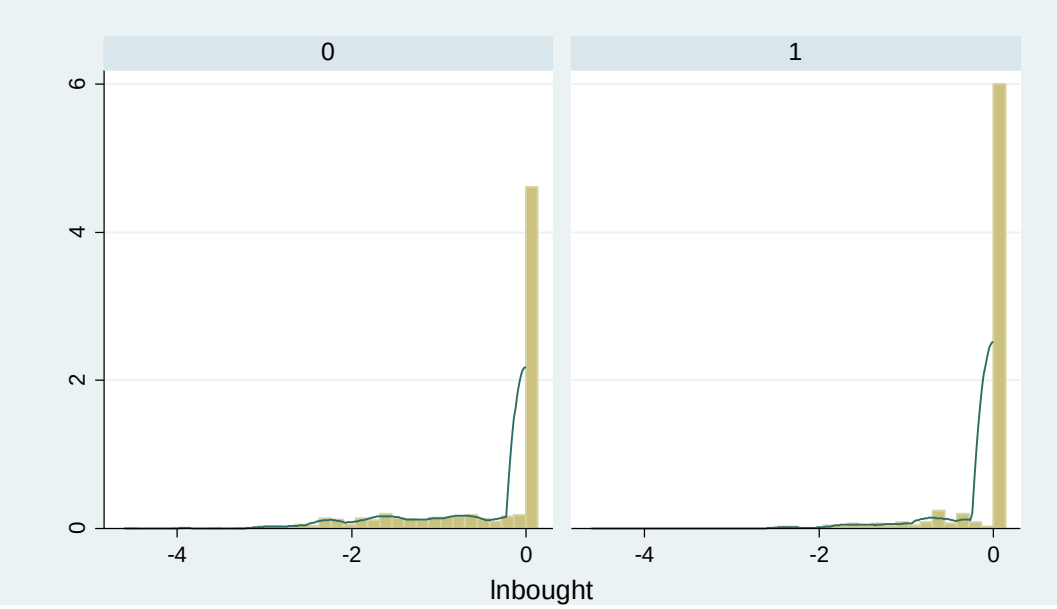

Figure 3.9

Histogram and kernel density of LNBOUGHT, for public (left) and private (right) firms

As we can see, all variables are much more normally distributed and are less dispersed for public companies than for private firms.

In a tabular form, the median values of the parameters that have just been reviewed are:

Table 3.3

Median values of the most relevant variables in the data set

	Price	Revenue	Net income	Assets	Net book value	D/E	% bought
All firms	$124m	$50.9m	$1.2m	$111m	$42.1m	0.92	100%
Private firms	$40m	$24.7m	$0.6m	$41m	$8.7m	1.69	100%
Public firms	$132m	$59.9m	$1.2m	$114.6m	$43.7m	0.91	100%

If we use average (mean) values instead:

Table 3.4

Average (mean) values of the most relevant variables in the data set

	Price	Revenue	Net income	Assets	Net book value	D/E	% bought
All firms	$754m	$509m	$16.2m	$941m	$238m	2.63	79.5%
Private firms	$573m	$492m	-$31.7m	$756m	$44.3m	7.85	91.4%
Public firms	$768m	$512m	$17.9m	$948m	$245m	2.44	77.7%

As it is evident from the statistics presented, there are important differences between the median and mean measures, what is an indication of the strong variability of the transactions performed in Canada.

Chapter 4

Results

4.1. Identification of the most relevant variables for the pricing models

In this chapter we will find a way to determine the price of Canadian private and public companies from the perspective of their acquirers. As it was mentioned in Chapter 1, this work does not deal with the value of companies, but with their prices, as the prices of companies tend to be influenced by factors beyond their objective values.

If we plot the relationship between the variable LNPRICE and every other relevant logarithmic variable, we can easily identify if they keep some obvious linear relationship. The explanatory models will work better the more linear they are.

Let's plot first LNPRICE vs. LNASSETS, LNNBV, LNREVENUE, LNNI, LNBOUGHT and LNDE for public companies. The dashed square indicates the combinations of variables that we will consider:

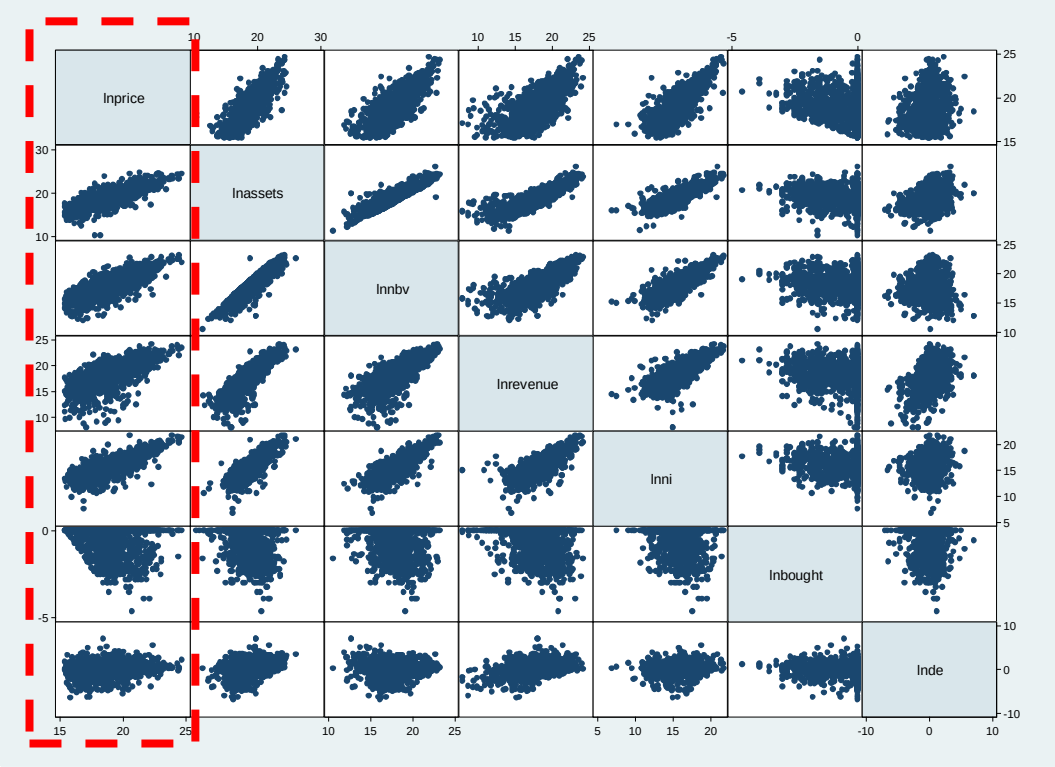

Figure 4.1

Relationships between variables in public companies

Visually it is obvious that LNPRICE keeps a linear relationship with LNASSETS, LNNBV, LNREVENUE and LNNI, and an apparently less linear connection to LNBOUGHT and LNDE.

The graphical result is confirmed when we run regressions of LNPRICE against each individual explanatory variable:

Table 4.1

Results of individual regressions of LNPRICE vs. each individual explanatory variable, for public companies

	LNASSETS	LNNBV	LNREVENUE	LNNI	LNBOUGHT	LNDE
P>|t|	0.00	0.00	0.00	0.00	0.00	0.00
Adjusted-R^2	69%	66%	49%	64%	4%	4%

The adjusted-R^2 of the individual regressions run between LNPRICE and LNASSETS, LNNBV, LNREVENUE and LNNI (i.e. the statistical measure of how well the price of the firm is predicted by each individual variable) are relatively high, while the role of LNBOUGHT and LNDE in predicting LNPRICE is anecdotic.

Performing now the same steps for private companies:

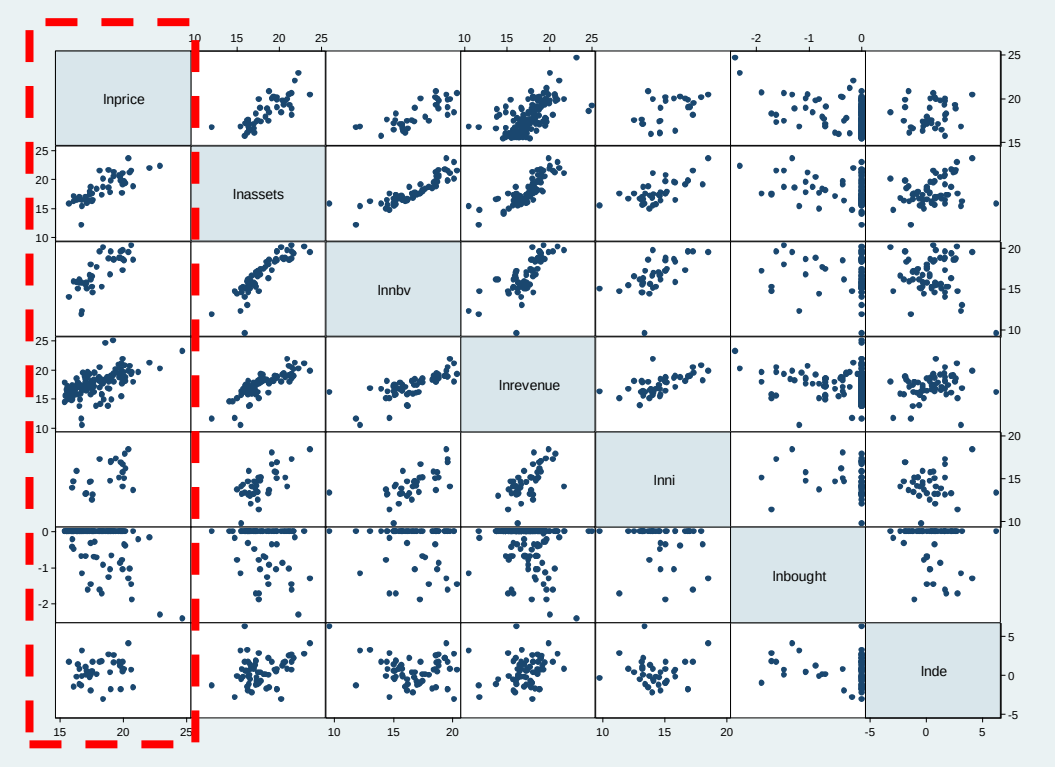

Figure 4.2

Relationships between variables in private companies

Table 4.2

Results of individual regressions of LNPRICE vs. each individual explanatory variable, for private companies

	LNASSETS	LNNBV	LNREVENUE	LNNI	LNBOUGHT	LNDE
P>|t|	0.00	0.00	0.00	0.01	0.00	0.37
Adjusted-R^2	63%	59%	37%	22%	18%	0%

We realize that the p-value of LNDE is high, meaning that the coefficient applicable to LNDE in a linear regression is not significantly different from 0. Furthermore, the adjusted-R^2 of the regression between LNPRICE and LNDE is very low, denoting that LNDE is irrelevant in explaining LNPRICE.

The adjusted-R^2 of the individual regressions run between LNPRICE and LNASSETS and LNNBV are relatively high. LNREVENUE, LNNI and LNBOUGHT have a limited effect on LNPRICE.

We can now plot side-by-side the adjusted-R^2 obtained when running the regressions of LNPRICE vs. each individual explanatory variable, differentiating by public and private firms:

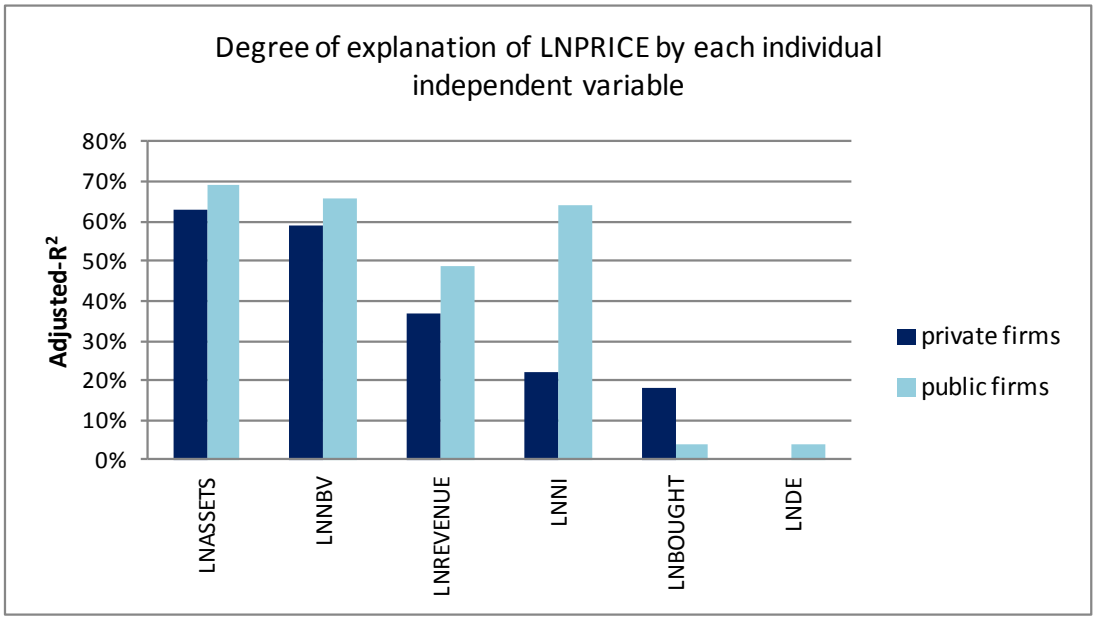

Figure 4.3

Degree of explanation of LNPRICE by each individual independent variable, given by the adjusted-R^2 of each individual regression

We can see that LNASSETS and LNNBV are able to explain most of the variability in price for private and public firms (adjusted-R^2 around 60%). LNREVENUE and LNNI also play an important role for public companies. LNBOUGHT, LNREVENUE and LNNI play a limited role in explaining LNPRICE for private firms. LNBOUGHT is irrelevant for public firms and LNDE is irrelevant for both types of companies.

With these preliminary results it seems evident that we don't need the same variables for the pricing of private and public firms, i.e. that a common model would not be valid to explain the price of both types of companies.

The next section tries to confirm whether we need one or two separate models.

4.2. Revisiting Silber's model

Silber (1991) proposed a simplified model to explain the price discount observed when selling private companies. The model defines the discount as:

$$1 - p^*/p = f(CR, M, CF, S) \qquad [3]$$

where p^* is the price of the privately traded stock, p is the price of the publicly traded stock, CR is the credit-worthiness of the firm, M is a measure of marketability of the restricted stock, CF is the cash flow of the company and S represents special relationships between the private and public investors.

While the price discount is a variable that we will use at a later stage, this Research Project deals primarily with the calculation of p and p* themselves. Once these two values are available, calculating the price discount is pretty straightforward.

With Silber's model as base, we can follow two different strategies to determine the models applicable to Canadian private and public companies:

a) To use a common model with a dummy variable (DPRIVATE) that differentiates between both types of firms.

   This is a feasible solution. However, following this strategy the discount factor would be a mere constant, independent of any combination of assets, book value, revenues, net income or size of the block acquired. This would go against the economic intuition, as we can imagine that the discount factor has to depend, at least, on the size of the firm.

   Thus, we will not follow this path.

b) The second alternative is to develop two separate models, one for private firms and another one for public firms.

   As we saw in Section 4.1, LNPRICE has a strong relationship with several of the explanatory variables. Some of the independent variables can explain up to 69% of the variation in LNPRICE. Given the good results of these initial individual regressions, this work will keep exploring this path, i.e. treating

differently the observations of private and public firms instead of using a dummy variable in a common model.

4.3. Regression analysis - Private companies

As we saw at the beginning of this Chapter, there is a strong relationship between LNPRICE and LNASSETS for private firms. The relationship between both variables explains 63% of the variability in LNPRICE.

When trying to introduce more variables in this simple model we could not get any substantial improvement in the reliability of the results. So, we will stick to the simple model.

Regressing both variables we obtain:

$$LNPRICE = 7.490 + 0.593\ LNASSETS \qquad [4]$$

And the following statistics delivered by Stata:

Source	SS	df	MS		Number of obs =	47
					F(1, 45) =	79.44
Model	85.330541	1	85.330541		Prob > F =	0.0000
Residual	48.3382145	45	1.07418255		R-squared =	0.6384
					Adj R-squared =	0.6303
Total	133.668756	46	2.90584251		Root MSE =	1.0364

lnprice	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnassets	.5927831	.0665093	8.91	0.000	.4588264	.7267397
_cons	7.490683	1.23611	6.06	0.000	5.001029	9.980337

Figure 4.4

Results of the regression of LNPRICE for private firms

Prob>F = 0, so not all model coefficients are 0.

The constant and the coefficient of LNASSETS are significantly different from 0.

The signs of the constant and the coefficient of LNASSETS are in accordance with the economic intuition, as the price of the firm increases with the value of its net assets.

The results found are also in line with the common practice of business valuators. For instance, the Canadian Institute of Chartered Business Valuators (Introductory Business & Securities Valuation, 2012, p. M2-6) states that, "in practice, pricing considerations for small and medium-sized businesses are often measured as an amount over the value of the net assets".

Furthermore, the adjusted-R^2 is 63%, substantially higher than the 41% obtained by Hertzel (1993) and the 29% obtained by Silber (1991). This means that the model developed in this Research Project is able to explain in a greater degree the variances of LNPRICE.

Visually, the fit between the observed prices and the prices predicted by the model looks good:

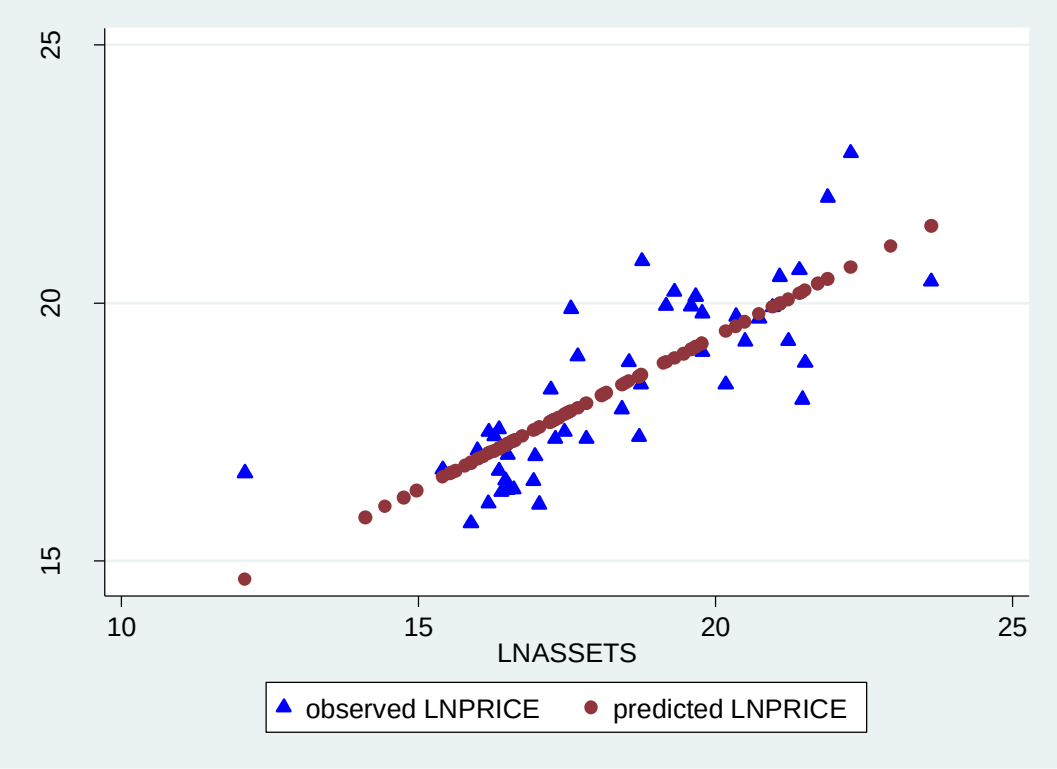

Figure 4.5

Results of the regression of LNPRICE for private firms

4.4. Regression analysis - Public companies

As we saw at the beginning of this Chapter, Silber (1991) uses a regression model with LNREVENUE, LNNI and LNBOUGHT as explanatory variables.

However, as we also saw in Section 4.1, the regressions that we have run between LNPRICE and each individual independent variable make us think that

LNBOUGHT plays a limited role in explaining LNPRICE -at least in Canada- and, instead, LNNI, LNASSETS, LNNBV and -to a lesser extent- LNREVENUE, are able to explain most of the variability of LNPRICE.

After some tries we find that the optimal model is:

Source	SS	df	MS	Number of obs =	1142
				F(3, 1138) =	1282.58
Model	2652.41823	3	884.139411	Prob > F =	0.0000
Residual	784.474372	1138	.689344791	R-squared =	0.7717
				Adj R-squared =	0.7711
Total	3436.89261	1141	3.01217582	Root MSE =	.83027

lnprice	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnni	.207838	.0225994	9.20	0.000	.1634968	.2521791
lnnbv	.2466907	.0396341	6.22	0.000	.1689266	.3244549
lnassets	.4096631	.0393062	10.42	0.000	.3325424	.4867838
_cons	3.562552	.2641049	13.49	0.000	3.044365	4.080739

Figure 4.6

Results of the regression of LNPRICE for public firms

Prob>F = 0, so not all model coefficients are 0.

The constant and the coefficient of LNASSETS, LNNI and LNNBV are significantly different from 0.

The signs of the constant and the coefficient of LNASSETS, LNNI and LNNBV are in accordance with the economic intuition, as the price of the firm increases with the value of its assets, net income and book value.

The model for public firms is thus:

LNPRICE = 3.563 + 0.410 LNASSETS + 0.208 LNNI + 0.247 LNNBV [5]

The adjusted-R^2 is 77%, substantially higher than the 41% obtained by Hertzel (1993) and the 29% obtained by Silber (1991). This means that the model developed in this research project is able to explain in a greater degree the variances of LNPRICE.

Visually, the fit between the real prices and the prices predicted by the model looks good:

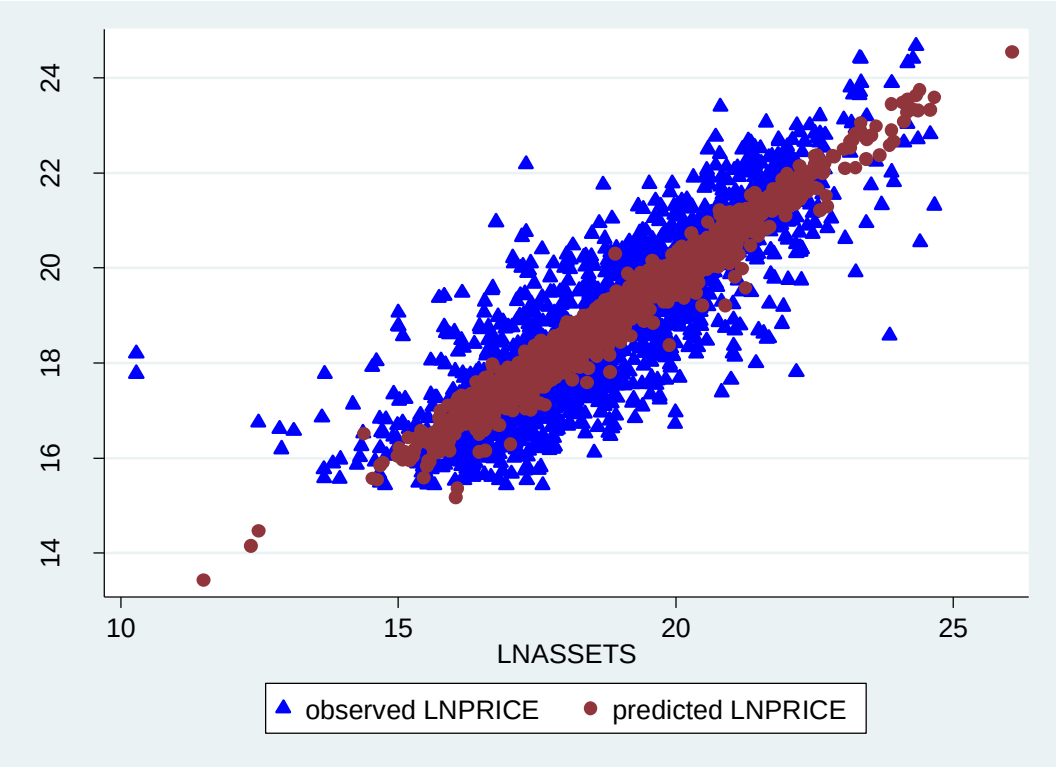

Figure 4.7

Results of the regression of LNPRICE for public firms

4.5. Double check of results

We will compare now the results of this study vs. the work of Silber, to double check that our findings are consistent with the common industry practice in the US.

We can take an average Canadian company and calculate what would be its price in private or public hands. Using the mean values listed on Table 3.4, and working with the regression Equations [4] and [5] we get:

$LNPRICE_{private} = 7.491 + 0.593\ LNASSETS$

$= 7.491 + 0.593\ \ln(\$941m)$

$= 19.744$

$\rightarrow PRICE_{private} = \$375.5m$

$LNPRICE_{public} = 3.563 + 0.410\ LNASSETS + 0.208\ LNNI + 0.247\ LNNBV$

$= 3.563 + 0.410\ \ln(\$941m) + 0.208\ \ln(\$16.2m) + 0.247\ \ln(\$238m)$

$= 20.252$

$\rightarrow PRICE_{public} = \$624.0m$

$$\text{Price discount} = 1 - PRICE_{private} / PRICE_{public} \quad [6]$$

$= 39.8\%$

For the average Canadian company, the price discount is 39.8%. This value is of the same level of magnitude -although different- than the 33.75% obtained by

Silber for the US, and close to results obtained by other researchers such as Solberg, Longstaff and Capron (see Table 2.1).

This check point reassures the validity of the models developed in this Research Project.

4.6. Magnitude of the price discounts

We saw in Section 4.5 what the expected price discount is for an average Canadian firm. Let's generalize now the calculation of the price discount for any combination of assets, net income and book value.

Using Equations [4], [5] and [6], the price discount can be calculated as:

$$\text{Price discount} = 1 - \text{PRICE}_{\text{private}} / \text{PRICE}_{\text{public}}$$

$$= 1 - [e^{(7.491 + 0.593\ \text{LNASSETS})}] / [e^{(3.563 + 0.410\ \text{LNASSETS} + 0.208\ \text{LNNI} + 0.247\ \text{LNNBV})}]$$

$$= 1 - e^{(3.928 + 0.183\ \text{LNASSETS} - 0.208\ \text{LNNI} - 0.247\ \text{LNNBV})} \qquad [7]$$

We notice that the price discount is a function of 3 variables. As it is not possible to plot 4 dimensions on a 2D surface, we have to fix one of the three independent variables to visualize the relationship between the price discount and the two remaining variables.

If we fix, for instance, the variable NI to the average of all Canadian companies, as given by Table 3.4 ($16.2m), we get the following relationship among the other three variables:

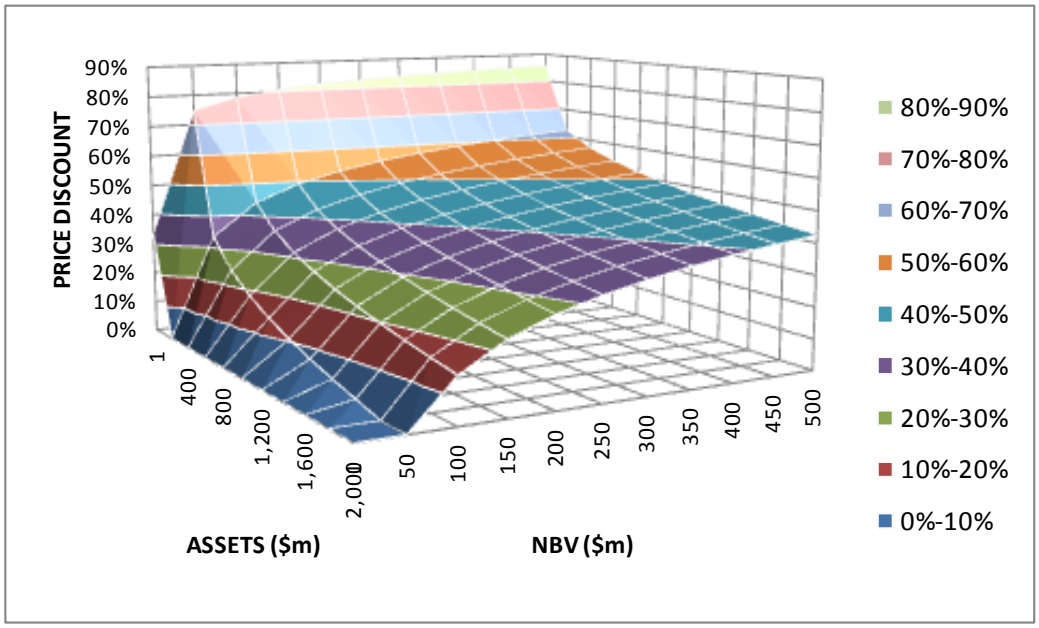

Figure 4.8

Price discount as a function of ASSETS and NBV, with NI = $16.6m

The discounts can be very substantial, depending on the combination of variables.

If we fix NBV instead of NI, making NBV equal to the average of all Canadian firms ($238m, as per Table 3.4), the relationship among the remaining three variables is this one:

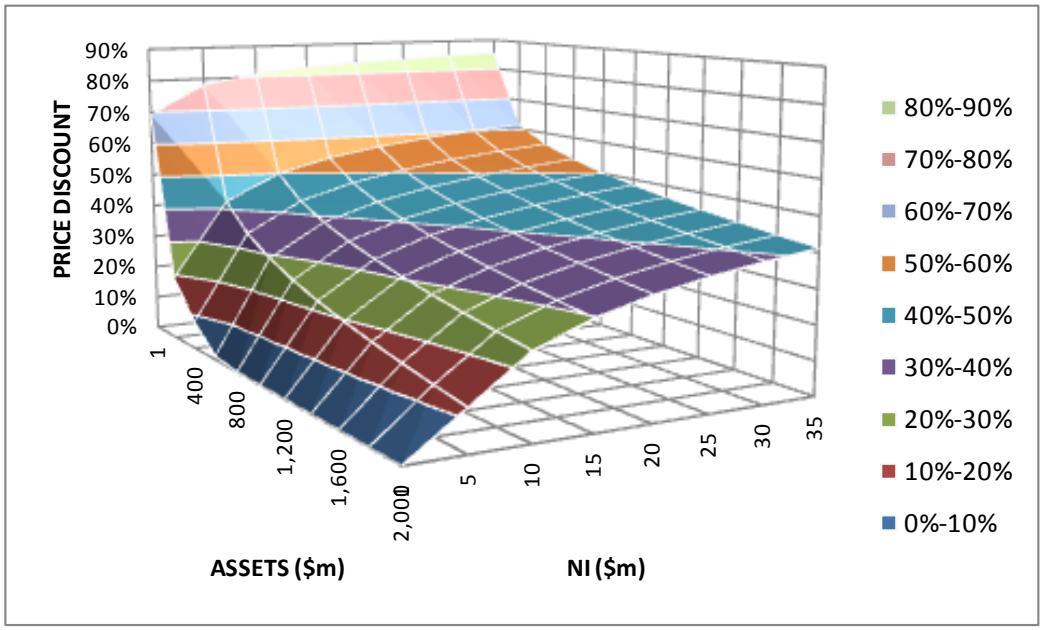

Figure 4.9

Price discount as a function of ASSETS and NI, with NBV = $238m

We can also fix 2 variables and determine what is the dependency of the price discount vs. the third variable.

For instance, let's use the average values of NI for Canada ($16.2m) and NBV ($238m). For different values of ASSETS the calculated price discount is:

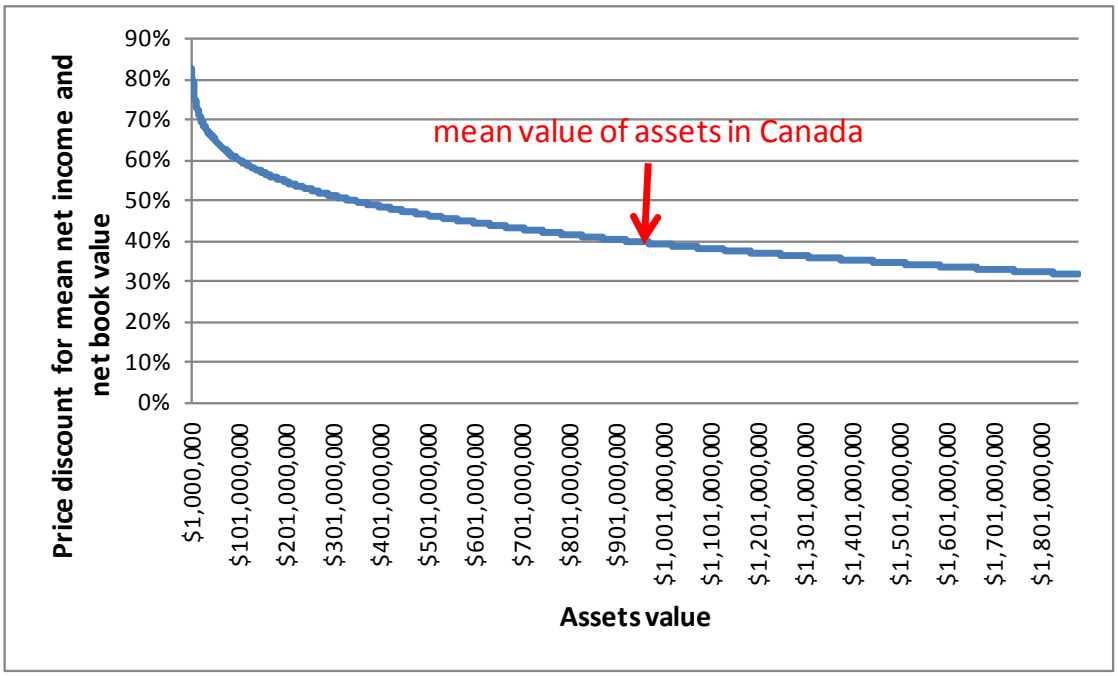

Figure 4.10

Price discount vs. assets value

If we fix ASSETS and NBV to their Canadian averages (ASSETS=$941m, NBV=$238m), the calculated price discount for different values of NI is:

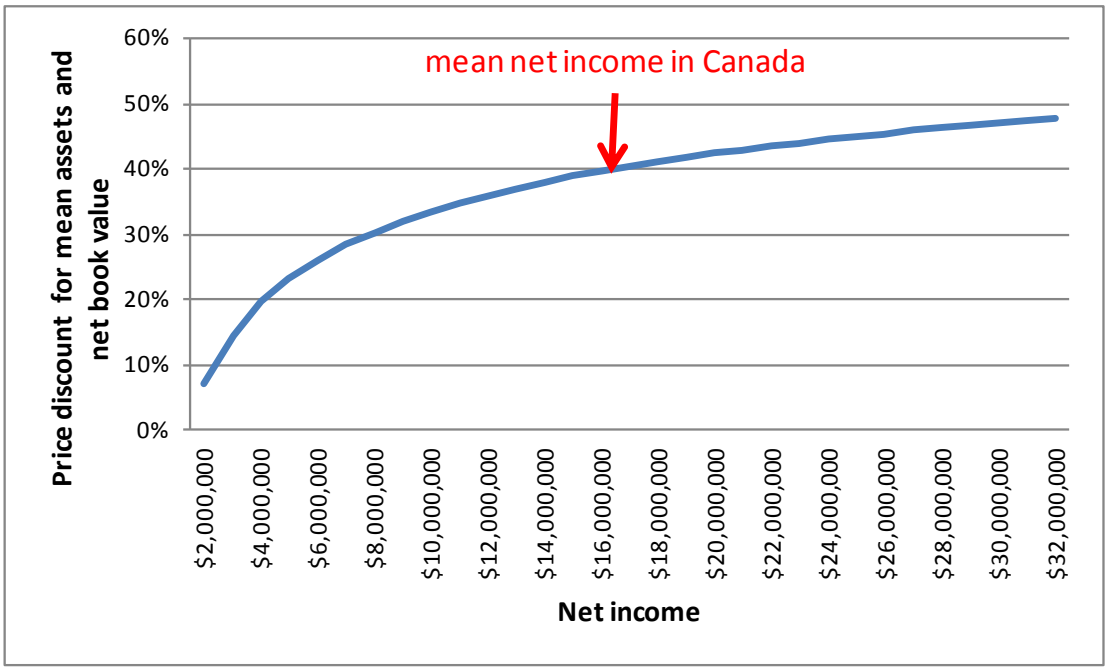

Figure 4.11

Price discount vs. net income

If we fix ASSETS and NI to their Canadian averages (ASSETS = $941m, NI = $16.2m), the calculated price discount for different values of NBV is:

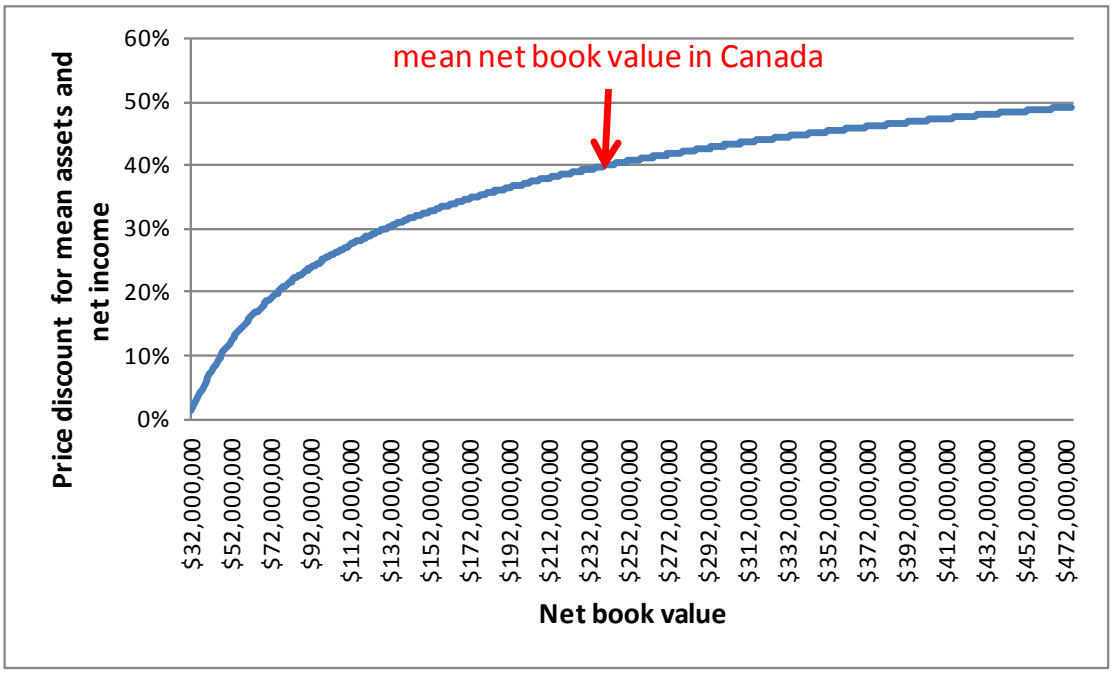

Figure 4.12

Price discount vs. net book value

4.7. Applicability of the pricing models

It is necessary to spend now some time in the applicability of the pricing equations to real life cases, to determine how we can benefit from their use.

4.7.1. Recommendations for private firms' owners

As the owner of a private company, if you want to sell your company, your main goal is to sell it for the highest possible price.

As we saw in Section 4.3, the price of a private company is based on the value of its assets.

Drawing the curve given by Equation [4] and a 45º line, we can appreciate that the price of the firm is greater than the value of its assets for companies with assets less than $100m. From that point on, the price of the firm is usually less than the value of its assets:

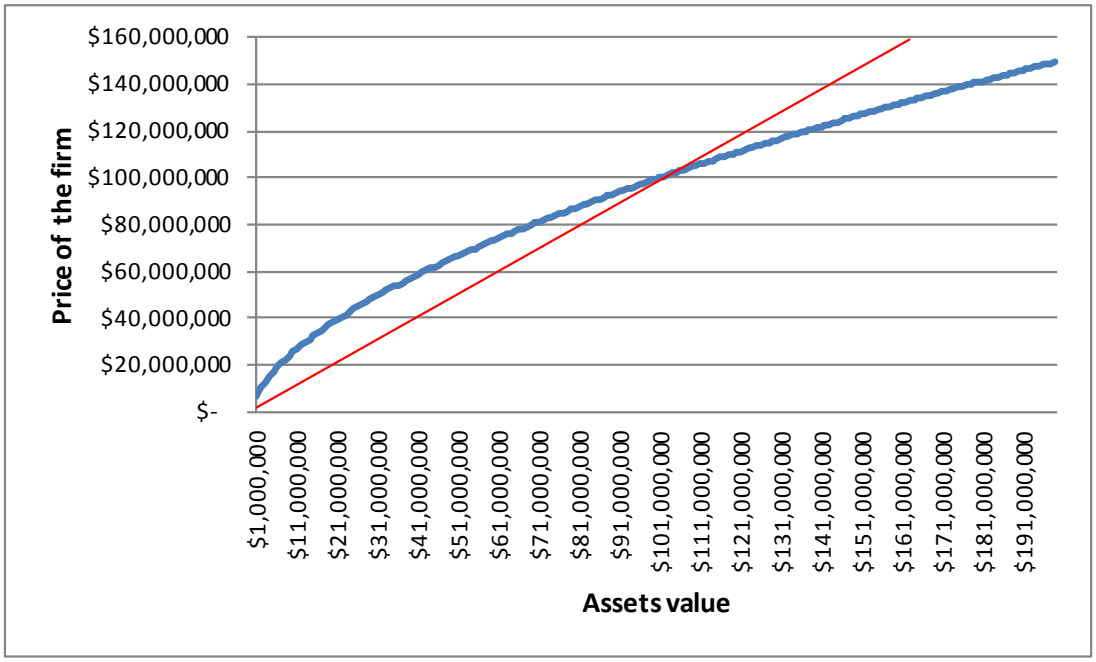

Figure 4.13

Price of a private firm as a function of the value of its assets

So, the ideal strategy for the owner of a private firm would be:

a) For private firms with assets value less than $100m, sell the company according to the price determined by the running business (Equation [4]).

b) For private firms with assets value higher than $100m, sell the company assets, disregarding their use in a continuing operation.

Graphically, this strategy can be explained by the following kinked curve:

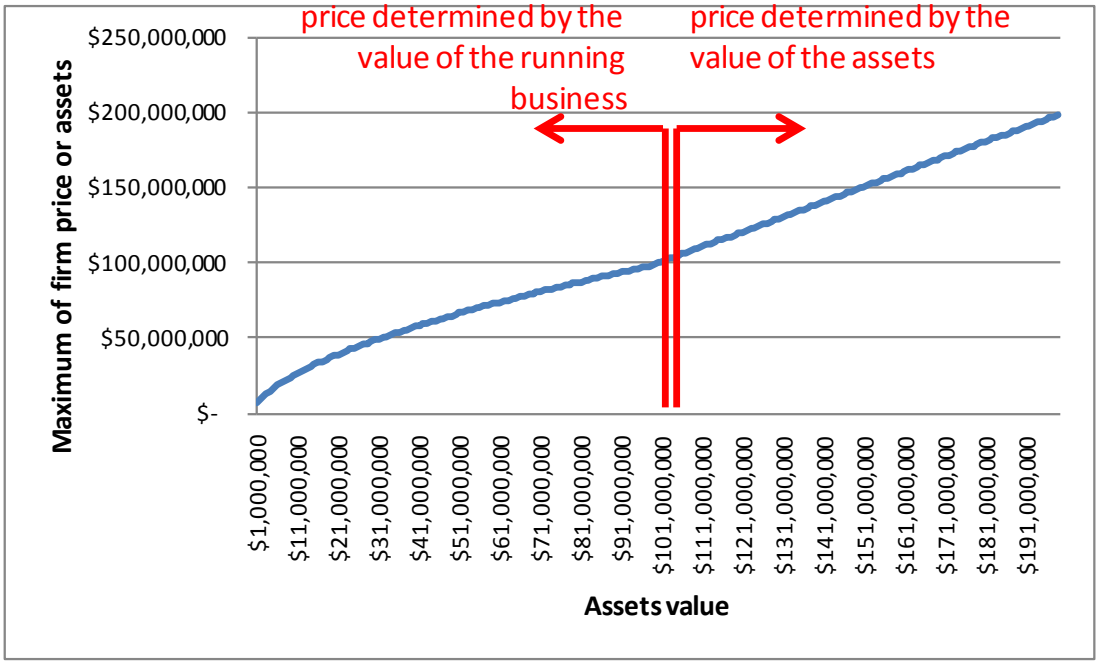

Figure 4.14

Maximum of firm price or assets

4.7.2. Recommendations for private equity firms, speculative investors

The main goal here is to buy cheap and quickly sell expensive.

The obvious strategy would be to buy a private firm and turn it public. As shown in Figures 4.6 through 4.10, the price discounts can be very substantial. Deducting the costs related to exchange fees, Securities Commission fees,

sponsorship fees, investment dealer fees and professional fees there's still room for a significant profit.

A different alternative is to take advantage of an arbitrage opportunity that arises from the different behaviour of purchasers of private companies with assets below and above $100m. As seen in Section 4.7.1 above, if the private company holds assets with a value of less than $100m, purchasers tend to pay a premium above the value of the assets, which most probably represents the value of the assets in a continuous operation. However, if the value of the assets is greater than $100m, acquirers tend to pay a price below the value of the assets.

With this in mind, a speculator could buy a private company with assets > $100m and chop it into smaller operational businesses, each of them with assets < $100m. Each of the resulting businesses would be priced at a premium.

4.7.3. Recommendations for public companies who want to acquire the portfolio of a private firm

The main goal of the acquirer is to buy a firm that will increase the price of his company by more than the price paid.

So, the condition to fulfil is:

$PRICE_{public} > PRICE_{private}$

$\rightarrow e^{(3.563 + 0.410\ LNASSETS + 0.208\ LNNI + 0.247\ LNNBV)} > e^{(7.491 + 0.593\ LNASSETS)}$

$\rightarrow e^{(-3.928 - 0.183\ LNASSETS + 0.208\ LNNI + 0.247\ LNNBV)} > 1$

$\rightarrow -3.928 - 0.183\ LNASSETS + 0.208\ LNNI + 0.247\ LNNBV > 0$ [8]

In a graphical form:

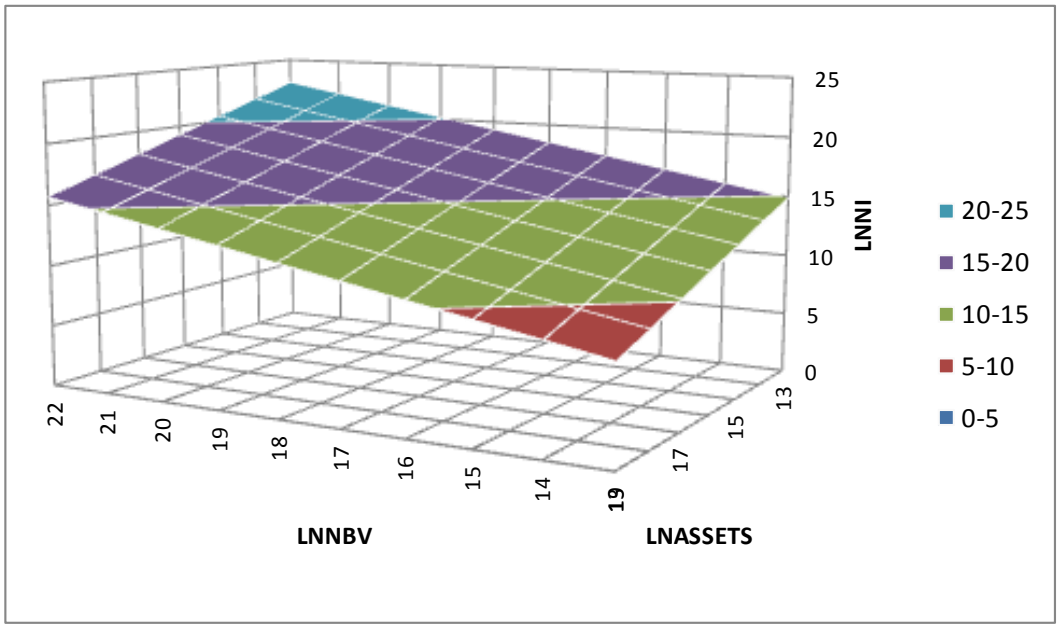

Figure 4.15

Surface that makes $PRICE_{private} = PRICE_{public}$

All points above the 3D surface displayed fulfil the condition and represent the constellation of private companies that are worth acquiring by a public company.

Chapter 5

Conclusion and Recommendations

5.1. Conclusion

The present work provides us with two models to estimate the market price of Canadian private and public companies, with a high degree of reliability of the results. The models are easy to use and specific for Canada.

For private companies, we have concluded that the price of a company is determined by a function of the value of its assets.

In the case of public firms, the price of the company is, again, a function of its assets, but also of the net income and net book value. The additional variables vs. the private firm case account for the additional variability observed in transactions of listed companies.

The pricing models allow us to confirm that an average Canadian private company is sold at a discount of 39.8% vs. a similar public company. This figure is in line with the findings of several researchers for the US.

Moreover, the models are used to develop applications aimed at extracting value from transactions of private firms, e.g. sales of private firms, acquisition of private firms by public companies and taking a private company public.

5.2. Directions of future research

While the benefits of this Research Project are clear, there is still room for further investigation.

Although the pricing of private and private companies is well explained by the models described, it is unclear what are the causes that underlie the models, e.g. why the price of a private company is mainly determined by a function of its assets value, disregarding most of the effects derived from the volume of sales or the profit of the firm. A field survey, interviewing M&A professionals, would be recommended to fill this gap.

Another question that remains for future study is whether the described equations -for both private and public firms- can be differentiated by sector of activity of the business. This question cannot be answered at this time for private companies, as the usable sample size is too small to extract significant information when performing analysis at the Standard Industry Classification (SIC) level.

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