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Magnetic deflection demonstrator

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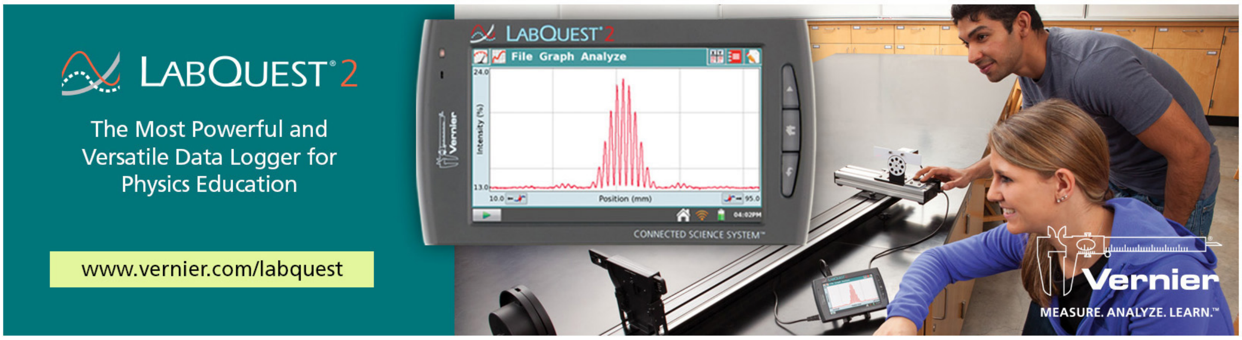

NOTE

Is Maxwell's Displacement Current a Current?

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This note is prompted by the *TPT* article "Visualizing Displacement Current—A Classroom Experiment,"[1] that describes an experiment of a type first reported by Silvanus P. Thompson in 1889.[2] A toroidal coil is placed between the plates of a capacitor that is connected to an alternating voltage source. An alternating voltage signal is developed between the ends of the toroid. Similar experiments have been reported by Van Cauwenberghe,[3] Meissner,[4] Carver and Rajhel,[5] Heller et al.,[6] and probably by others. It is often claimed that such experiments demonstrate the magnetic field produced by Maxwell's displacement current—a quantity introduced by him in 1864 to provide completeness and consistency to the basic theory of electromagnetism. The purpose of this note is to examine that claim, not to discuss the details of the experiment of Ref. 1. The key question is: Does the displacement current act as a source of magnetic field in the same way as a current in a wire? (We are limiting ourselves to the case where the region between the capacitor plates contains negligible numbers of electric charges—ideally a vacuum, but air at normal pressure comes close enough.) Our answer—although it is not new—may surprise some readers.

Before proceeding we should point out that nothing said here is intended to question the importance of displacement current for electromagnetic theory and its absolutely central role in the theory of electromagnetic waves. We are concerned only with the simple case of a capacitor in a circuit being charged or discharged relatively slowly—what is called a quasi-static situation. It is called quasi-static because the time of any change is very long compared with the time for an electromagnetic wave to traverse the dimensions of the circuit. Even the 1-MHz oscillating voltage in Ref. 1 is quasi-static by this criterion.

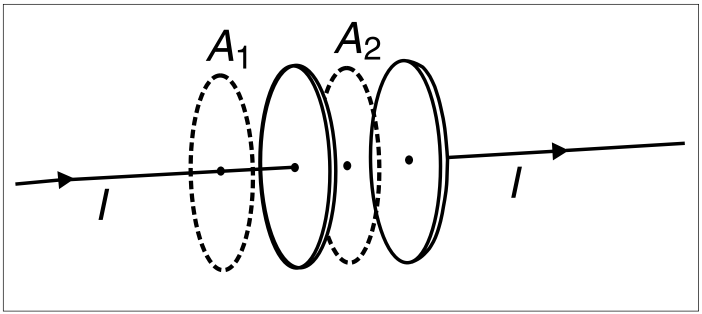

Fig. 1. Parallel-plate capacitor being charged by current *I* in a long straight wire. The circumferential magnetic field is considered at the positions of two circles whose radii are equal to the radii of the capacitor plates.

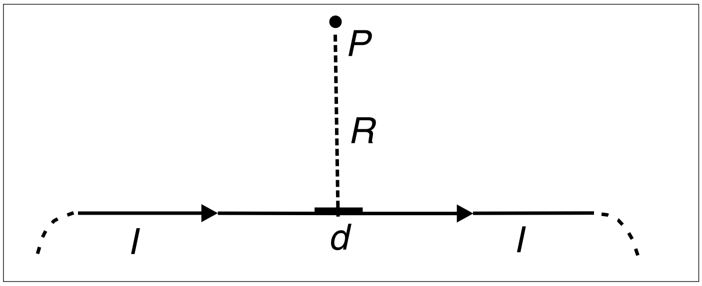

Fig. 2. Diagram for considering the contribution to the magnetic field at a point *P* due to one element, of length *d*, in a complete circuit carrying a current *I*.

First we shall consider what can be inferred if the displacement current is assumed to be a source of B in the same way as a conduction current. Then we shall examine the justification for that assumption.

The subject of Maxwell's displacement current appears in most elementary texts on electricity and magnetism, and it is usually introduced by considering the charging of a parallel-plate capacitor in a dc circuit (Fig. 1). Readers will be familiar with the argument. First apply Ampère's circuital law to a circular area A_1 that is pierced by the current I in the connecting wires. Then relate the line integral of the magnetic field B around the circle to the current by what we know as Ampère's circuital law:

$$\oint B \cdot ds = \mu_o I \qquad (1)$$

However, if we consider a similar (and nearby) circular area A_2 lying between the capacitor plates, the right-hand side of the above equation would be zero; yet it can be easily verified experimentally that the strength of the magnetic field around the circumference of A_2 is much the same as for A_1. To rectify the situation, it is argued (following Maxwell) that the changing flux ϕ_E of the electric field between the capacitor plates provides, in effect, a continuation ($I_{\text{displ.}}$) of the conduction current in the connecting wires. If, in particular, the radius of the circular area A_2 is equal to the radius of the capacitor plates, it is then argued that (ignoring fringing electric fields) we can put $I_{\text{displ.}} = \epsilon_o(\partial \phi_E/\partial t)$; this relation follows directly from equating $I_{\text{displ.}}$ to the rate of change of charge on the positive capacitor plate. From this point of view (again ignoring the fringing electric field of the capacitor) the current through the surface A_1 in Fig. 1 is 100% conduction current, and through the surface A_2 it is 100% displacement current. Most textbooks go on to discuss how, in the more general application of Eq. (1) (where I now includes both kinds of current), the line integral of $\boldsymbol{B}$ around a closed loop can be equated to μ_o times the sum of conduction current and displacement current through *any* surface bounded by the loop. This means that we cannot simply take it for granted that the local magnetic field is caused by the local current. Thus, for a loop between the capacitor plates, instead of taking a surface in the plane of the loop we can equally well take a surface that goes outside the capacitor and is pierced by the current in the wire; in this case the only contribution from the displacement current comes from the (usually neglected) fringing field of the capacitor. It follows that the magnetic field in the region between the plates (where the pick-up coil of Ref. 1 and other such experiments is placed) cannot be unambiguously attributed to the displacement current.

Further insight into the situation is provided if we consider the magnetic field on the basis of what most textbooks call the law of Biot and Savart.[7] In this approach, $\boldsymbol{B}$ is calculated at any point by adding up the contributions ascribable to all the individual elements of current in a circuit:

$$\boldsymbol{B} = \frac{\mu_o}{4\pi} \oint \frac{I\, \mathrm{d}\boldsymbol{l} \times \mathbf{r}}{r^3} \qquad (2)$$

Even if we assume that the displacement current can act like a current in a wire in contributing to the magnetic field, it would still be true that most of the contributions to the Biot-Savart integral would come, not from displacement current, but from the real conduction currents in the connecting wires, plus radial currents in the capacitor plates. Consider, for example, a circuit (Fig. 2) of which one portion consists of a long straight wire and the rest of the circuit is far away. Because the Biot-Savart law is an inverse-square law, the rest of the circuit contributes little to the magnetic field at a point P distance R from the straight wire. And, using either Ampère's law or the Biot-Savart law, we have the familiar result for an effectively infinite straight wire:

$$B(P) = \frac{\mu_o I}{2\pi R}$$

The contribution to $B(P)$ from a short segment of wire (of length d) closest to P is given (from the Biot-Savart law) by:

$$\Delta B = \frac{\mu_o I d}{4\pi R^2}$$

Thus $\Delta B/B = d/2R$.

Now imagine that the short segment of wire is replaced by the capacitor of Fig. 1, with plates of radius R separated by the distance d. Then, even if displacement current across the gap *did* produce a magnetic field, its fractional contribution to the net magnetic field at P would only be equal to $\Delta B/B = d/2$R, which would be small if the spacing of the plates was small compared to their diameter. Admittedly, at a radius r smaller than the radius of the capacitor plates (e.g., at r = R/2, the approximate radius of the toroid in Ref. 1), the fractional contribution to the total magnetic field from displacement current, as calculated from Biot-Savart, would be larger, but it would never account for the whole of B between the capacitor plates.

This, however, is not the end of the matter. Indeed, it is somewhat of a diversion, because ever since Maxwell introduced his displacement current, it has been asked whether this current does in fact generate a magnetic field in the same way as a conduction current. In 1881 G. F. Fitzgerald (of the Fitzgerald-Lorentz contraction in relativity) wrote: "It may be worth remarking that no effect except light has ever yet been traced to the displacement currents assumed by Maxwell.... It has not, as far as I am aware, been ever actually demonstrated that open circuits... produce exactly the same effects as closed circuits...."[8] I believe that this statement still holds good. It turns out, in fact, that the magnetic field around the toroidal coil placed between the capacitor plates of Fig. 1 can be ascribed entirely to conduction currents; the integrated effect of the displacement current, when we include the fringing field, is zero!

This point is discussed in detail in Purcell's famous textbook on electricity and magnetism.[9] One consequence of this is that, in a setup such as that illustrated in Fig. 1, the magnetic field just outside the capacitor gap (e.g., at the position of the symbol A_2) is in fact a little weaker than at a similar point (A_1) farther away from the plates, because the absence of a short length of true current between the plates makes more of a difference at A_2 than at the circle farther from the capacitor. This effect has been semi-quantitatively confirmed by Heller.[10] A detailed historical review of the whole question has been published by Roche.[11]

Where does that leave us? As said at the beginning, this note does not question the essential role of the vacuum displacement current $\partial \boldsymbol{E}/\partial t$ in the fundamental equations of electromagnetism. This quantity, and its magnetic counterpart $\partial \boldsymbol{B}/\partial t$, underlie the whole theory of electromagnetic waves. But I believe that there are two unfortunate aspects of talking about these matters. The first comes from Maxwell's own belief that $\partial \boldsymbol{E}/\partial t$ represented a real flow of electricity across an insulating gap, and so he called it a current. (It must be remembered that little was known about the true nature of electricity in Maxwell's time, and also that there was a firm belief in the "luminiferous ether," a dielectric medium that filled space, even in the absence of ordinary matter.) The second is the loose use of the word "cause." Although most textbooks say that a changing magnetic field *causes* an electric field, and that correspondingly a changing electric field causes a magnetic field (and it is very convenient to use this language) all that anyone may legitimately claim (and all that Maxwell's equations express) is that the two effects are associated or correlated. The real cause of both phenomena is the motion of electric charges somewhere—often far away. This imprecision of language does not matter much for most purposes, but it does lead to a misconception of what is being demonstrated when someone measures the magnetic field between the plates of a charging or discharging capacitor. What happens there is explainable as a matter of ordinary electric currents and their magnetic fields. There is no need to invoke Maxwell's displacement current in this context. In other words, the experiment does not show what it is commonly alleged to show. Considering the question of displacement current in general—not limited to situations like Ref. 1—the "bottom line" is that the vacuum displacement current is not a flow of charge; nor is it, to my mind, a physical source of magnetic fields. Therefore, in my opinion, it does not qualify to be called a current. Years ago, Rosser[12] presented essentially the same conclusion, but it seems to have been ignored.

A reminder: As stated at the beginning, our discussion assumes an effective vacuum between the capacitor plates. If part or all of this region is occupied by a polarizable dielectric, then this provides mobile (though not free) charges, whose oscillatory motion under an applied alternating electric field can be assumed to supplement the magnetic field due to the rest of the circuit.

Acknowledgment

I wish to thank Peter Heller, John G. King, and John Roche for extensive discussions on this topic. (In 1962/3, Professor King and I were engaged in doing an experiment of the type described in Ref. 1 when we realized that it demonstrated nothing about magnetic fields caused by displacement currents, so we discontinued it.)

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Magnetic Deflection Demonstrator

"No!!"

This thought-provoking response came from the school's "Guardian of the Purse Strings" recently when we submitted a purchase order to acquire a replacement e/m tube for a well-known apparatus. Unfortunately, the price tag on a replacement tube is almost $700, not quite petty cash.

Well, necessity being what it is, the thought occurred to us that there was an oscilloscope that happened to be collecting dust in one of the cupboards. It would do nicely as a magnetic deflection demonstrator if the cathode ray tube (CRT) could be removed from the oscilloscope proper and placed on top of the enclosure. A brief examination of the insides of the oscilloscope indicated that it would be necessary to add approximately 20 cm or so to each of the wire leads going from the oscilloscope chassis to the base of the CRT and the modification would be complete.

A couple of hours later, the wires had been lengthened and the CRT was mounted on top of the oscilloscope enclosure, fastened by a couple of plastic straps. We had taken the precaution of wrapping plastic tape around the conical part of the CRT in the event of implosion. The neck of the CRT, however, was left uncovered so that the students could see the filament structure. Since the oscilloscope was still operational when we decided on the modification, then the intensity, focus, vertical, and horizontal positioning controls were still working and thus enabled us to control the placement of the "spot" anywhere on the face of the tube. Our particular model had provisions for x-y operation, eliminating any need to disable the horizontal sweep circuit.

The modifications having been completed, magnetic fields could be placed wherever desired and the motion of the spot explained to the students in terms of the right-hand rule (taking into account the negatively charged electron beam inside the tube).

You too can turn that thought-provoking "No!!" into a useful magnetic deflection demonstrator.

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